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Асимптотика м'якого розв'язку хвильового рівняння у тривимірному просторі, керованого загальною стохастичною мірою

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Asymptotics of the mild solution of a wave equation in three-dimensional space driven by a general stochastic measure

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Анотація. Розглянуто задачу Коші для хвильового рівняння у тривимірному просторі, керованого загальною стохастичною мірою. Доведено, що за певних умов розв'язок прямує до нуля майже напевно, якщо абсолютна величина просторової змінної прямує до нескінченності.

Ключові слова: асимптотична поведінка, стохастична міра, стохастичне хвильове рівняння, м'який розв'язок, простір Бєсова.

Abstract. We study the Cauchy problem for a wave equation in three-dimensional space driven by a general stochastic measure. Under some assumptions, we prove that the mild solution tends to zero almost surely as the absolute value of the spatial variable tends to infinity.

Key Words: asymptotic behavior, stochastic measure, stochastic wave equation, mild solution, Besov space.

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1 Introduction

We continue investigation of the wave equation driven by a general stochastic measure. Namely, we study the asymptotic behavior of the mild solution in the case of three-dimensional space.

Let $\mathsf{L}_0(\Omega, \mathcal{F}, \mathsf{P})$ be the set of all real-valued random variables defined on complete probability space $(\Omega, \mathcal{F}, \mathsf{P})$, X be an arbitrary set and $\mathcal{B}(\mathsf{X})$ be a σ-algebra of Borel subsets of X. Let μ be a stochastic measure on $\mathcal{B}(\mathsf{X})$, i.e. a σ-additive mapping $\mu : \mathcal{B}(\mathsf{X}) \to \mathsf{L}_0(\Omega, \mathcal{F}, \mathsf{P})$. Also, such μ is called a general stochastic measure (see, for example, [1, Section 7]). This underlines the fact that we do not require the fulfillment of any other assumptions except σ-additivity.

Consider the following Cauchy problem

$$\begin{cases} \dfrac{\partial^2 u(t,x)}{\partial t^2} = a^2 \Delta_x u(t,x) + \\ + f(t,x,u(t,x)) + \sigma(t,x)\,\dot{\mu}(t), \\ u(0,x) = u_0(x); \quad \dfrac{\partial u(0,x)}{\partial t} = v_0(x), \end{cases} \tag{1}$$

where $(t,x) \in [0,T] \times \mathbb{R}^d$, $d = \overline{1,3}$, $T > 0$, $a > 0$, and μ is a stochastic measure defined on the Borel σ-algebra $\mathcal{B}([0,T])$. The solution of equation (1) is understood in the following mild sense

$$\begin{aligned} u(t,x) &= \int_{\mathbb{R}^d} \mathcal{S}_d(t, x-y) v_0(y)\, dy + \\ &+ \frac{\partial}{\partial t}\left(\int_{\mathbb{R}^d} \mathcal{S}_d(t, x-y) u_0(y)\, dy\right) + \\ &+ \int_0^t ds \int_{\mathbb{R}^d} \mathcal{S}_d(t-s, x-y) f(s,y,u(s,y))\, dy + \\ &+ \int_0^t d\mu(s) \int_{\mathbb{R}^d} \mathcal{S}_d(t-s, x-y)\sigma(s,y)\, ds\,, \end{aligned} \tag{2}$$

where $\mathcal{S}_d$ is the fundamental solution of the wave equation in $\mathbb{R}^d$ (see, for example, [2]).

The integrals of random functions with respect to dy and ds are considered for every fixed $\omega \in \Omega$. Such integrals are studied in [3], for example.

The existence, uniqueness and Hölder regularity of the solution of equation (2) are proved in [4, 5, 6] (for $d = 1, 2, 3$ respectively).

In the article [8] the asymptotic behavior of the mild solution is investigated in two cases: the equation is given on the line and on the plane.

Also, the asymptotics of the mild solutions of heat equations with general stochastic measures is considered in [7, 9, 10]. In the papers [11, 12] the asymptotic behavior of solutions of wave equations with white noise is studied. The long time behavior of the solutions to stochastic wave equations driven by a Lévy process is investigated in [13].

Our aim here is to prove that the solution of problem (2) tends to zero almost surely as the absolute value of the spatial variable tends to infinity in the case when $d = 3$.

