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PARAMETRIC IDENTIFICATION OF ASSET DYNAMIC MODELS

The article describes the mathematical formulation of the problem of constructing the optimal structure of a securities portfolio on the stock market. The most important mathematical problems in decision-making by investors are the problems of constructing a forecast for a single share and optimal diversification of the portfolio structure. In applied mathematics, such a problem is called a mathematical problem of optimal control. In this work, mathematical models are used that describe the dynamics of the formation of the market value of a single share and a portfolio. The corresponding models are written in the class of ordinary differential equations with parameters. The procedure for constructing a dynamic model of the formation of the market value of a single share is based on the application of the market model of W. Sharpe and the fundamental theory of H. Markowitz.

The paper formulates a problem of mathematical models identifying the parameters of dynamic processes that can be described by ordinary differential equations and systems. Using the example of mathematical models of dynamic formation of the market value of a single share and a portfolio of shares, algorithms for constructing optimal values of parameters of such models have been developed. Parametric identification and optimization algorithms are based on iterative procedures that allow, at each step, to form the "best" values of model parameters from the point of view of selected quality criteria. An algorithm for constructing guaranteed parameter estimates in the class of ellipsoidal sets has been developed. The mathematical models and algorithms presented in this study make it possible, together with the methods of technical analysis, to develop effective tactics and strategies for optimal investment in securities. The results obtained should be considered as one of the alternative approaches to modeling the dynamics of market assets and their portfolios.

K e y w o r d s : *mathematical modeling, asset dynamics models, parameter identification, stock market.*

AMS 2020 classification: 65K10, 93B30.

Introduction

This article is devoted to the construction of algorithms for determining the parameters of mathematical models of the dynamics of risky assets of the stock market formation. The relevance of this scientific direction is confirmed by the fact that a large number of scientific studies have been devoted to the development of such models and algorithms, which were carried out by H. Markowitz, W. Sharpe, K. Ito, F. Black, M. Scholes, and R. Merton. The object of the study is parametrically specified mathematical models of the stock market assets.

Modern applied mathematical modeling operates with various approaches to calculating parameters with a certain structure of mathematical models. The most well-known and widespread methods for identifying parameters of discrete and continuous models include: methods of covariance-dispersion analysis; methods of simulation modeling; approaches based on the principles of stability theory; approaches based on methods of sensitivity analysis of solutions; methods of minimax identification.

However, due to the complexity of the model structure, the nonlinear nature of the processes being modeled, as well as other factors, it is often necessary to turn to computer modeling, which makes it possible to build computer images of models by searching through a large number of candidate models using self-organization or simulation modeling methods (Xin-Yu Guo, & Sheng-En Fang, 2023). The complexity of a parametrically specified model will be understood as the dimensionality of the parameter vector, and the main principle of simulation modeling will be: "the more complex the model, the more accurate it is", in contrast to the main principle of model self-organization about "the existence of a of optimal complexity model" (Elder, Zehetner, & Kunze, 2016). Such approaches, which include methods such as neural networks or group consideration of arguments, are often used in solving problems of modeling complex and interconnected economic processes, including mathematical problems of the stock market and the banking sector of the economy (Rios, Efimov, & Moreno, 2017).

The approaches to building mathematical models of the formation of price dynamics of a single share and a portfolio of shares are based on the principles of portfolio theory, formulated by H. Markowitz and W. Sharpe. H. Markowitz proposed approaches to measuring risk and basic principles of portfolio investment (Markowitz, 1952). This laid the theoretical foundation for research on a portfolio of risky investments. Sharpe's capital asset pricing model (Sharpe, 1964) is also actively used by investors in solving problems of modeling the dynamics and building short-term forecasts of market prices of assets.

