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Математичне моделювання агрегації та осідання частинок в похилих трубках

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Mathematical modelling of particle aggregation and sedimentation in the inclined tubes

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Осідання частинок, які агрегують у гравітаційному полі, широко використовується як простий і дешевий тест на стабільність суспензії різних технічних сумішей, крові та нанорідин. Встановлено, що нахил трубки робить тест набагато швидшим, що відомо як ефект Бойкотта. Залежність швидкості осідання від кута нахилу є складною і мало вивченою задачею. У цій роботі узагальнено двофазну модель суспензії частинок, які агрегують у похилих трубках. Задача розглядається у двовимірному випадку, що відповідає вузьким прямокутним ємностям або зазорам віскозиметрів конусоподібного типу. У припущенні малих кутів нахилу рівняння усереднюються по поперечній координаті, а отримана гіперболічна система рівнянь розв'язується методом характеристик. Чисельні розрахунки виявили, що збільшення початкової концентрації частинок, їх швидкості агрегації, зовнішньої рівномірної сили і кута нахилу прискорюють осідання, а будь-яке зростання в'язкості рідини сповільнює його, що є фізично доречним. Так чи інакше, поведінка прискорення різна. На основі отриманих результатів запропоновано новий метод оцінки стійкості суспензії.

Ключові слова: ефект Бойкотта, суспензія, агрегація, седиментація, медична діагностика.

Sedimentation of the aggregating particles in the gravity field is widely used as an easy and cheap test of the suspension stability of different technical suspensions, blood and nanofluids. It was established the tube inclination makes the test much faster that is known as the Boycott effect. The dependence of the sedimentation rate on the angle of inclination is complex and poorly understood yet. In this paper the two phase model of the aggregating particles is generalized to the inclined tubes. The problem is formulated in the two-dimensional case that corresponds to the narrow rectangle vessels or gaps of the viscosimeters of the cone-cone type. In the suggestion of small angles of inclination the equations are averaged over the transverse coordinate and the obtained hyperbolic system of equations is solved by the method of characteristics. Numerical computations revealed the increase in the initial concentration of the particles, their aggregation rate, external uniform force and inclination angle accelerate the sedimentation while any increase in the fluid viscosity decelerates it that is physically relevant. Anyway, the behaviors of the acceleration are different. Based on the results, a novel method of estimation of the suspension stability is proposed.

Key words: Boycott effect, suspension, aggregation, sedimentation, medical diagnostics.

Статтю представив д.ф.-м.н., проф. Жук Я. О.

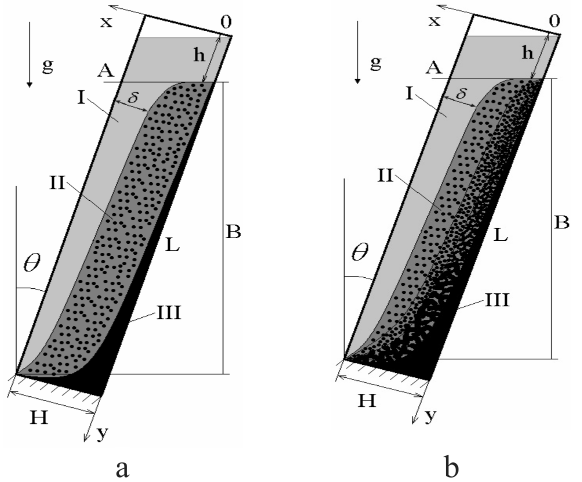
Introduction

Influence of inclination of the vessel in which a suspension of particles sediments in the gravity field was first discovered by Arthur Boycotte in 1920 on the red blood cell (RBC) sedimentation in thin long vertical tubes [1], and now it is known as the Boycotte effect. On that time the RBC sedimentation test was recognized as the most powerful medical diagnostic means on general pathology, and many researchers were seeking for more benefit test conditions, including the usage of the inclined tubes. This sedimentation technique is also widely used for the waste water cleaning, drinking water purification, treatment of mixtures in industry and manufacture, and the high reservoirs or deep wells are needed for the successful processing. The settling of particles in the high containers is limited by its width, and tilting of the reservoir or well increases the efficient area of sedimentation, decreases the distance that each particle must travel before impacting a wall, and, therefore, enhances the sedimentation rate in orders of magnitude [2].

The Boycotte effect is used in the oil industry because at certain inclination angles (40-50°) of the tube the clearest separation of the suspension for oil-well cementation is observed [3]. The effect is also used for mixing of the granular matters [4]. It may be responsible for specific sediment distribution along inclined ocean bottom at the water stratification conditions [5], and for the pattern formation at the inclined surfaces of the sand-dunes [2]. All suspensions settle faster in the inclined vessels and exhibit clear separation of the layers of different optical density, but the problem on determination of the optimal angels remains still unsolved.