2 Some results needed for the proof

Consider the Besov space $B^{\alpha}_{22}([b,c])$, $\alpha \in (1/2,1)$, $b,c \in \mathbb{R}$, that is, the space of functions $g : [b,c] \to \mathbb{R}$ such that the norm

$$\|g\|_{B^{\alpha}_{22}([b,c])} = \|g\|_{\mathsf{L}_2([b,c])} + \\ +\Big(\int_0^{c-b} \big(w_{2,[b,c]}(g,r)\big)^2 r^{-2\alpha-1}\,dr\Big)^{\frac{1}{2}},$$

is finite where

$$w_{2,[b,c]}(g,r) = \sup_{0\leqslant h\leqslant r}\left(\int_b^{c-h} |g(s+h)-g(s)|^2\,ds\right)^{\frac{1}{2}}.$$

For an arbitrary $t \in (0,t]$, put

$$\Delta^{(t)}_{kn} = ((k-1)2^{-n}t, k2^{-n}t],\ n \geqslant 0,\ 1 \leqslant k \leqslant 2^n.$$

Let a function $g(z,s) : Z \times [0,t] \to \mathbb{R}$ be such that $\forall z \in Z :\ g(z,\cdot)$ is continuous on $[0,t]$ for all $z \in Z$. Here the symbol Z denotes an arbitrary set. Put

$$g_n(z,s) = g(z,0)\mathbb{1}_{\{0\}}(s) + \\ +\sum_{1\leqslant k\leqslant 2^n} g(z,(k-1)2^{-n}t)\mathbb{1}_{\Delta^{(t)}_{kn}}(s).$$

Then the random function

$$\eta(z) = \int_{(0,t]} g(z,s)d\mu(s), \quad z \in Z,$$

has a version

$$\widetilde{\eta}(z) = \int_{(0,t]} g_0(z,s)d\mu(s) + \\ +\sum_{n\geqslant 1}\left(\int_{(0,t]} g_n(z,s)d\mu(s) - \int_{(0,t]} g_{n-1}(z,s)d\mu(s)\right), \tag{3}$$

such that

$$|\widetilde{\eta}(z)| \leqslant |g(z,0)\mu((0,t])| + C\|g(z,\cdot)\|_{B^{\alpha}_{22}([0,t])}\times \\ \times\left\{\sum_{n\geqslant 1} 2^{-n\varepsilon}\sum_{1\leqslant k\leqslant 2^n}\left|\mu\left(\Delta^{(t)}_{kn}\cap(0,t]\right)\right|^2\right\}^{\frac{1}{2}}, \tag{4}$$

where $\alpha = \varepsilon/2 + 1/2$ (see [6]). Note that the version (3) is the same for all $z \in Z$, while the constant C depends on α and t and does not depend on z, ω.

3 Main result

According to [14, § 12] the fundamental solution of wave equation (1) ($d = 3$) is given by

$$\mathcal{S}_3(t,x) = \frac{1}{2\pi a}\delta\left(a^2t^2 - |x|^2\right)\mathbb{1}_{\{t>0\}} = \\ = \frac{1}{4\pi a^2 t}\delta_{S_{at}}\mathbb{1}_{\{t>0\}},$$

where δ is the Dirac function (correspondingly, $\delta_{S_{at}}$ is the uniform measure on a sphere of radius at that has the full mass $4\pi a^2t^2$), and where $|\cdot|$ denotes the Euclidean norm.

Thus for any test function φ on $\mathbb{R}^4$ (that is any infinitely differentiable function with a finite support in $\mathbb{R}^4$) we have

$$(\mathcal{S}_3,\varphi) = \frac{1}{4\pi a^2}\int_0^{\infty}\frac{1}{t}\int_{|x|=at}\varphi(x,t)\,dS(x)\,dt = \\ = \frac{1}{4\pi a^2}\int_{\mathbb{R}^3}\frac{1}{|x|}\varphi\left(x,\frac{|x|}{a}\right)dx\,, \tag{5}$$

where $dS(x)$ is a differential of the area in the sphere $\{x \in \mathbb{R}^3 : |x| = at\}$.