Among other important approaches to solving applied portfolio theory problems, the following can be noted:

- Evolutionary and Memetic Computing for Project Portfolio Selection (Kolish, & Fliedner, 2022; Yue, Jing, & Liu, 2025);
- An exploration of meta-heuristic approaches for the project portfolio selection (Harrison et al., 2020);
- A simheuristic method for project portfolio selection under uncertainty (Panadero et al., 2020);
- A bi-objective fuzzy portfolio selection model (Kar et al., 2019);
- A genetic programming for portfolio selection problem solution (Lin, Zhu, & Gao, 2020);

• An alternative mathematical method for the portfolio selection problem (Mohagheghi, Meysam Mousavi, & Mohagheghi, 2020; Li et al., 2020);

• Project portfolio selection and scheduling optimization based on risk measure (Dixit, & Tiwari, 2020).

The first part of this study is devoted to the consideration and analysis of mathematical models of the market value dynamics of a single share and a portfolio of shares (Kulian, Yunkova, & Korobova, 2022). An important feature of models of the formation of stock market asset dynamics is their parametric representation. In this regard, any applied problem that can be formulated for such models requires preliminary determination of parameter values. Constructing a short-term forecast or optimal diversification of an investment portfolio is a most important applied problem (Kulian, Yunkova, & Korobova, 2021). The second part of the study is devoted to constructing mathematical algorithms that can be used for solving the problem of parametric identification of mathematical models of the formation of asset dynamics. The third part of the study considers an application of the constructed algorithms to solve applied problems of modeling the asset dynamics in the real stock market.

1. Analysis of stock price formation models in the stock market

As shown in (Kulian, Yunkova, & Korobova, 2021), based on the market model of W. Sharpe, generalized deterministic mathematical models of the formation of the market value of a single stock and a portfolio of stocks can be written in the form

$$\dot{r}_i = \frac{dr_i}{dt} = f(r_i, t, \alpha),\ \ r_i(t_0) = r_0, t \in [t_0, T],\ i = \underline{1,n} \tag{1}$$

and

$$\dot{r}_p = f_p\left(r_p, x_i, \dot{x}_i, r_i, \dot{r}_i, t\right) \tag{2}$$

respectively, and are given parametrically. Here, r_i is the expected market value of the stock; r_p is the expected market value of the investment portfolio; x_i is a share of the i-type asset in the portfolio; t is time.

An important mathematical problem, the solution of which is used to correctly formulate the problem of constructing the optimal structure of an investment portfolio, is the problem of identifying the optimal values of the parameters α of mathematical models of the formation of the market value of one share (1) and a portfolio of shares (2). The problem of optimizing a portfolio of shares from the point of view of its profitability in the static case has the form

$$r_p = \sum_{i=1}^{n} x_i r_i \to \max_x .$$

In practical investing, it can be considered in the following formulation

$$r_p(T) = \sum_{i=1}^{n} x_i(T) r_i(T) \to \max_{x \in X(t)},\ \ r_p(t_0) = r_{p_0},\ t \in [t_0, T]$$

and consists in constructing an investment portfolio with optimal profitability at the final point T. Here X(t) is a bounded closed set of stocks. The right-hand sides of the equations of the system (1) are linear and depend on the parameters. To ensure the possibility of its integration

$$\frac{dr_i}{dt} = \left(\alpha_1 SM_{ind}(t) + \alpha_2 I(t)\right) r_i(t) + \alpha_3\left(r_i(t), \rho_{ij}\right),\ \ i, j = \underline{1,n}$$

we will build procedures for their determination. Here, $SM_{ind}(t)$ – Stock Market Index; I(t) – Inflation index; ρ_{ij} – correlation coefficient between i and j shares.

In different formulations of the problem, the vectors $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ or $\alpha = (\alpha_1, \alpha_2)$ can be considered as the parameter vector. The first representation of the parameter vector will be used in the case when the information about the market value of shares is "noisy". In the case when the market value of shares adequately reflects the market dynamics, the second representation is used.

2. Algorithms for constructing optimal parameters of the models

Algorithm 1. Another approach to solving the parametric identification problem is to use sensitivity functions. Consider the problem of constructing sensitivity functions for pricing models of a single stock and a portfolio of stocks.