Therefore, a mild solution of problem (1) (for $d = 3$) is a measurable random function $u(t,x) = u(t,x,\omega) : [0,T]\times\mathbb{R}^3\times\Omega \to \mathbb{R}$ that almost surely satisfies the following integral equation

$$u(t,x) = \frac{1}{4\pi a^2 t}\int\limits_{|y-x|=at} v_0(y)\,dS(y) + \\ +\frac{1}{4\pi a^2}\frac{\partial}{\partial t}\left(\frac{1}{t}\int\limits_{|y-x|=at} u_0(y)\,dS(y)\right) + \\ +\frac{1}{4\pi a^2}\int_0^t ds\int\limits_{|y-x|=a(t-s)}\frac{f(s,y,u(s,y))}{t-s}\,dS(y) + \\ +\frac{1}{4\pi a^2}\int_0^t\frac{1}{t-s}d\mu(s)\int_{|x-y|=a(t-s)}\sigma(s,y)\,dS(y)\,. \tag{6}$$

Here $dS(y)$ is a differential of the area in the sphere $\{y \in \mathbb{R}^3 : |y-x| = at\}$ and in the sphere $\{y \in \mathbb{R}^3 : |y-x| = a(t-s)\}$ for the corresponding integrals.

The following conditions are assumed throughout the paper.

A1. The functions $u_0(y) = u_0(y,\omega) : \mathbb{R}^3\times\Omega \to \mathbb{R}$, $v_0(y) = v_0(y,\omega) : \mathbb{R}^3 \times \Omega \to \mathbb{R}$ are measurable and, for all $\omega \in \Omega$,

$$|u_0(y,\omega)| \leqslant C_{u_0}(\omega), \quad \left|\frac{\partial u_0}{\partial y_i}(y)\right| \leqslant C_{u_0}(\omega),$$
$$|v_0(y,\omega)| \leqslant C_{v_0}(\omega)\,.$$

A2. The functions $v_0(y), u_0(y), \frac{\partial u_0(y)}{\partial y_i}, i = 1,2,3$, are Hölder regular, namely

$$\left|v_0(y') - v_0(y'')\right| \leqslant L_{v_0}(\omega)\left|y' - y''\right|^{\beta(v_0)},$$
$$0 < \beta(v_0) \leqslant 1;$$
$$\left|u_0(y') - u_0(y'')\right| \leqslant L_{u_0}(\omega)\left|y' - y''\right|^{\beta(u_0)},$$
$$0 < \beta(u_0) \leqslant 1\,;$$
$$\left|\frac{\partial u_0}{\partial y_i}(y') - \frac{\partial u_0}{\partial y_i}(y'')\right| \leqslant L_{u_0}(\omega)\left|y' - y''\right|^{\beta(u_0)}\,.$$

A3. The function $f(s,y,v) : [0,T]\times\mathbb{R}^3\times\mathbb{R} \to \mathbb{R}$ is measurable and bounded, that is

$$|f(s,y,v)| \leqslant C_f.$$

A4. The function $f(s,y,v)$ is uniformly Lipschitz with respect to $y \in \mathbb{R}^3, v \in \mathbb{R}$, that is

$$\left|f(s,y',v') - f(s,y'',v'')\right| \leqslant L_f\left(\left|y'-y''\right| + \left|v'-v''\right|\right).$$

A5. The function $\sigma(s,y) : [0,T] \times \mathbb{R}^3 \to \mathbb{R}$ is measurable and bounded, that is

$$|\sigma(s,y)| \leqslant C_\sigma.$$

A6. The function $\sigma(s,y)$ is Hölder regular, namely

$$\left|\sigma(s',y') - \sigma(s'',y'')\right| \leqslant$$
$$\leqslant L_\sigma\left(\left|s'-s''\right|^{\beta(\sigma)} + \left|y'-y''\right|^{\beta(\sigma)}\right),$$
$$1/2 < \beta(\sigma) \leqslant 1.$$

A7. $|u_0(y)| \to 0$, $\left|\frac{\partial u_0}{\partial y_i}(y)\right| \to 0$, $|v_0(y)| \to 0$, $\sup\limits_{s\in[0,T],\, z\in\mathbb{R}} |f(s,y,z)| \to 0$, $\sup\limits_{s\in[0,T]} |\sigma(s,y)| \to 0$, as $|y| \to \infty$.

The symbols C and $C(\omega)$ denote a nonrandom and random constants, respectively, that can be different in different places and whose precise values do not matter for our reasoning.

Theorem 3.1. *Suppose Assumptions* A1 – A7 *hold. Then the solution of equation* (6) *has a version $u(t,x)$ such that for all $t \in [0,T]$, $\omega \in \Omega$,*

$$|u(t,x)| \to 0, \quad |x| \to \infty.$$

Proof. According to Theorem 2.1 of [6], under Assumptions A1 – A6 for all $t \in [0,T]$, $x \in \mathbb{R}^3$, equation (6) has the unique solution $u(t,x)$.