For the convenience of transformations, we denote the right-hand side by $f(t, r, \alpha)$, where $\alpha = (\alpha_1, \alpha_2)$. We will also assume that the initial conditions $r(t_0) = r_0 = r_0(\alpha)$, $t_0 = t_0(\alpha)$ and that α is a scalar and $\alpha = \alpha_1$, since in short-term modeling of the dynamics of stock price formation α_2 can be considered constant. In such a statement, ρ_{ij} are the correlation coefficients between stocks, which can be calculated based on known statistical information. According to (Kulian, Yunkova, Korobova, 2021), provided that the right-hand side of (1) is continuous in its arguments and continuously differentiable in r, α and also under the continuous differentiability of the functions r_0, t_0in α_1, there is a continuous derivative

$$u(t, \alpha_1) = \frac{\partial r(t, \alpha_1)}{\partial \alpha_1}.$$

This derivative satisfies the differential equation

$$\frac{du}{dt} = \frac{\partial f(r, t, \alpha_1)}{\partial r} u + \frac{\partial f(r, t, \alpha_1)}{\partial \alpha_1}$$

under the initial condition

$$u(t_0) = \frac{dr_0(\alpha_1)}{d\alpha_1} - f\left(r_0(\alpha_1), t_0(\alpha_1), \alpha_1\right) \frac{dt_0(\alpha_1)}{d\alpha_1}. \tag{3}$$

Equation (3) is obtained by direct differentiation with respect to α_1, and the initial condition is obtained from the differentiation of the integral equation

$$r(t, \alpha_1) = \int_{t_0(\alpha_1)}^{t} f\left(r(\tau, \alpha_1), \tau, \alpha_1\right) d\tau + r_0(t_0, \alpha_1)$$

and looks like

$$u(t, \alpha_1) = \frac{\partial r(t, \alpha_1)}{\partial \alpha_1} = \int_{t_0(\alpha_1)}^{t} \left[\frac{\partial f(r(\tau, \alpha_1), \tau, \alpha_1)}{\partial r} u(\tau, \alpha_1) + \frac{\partial f(r(t, \alpha_1), \tau, \alpha_1)}{\partial \alpha_1}\right] d\tau +$$
$$+ \frac{dr_0(\alpha_1)}{d\alpha_1} - f\left(r(t_0(\alpha_1), \alpha_1), t_0(\alpha_1), \alpha_1\right) \frac{dt_0(\alpha_1)}{d\alpha_1}.$$

The initial condition for the sensitivity function $u(t_0)$ is obtained by putting $t = t_0$ in (3). Therefore, for (1), we have defined the initial condition and the sensitivity function, which describes the dependence of the stock price on the parameter α_1. Let us apply the sensitivity function to identify the parameters α_1, α_2 of the mathematical model (1). The mathematical problem has the following form

$$\min_{\alpha_1,\alpha_2}(r(t,\alpha_1,\alpha_2) - z(t))^2,\ t \in [t_0, T].$$

Here z(t) – observation of the share price behavior in the market.

Algorithm 2. The vector of model parameters is written in the form $\alpha = (\alpha_1, \alpha_2)$ provided $r(t_0) = r_0$ at $t \in [t_0, t_1]$.

Consider the point α_0 in the parameter space of the mathematical model

$$\alpha_0 = (\alpha_{0_1}, \alpha_{0_2}).$$

And let this value of parameters be the solution of the problem of parametric identification of the model (1). Around the point α_0 we describe a circle of unit radius

$$(\alpha_1 - \alpha_{0_1})^2 + (\alpha_2 - \alpha_{0_2})^2 = 1$$

with the center in p. $(\alpha_{0_1}, \alpha_{0_2})$. The length of the circle will then be $l = 2\pi$.

Let us divide this line into n equal parts, and the value of n is chosen depending on the accuracy of the obtained solution to the problem of constructing a guaranteed multiple-parameter estimate. Consider an arbitrary point $\alpha_k = (\alpha_{k_1}, \alpha_{k_2})$.

Let's draw a tangent to the circle at this point. Its equation will be $F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1}) = F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_2 - \alpha_{k_2})$. The equation of the normal to the tangent at this point will be

$$F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})(\alpha_2 - \alpha_{k_2}) = F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1}).$$

We will search for new values of the parameters α that satisfy the conditions for the software functioning of the system on the normal. From the equation of the normal we determine α_2

$$\alpha_2 = \frac{F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1})}{F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})} + \alpha_{k_2}.$$

The new position of the coordinate α_2 is determined by changing the position of the coordinate α_1

$$\alpha_{N_1} = \alpha_1 + \delta\alpha_1,\ \delta > 0,$$

$$\alpha_{N_2} = \frac{F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 + \delta\alpha_1 - \alpha_{k_1})}{F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})} + \alpha_{k_2}.$$

Thus, a new position of the parameter value $\alpha_N \in A$, is obtained, where A is a bounded closed set of parameters of the mathematical model

$$\alpha_N = (\alpha_{N_1}, \alpha_{N_2}).$$

The above procedure implements the movement of the parameter α along the normal until one of the selected criteria above is met. By performing such steps for each of the previously defined points, we will obtain new values of the coordinates of the vector α in the parameter space of the mathematical model.

Table 1 below presents the results of a computational experiment for the practical application of Algorithm 2 for model (1).

Table 1

Results of modeling the dynamics of two stocks

Characteristic	Day								
	1	2	3	4	5	6	7	8	9
$\underline{r_1}$, $	42.22	43.74	43.60	44.13	43.11	42.87	42.59	42.57	42.08
$\underline{r_2}$, $	3046.01	3106.89	3133.15	3190.00	3140.02	3125.71	3115.51	3088.99	3087.67
SM_{ind}	5133.07	5170.97	5196.52	5204.08	5162.25	5082.77	5056.90	5068.05	5080.94
Δ_1	0.047	0.054	0.034	0.035	0.081	0.062	0.054	0.043	0.040
Δ_2	0.031	0.035	0.047	0.034	0.096	0.071	0.059	0.037	0.032

In the table $\underline{r_1}$ denotes the market value of the first company's stock, and $\underline{r_2}$ denotes the market value of the second company's stock. Through Δ_1 indicates the deviation between actual and calculated trajectories of the first stock, and through $\Delta_2$indicates the deviation between actual and calculated trajectories of the second stock.

To calculate deviation Δ_1 of trajectories were used α_1 = 2.348, α_2 = 0.017, α_3 = - 0.76. To calculate deviation Δ_2 of trajectories were used α_1 = 1.981, α_2 = 0.032, α_3 = - 0.489. I(t)= const. Correlation dependencies ρ_{ij} and model (1) parameters α_1, α_2, α_3 were calculated based on known statistical information of the S&P 500 market using Algorithm 2. Analysis of the results in Tab. 1 shows the correlation between the dynamics of the market index and the dynamics of market prices of shares of two companies. Analysis of computational experiment results shows that the application of Algorithm 2 in areas of a stable positive or negative trend allows obtaining a 2–5 % deviation of the real and calculated trajectories, while in areas of a change in the direction of dynamics, an increase in the deviation of the trajectories to 8–9 % is observed.

Algorithm 3. When solving forecasting or management problems in the above application areas, it is necessary to develop algorithms and methods to obtain guaranteed financial results. An example of such an approach can be the development of the Value at Risk methodology, and confirmation of its relevance is its general recognition as a reliable mathematical tool when making investment decisions. Let us turn to the problem of constructing a guaranteed multiple-parameter estimate for a mathematical model of general form (1). Let us formulate a procedure for constructing an optimal ellipsoidal parameter estimate of the mathematical model (1).

After constructing and analyzing solutions to the problem of identifying the parameters of the mathematical model of the formation of the market value of a stock (or a portfolio of stocks), we will determine the point that is farthest from the point $D(\alpha_1^c, \alpha_2^c)$, the coordinates of which will be determined using the formulas

$$\alpha_1^c = \frac{1}{k}\sum_{j=0}^{k} \quad \alpha_1^j,\quad \alpha_2^c = \frac{1}{k}\sum_{j=0}^{k} \quad \alpha_2^j.$$

To do this, we will solve the one-dimensional optimization problem

$$R = \max_i L(\alpha_i, \alpha^c),$$

where $L(p,q)$ is a function of the distance between points p and q.