We have

$$|u(t,x)| \leqslant \left|\frac{1}{4\pi a^2 t}\int\limits_{|y-x|=at} v_0(y)\,dS(y)\right| +$$
$$+\left|\frac{1}{4\pi a^2}\frac{\partial}{\partial t}\left(\frac{1}{t}\int\limits_{|y-x|=at} u_0(y)\,dS(y)\right)\right| +$$
$$+\left|\frac{1}{4\pi a^2}\int_0^t ds \int\limits_{|y-x|=a(t-s)} \frac{f(s,y,u(s,y))}{t-s}\,dS(y)\right| +$$
$$+\left|\frac{1}{4\pi a^2}\int_0^t \frac{1}{t-s}d\mu(s)\int_{|x-y|=a(t-s)} \sigma(s,y)\,dS(y)\right| =$$
$$= |J_1(t,x)| + |J_2(t,x)| + |J_3(t,x)| + |J_4(t,x)|. \quad (7)$$

Now each of terms is treated separately. First we consider the stochastic integral. Similarly to proof of [6, Lemma 4.1] we use equality (5), pass from the surface integral to the multiple integral and obtain

$$J_4(t,x) = \frac{1}{4\pi}\int_{(0,t]} h(t,x,s)d\mu(s),$$

where

$$h(t,x,s) = (t-s)\int_0^{2\pi} d\varphi \int_0^\pi \sigma\big(s, x_{t-s}\big)\sin\theta\, d\theta\,,$$

and

$$x_{t-s} = (x_1 + a(t-s)\cos\varphi\sin\theta, x_2 +$$
$$+ a(t-s)\sin\varphi\sin\theta, x_3 + a(t-s)\cos\theta),$$
$$x = (x_1, x_2, x_3).$$

Let arbitrary $t \in [0,T]$, $x \in \mathbb{R}^3$, be fixed. Then the random function $J_4(t,x)$ has a version (3) that admits bound (4). This version is denoted by the same symbol J_4. To use this bound we estimate the norm of the function $h(t,x,s)$ in the Besov space.

By Assumption A7 for all $\varepsilon_0 > 0$ there exists $\delta_0 > 0$ such that for all $|y| > \delta_0$,

$$\sup_{s\in[0,T]} |\sigma(s,y)| < \varepsilon_0, \quad \sup_{s\in[0,T],\, z\in\mathbb{R}} |f(s,y,z)| < \varepsilon_0,$$
$$|u_0(y)| < \varepsilon_0,\ \left|\frac{\partial u_0}{\partial y_i}(y)\right| < \varepsilon_0,\ |v_0(y)| < \varepsilon_0. \quad (8)$$

Then $\forall s \leqslant t, \forall |x| > \delta_0 + aT$ there exists a point y_0, $|y_0 - x| \leqslant a(t-s)$, such that

$$|h(t,x,s)| = (t-s)\left|\int_0^{2\pi} d\varphi \int_0^{\pi} \sigma\big(s, x_{t-s}\big)\sin\theta d\theta\right| = \\ = (t-s)|\sigma(s,y_0)|\int_0^{2\pi} d\varphi\int_0^{\pi}\sin\theta d\theta < 4\pi T\varepsilon_0. \quad (9)$$

Here we use the Mean Value Theorem, the Squeeze Theorem and the first inequality of (8).

Hence

$$|h(t,x,s)| < C\varepsilon_0,$$
$$\|h(t,x,s)\|_{L_2([0,t])} = \left(\int_0^t |h(t,x,s)|^2 ds\right)^{\frac{1}{2}} < C\varepsilon_0,$$
$$|h(t,x,s+h) - h(t,x,s)| < 8\pi T\varepsilon_0 = C\varepsilon_0.$$

Therefore, for $s + h \leqslant t$,

$$w_{2,[b,c]}(g,r) < C\varepsilon_0\sqrt{t} \leqslant C\varepsilon_0 \quad (10)$$

On the other hand, by A5 and A6 we have

$$|h(t,x,s+h) - h(t,x,s)| = \Bigg|(t-s)\times \\ \times\int_0^{2\pi} d\varphi\int_0^{\pi}\big(\sigma(s,x_{t-s-h}) - \sigma(s,x_{t-s})\big)\sin\theta\, d\theta - \\ - h\int_0^{2\pi} d\varphi\int_0^{\pi}\sigma(s,x_{t-s-h})\sin\theta\, d\theta\Bigg| \leqslant \\ \leqslant (t-s)4\pi L_\sigma h^{\beta(\sigma)} + h4\pi C_\sigma \leqslant Ch^{\beta(\sigma)}.$$