On the boundary of the circle $S(1,D)$ of radius 1, with the center at the point $D(\alpha_1^c, \alpha_2^c)$ we form an arbitrary δ -grid $\{y_i\}_{i \in K_s}$ with nodes y_i. At each node y_i we construct the external normal vector $\bar{n}_{e_i}$ to the circle according to the rules described above, and we solve the auxiliary problem of constructing new values of the parameters α analogously to the approach described in Algorithms 1, 2.

Having performed the above procedure for each of the vectors $n_i, i \in K_n$, we will obtain a new set of points $\alpha_{e_n} \in A$. Let us describe the ellipsoid of the smallest volume (the ellipse of the smallest area in the plane) around the points thus constructed, which we will call the "guaranteed ellipsoidal estimate" of the parameters of the model (1), constructed on the defined criteria

$$\min_{\alpha \in A} \int_{t_0}^{T} (r^0(t_0, t_1, r_0, \alpha) - r_{exp}(t))dt,$$
$$\min_{\alpha \in A} \max_{t \in [t_0, T]} \|(r^0(t_0, t_1, r_0, \alpha) - r_{exp}(t)\|.$$

Here $r^0(t_0, t_1, r_0, \alpha)$ is the solution of the Cauchy problem for model (1) under the condition $r^0(t_0, t_1, r_0, \alpha)$, r_{exp} – the experimental or program trajectory.

Based on the solutions of the parametric identification problem of the mathematical model (1) and known observations, we will construct a guaranteed multiple estimate of the parameters in the class of ellipsoidal sets

$$Q(B,d): \{(B(\alpha - d), \alpha - d) \le 1\}, \tag{4}$$

where B is a symmetric positive-definite matrix that specifies the geometry of the multiple estimate in the parameter space; d *is* geometric center of the ellipsoid, $\alpha, d \in R^n$. The set (4) will be called a "guaranteed multiple estimate" if each value of the parameter α obtained for arbitrary observations belongs to $Q(B,d)$.

The iterative procedure of the method for constructing a guaranteed multiple estimate implements the refinement of the elements of the matrix B and the vector d at each step. The initial matrix B is chosen to be the unit $B^0 = E$, and the initial position of the center d is defined as the center of mass

$$d^0 = \frac{1}{k} \sum_{j=1}^{k} \alpha_j,$$

where k is the number of points.

At the $k+1$- step of the procedure, the formulas for determining $B^{k+1}$and d^{k+1} have the form

$$B^{k+1} = B^k + \lambda_1^{k+1} \nabla_B\{(B^k(\alpha - d^k), \alpha - d^k)\},$$
$$d^{k+1} = d^k + \lambda_2^{k+1} \nabla_d\{(B^k(\alpha - d^k), \alpha - d^k)\},$$

where $\lambda_1^{k+1}$and λ_2^{k+1} are such that

$$(B^{k+1}(\lambda_1^{k+1})(\alpha_{\underline{i}} - d^{k+1}(\lambda_2^{k+1}), \alpha_{\underline{i}} - d^{k+1}(\lambda_2^{k+1})) <$$
$$< \left(B^k(\alpha_{\underline{i}} - d^k), \alpha_{\underline{i}} - d^k\right), \text{ where } \underline{i} = \arg\arg\max_i (B(\alpha_i - d), \alpha_i - d).$$
$$\left(B^j(\alpha - d^j), \alpha - d^j\right) < 1.$$

On the boundary of the sphere $S(1, d^j)$ with the center at the point d^j (where the j -number of the procedure step for which the inequality $\left(B^j(p - d^j), p - d^j\right) < 1$ is satisfied), we form an arbitrary δ -grid $\{y_i\}_{i \in K_s}$ with nodes y_i.