Consequently,

$$w_{2,[b,c]}(g,r) < Cr^{\beta(\sigma)}.$$

Raise the latter inequality to the power θ and multiply by inequality (10) raised to the power $1-\theta$ for an arbitrary $\theta \in (0,1)$. We obtain

$$w_{2,[b,c]}(g,r) \leqslant Cr^{\theta\beta(\sigma)}\varepsilon_0^{1-\theta}.$$

Thus there exists $\alpha < \theta\beta(\sigma)$ such that

$$\|h(t,x,s)\|_{B_{22}^{\alpha}([0,t])} < C\varepsilon_0 + \\ + C\varepsilon_0^{1-\theta}\left(\int_0^t r^{2\theta\beta(\sigma)-2\alpha-1}dr\right)^{\frac{1}{2}} \leqslant \\ \leqslant C\big(\varepsilon_0 \vee \varepsilon_0^{1-\theta}\big),$$

where $a \vee b$ is a maximum of numbers a and b.

Now the bounds obtained above are substituted to (4). Finally, we obtain

$$|J_4(t,x)| < C\big(\varepsilon_0 \vee \varepsilon_0^{1-\theta}\big)\times \\ \times\left(|\mu((0,t])| + \left\{\sum_{n\geqslant 1} 2^{-n\varepsilon}\sum_{1\leqslant k\leqslant 2^n}\left|\mu\left(\Delta_{kn}^{(t)}\right)\right|^2\right\}^{\frac{1}{2}}\right) \leqslant \\ \leqslant C(\omega)\big(\varepsilon_0 \vee \varepsilon_0^{1-\theta}\big), \quad (11)$$

where the sum with the stochastic measure is finite according to [15, Lemma 3.1].

Next, by equality (22) of [6] we get the following representation of J_1

$$J_1(t,x) = \frac{t}{4\pi}\int_0^{2\pi} d\varphi\int_0^{\pi} v_0\big(x_t\big)\sin\theta\, d\theta\,.$$

The same reasoning as that used in obtaining bound (9) proves, $\forall |x| > \delta_0 + aT$,

$$|J_1(t,x)| < T\varepsilon_0\,. \quad (12)$$

Analogously, since by (23) of [6]

$$J_2(t,x) = \frac{1}{4\pi}\int_0^{2\pi} d\varphi\int_0^{\pi} u_0\big(x_t\big)\,\sin\theta\, d\theta + \\ + \frac{t}{4\pi}\int_0^{2\pi} d\varphi\int_0^{\pi}\frac{\partial}{\partial t}u_0\big(x_t\big)\,\sin\theta\, d\theta\,,$$

we have, $\forall |x| > \delta_0 + aT$,

$$|J_2(t,x)| < (1+T)\varepsilon_0\,. \quad (13)$$

Then we obtain similarly to the preceding bounds by representation (5) and the second inequality of (8) that for all $|x| > \delta_0 + aT$,

$$|J_3(t,x)| = \frac{1}{4\pi a^2}\times \\ \times\left|\int_{|y|\leqslant at}\frac{f\left(t - \frac{|y|}{a}, y + x, u\left(t - \frac{|y|}{a}, y + x\right)\right)}{|y|}dy\right| < \\ < \frac{\varepsilon_0}{4\pi a^2}\left|\int_{|y|\leqslant at}\frac{1}{|y|}dy\right| = \frac{t^2\varepsilon_0}{2} \leqslant \frac{T^2}{2}\,\varepsilon_0\,. \quad (14)$$

Finally, we substitute estimations (11)–(14) to relation (7). For all $\varepsilon > 0$ put $\varepsilon_0 = \varepsilon C^{-1}(\omega)$ if $\varepsilon \geqslant 1$, and $\varepsilon_0 = \big(\varepsilon C^{-1}(\omega)\big)^{\frac{1}{1-\theta}}$ if $\varepsilon < 1$. Then there exists a set $\Omega_0 \subset \Omega$, $\mathbf{P}(\Omega_0) = 1$, such that for all $t \in [0,T]$, $\omega \in \Omega_0$,

$$\forall\varepsilon > 0 \ \ \exists\delta = \delta_0 + aT \quad \forall |x| > \delta: \quad |u(t,x)| < \varepsilon.$$

$\square$

4 Conclusions

We investigate the equation that describes a wave in some medium with random and non-random sources of perturbations. Namely, the situation when influence of the sources vanishes as the spatial variable tends to infinity is studied. It is shown that in this case the wave also fades with unlimited growth of the absolute value of the spatial variable.

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