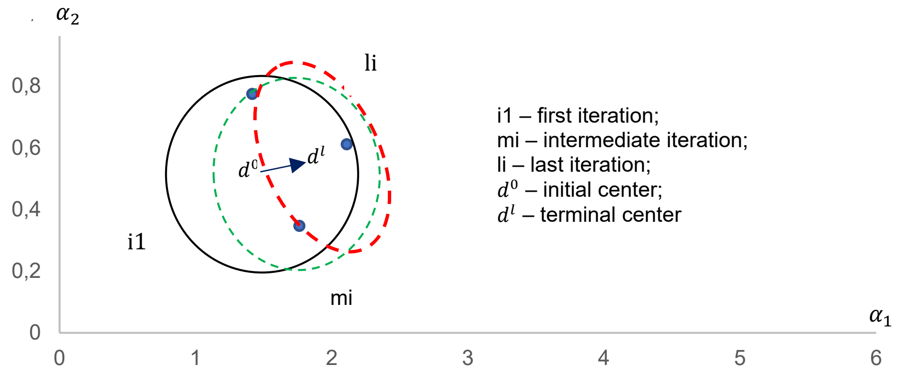

Fig. 1. Construction of multiple evaluation

As shown in Fig.1, at each node, using the described method of constructing the ellipsoid of the smallest volume around the given points, we construct a guaranteed multiple parameter estimate, which, as determined by Algorithm 3, ensures the fulfillment of the selected quality criteria. The set constructed in this way allows us to form an admissible set of parameters, and with further optimization of this multiple estimate, the logical next step is to formulate and solve the problem of constructing an "effective" by H. Markowitz multiple estimate.

Discussion and conclusions

The presented mathematical procedures make it possible for dynamic mathematical models of a single stock and a portfolio of stocks to solve the problems of parametric and guaranteed multiple identification of parameters and, based on the developed algorithms for determining the admissible set of parameters, to find solutions to the problems of constructing a market value trajectory for mathematical model (1) and optimal diversification of an investment portfolio of risky securities based on model (2).

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ІДЕНТИФІКАЦІЯ ПАРАМЕТРІВ МОДЕЛЕЙ ДИНАМІКИ АКТИВІВ

Описано математичну постановку задачі побудови оптимальної структури портфеля цінних паперів. Найважливішими прикладними проблемами в ухваленні рішень про інвестування є математичні задачі про побудову короткострокового прогнозу для окремої акції та задача про оптимальну диверсифікацію структури портфеля. Під час формулювання та розв'язання задач моделювання динаміки активів фондового ринку розглянуто математичні моделі формування ринкових цін однієї акції та портфеля акцій. Відповідні моделі сформульовано у класі звичайних диференціальних рівнянь з параметрами. Параметрами таких моделей є коефіцієнти чутливості ринкової вартості до зміни факторів, що на неї впливають. Процедура побудови динамічної моделі формування ринкової вартості окремої акції ґрунтується на застосуванні ринкової моделі В. Шарпа й елементів фундаментальної теорії Г. Марковиця. У дослідженні сформульовано загальну задачу ідентифікації параметрів динамічних процесів, що можуть бути описані звичайними диференціальними рівняннями та системами. На прикладі математичних моделей формування динаміки ринкової вартості однієї акції та портфеля акцій розроблено алгоритм визначення оптимальних значень параметрів таких моделей. Алгоритм побудовано на основі ітераційної процедури, яка дає змогу на кожному кроці формувати "оптимальні", з позиції обраних критеріїв якості, значення параметрів моделі. Розроблений алгоритм дозволяє визначати такі оцінки параметрів у класі еліпсоїдальних множин. На прикладі математичних задач побудови оптимальної структури портфеля ризикованих активів розроблено формальний підхід до формування "гарантованих" фінансових показників інвестиційної діяльності. Математичні моделі й алгоритми, представлені у пропонованому дослідженні, дають змогу разом із методами технічного аналізу розробити ефективні тактику та стратегію оптимального інвестування у цінні папери. Результати дослідження варто розглядати як один з альтернативних підходів фундаментального аналізу до моделювання динаміки ринкових активів.

Ключові слова: ***математичне моделювання, динаміка активу, ідентифікація параметрів, фондовий ринок.***

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