Problem formulation

The steady sedimentation of particles in the narrow channel of the width R and the length $L\ (R/L \ll 1)$, inclined at the angle θ is considered (Fig.1a,b). The channel corresponds to the gap between the walls of the rotation viscosimeter of the cone-cone type or a rectangle vessel with the depth D>>R.



a b

Fig.1. A sketch of the particle sedimentation in the inclined tube for the non-aggregating (a) and aggregating (b) particles.

The two phase approach to the suspension of the aggregating particles [6,7] is used. Neglecting the inertia forces as compared with the viscous forces, the equations of the quasi-steady motion can be written in the form

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}Nu_x^1+\frac{\partial}{\partial y}Nu_y^1=\varphi, \qquad (1)$$

$$\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}Hu_x^1+\frac{\partial}{\partial y}Hu_y^1=0, \qquad (2)$$

$$\begin{aligned}&\frac{\partial}{\partial x}\left[Hu_x^1+(1-H)u_x^2\right]+\\&\frac{\partial}{\partial y}\left[Hu_y^1+(1-H)u_y^2\right]=0\end{aligned}, \qquad (3)$$

$$H\frac{\partial p}{\partial y}=-F(u_y^1-u_y^2)+H\rho_s G\cos(\theta), \qquad (4)$$

$$(1-H)\frac{\partial p}{\partial y}=F\left(u_y^1-u_y^2\right)+(1-H)\rho_f G\cos(\theta), \qquad (5)$$

$$H\frac{\partial p}{\partial x}=-F\left(u_x^1-u_x^2\right)+H\rho_s G\sin(\theta), \qquad (6)$$

$$(1-H)\frac{\partial p}{\partial x}=F\left(u_x^1-u_x^2\right)+(1-H)\rho_f G\sin(\theta), \qquad (7)$$

where (u_x^1, u_y^1) and (u_x^2, u_y^2) are components of the velocity vectors for the particles (phase 1) and fluid (phase 2), (0xy) is the Cartesian coordinate system connected with the ρ_s, ρ_f are densities of the solid and fluid materials, p is the hydrostatic pressure, H and N are the mass and numerical concentrations of the particles, F is the phenomenological coefficient for the viscous drag forces acting on the particle from the viscous fluid, φ the aggregation rate, G is the mass force that can be chosen as $G = ng$, n is the magnification factor when the sedimentation is carried out in a centrifuge [7].

The equation (1) describes the kinetics of the particle aggregation due to the collisions, decompositions or exchange interactions [6]. The mass continuity conditions for the phases (2), (3) and the projections of the momentum equations for the phases on the axis coordinate (4)-(7) give the system of PDE for the velocities, pressures and numerical concentration of the aggregates. The same model in one-dimensional formulation has been used for the RBC sedimentation modeling in the vertical tubes, in thin gaps between the walls of the rotational viscosimeter of the cylinder-cylinder type [8] and in the centrifugal force field [9]. In the two-dimensional formulation it has been applied to the RBC sedimentation in the circular tubes in the external magnetic field [10]. The system (1)-(7) can be solved by numerical methods, but in order to derive more convenient half-analytical estimations, in this paper a simplified one-dimensional model will be obtained by averaging of the equations (1)-(7) over the transverse coordinate x.

One-dimensional approximation for small inclination angles

Based on the dimension theory, the expressions for F and φ have been found in the form [6,8-10]:

$$F = \mu_f H(1-H)^{-2,5} w^{-2/3}, \quad \varphi = -kH^2 w^{-2}, \qquad (8)$$

where $w = H/N$ is the average volume of the aggregates, k is the empirical constant that determines the aggregate formation.

By excluding pressure in (4)-(7) and using the impermeability condition at the walls of the tube [10], one can obtain

$$u_x^1 = -\Theta(H)\sin(\theta), \quad u_y^1 = \Theta(H)\cos(\theta),$$
$$u_x^2 = \frac{H\Theta(H)ng\delta\rho\sin(\theta)}{(1-H)F}, \qquad (9)$$
$$u_y^2 = -\frac{H\Theta(H)ng\delta\rho\cos(\theta)}{(1-H)F},$$

where $\Theta(H) = H(1-H)^2\delta\rho G/F(H)$, $\delta\rho = \rho_s - \rho_f$.

Let us assume, the aggregation at the expense of the lateral motion of the particles is essential if during the time T of the particle sedimentation along the distance L their radial displacement δ will be of the order of magnitude of R. Since $\delta \approx |u_x^1|T, T \approx L/|u_y^1|$, this condition accounting for (9) can be written as

$$\frac{\delta}{R} \approx \frac{L|u_x^1|}{R|u_y^1|} = \frac{L}{R}\tan(\theta). \qquad (10)$$

For the averaging purposes let us introduce the following designations:

$x \in [0; x_s]$, where x_s is the coordinate of the interface between the layers I and IIa (Fig.1b)

$$N(t,x,y) = \begin{cases} N(t,y), x \le x_s \\ 0, \quad x_s < x \le s \end{cases}$$
$$H(t,x,y) = \begin{cases} H(t,y), x \le x_s \\ 0, \quad x_s < x \le s \end{cases} \qquad (11)$$

where s is the width or the area occupied by the particles and aggregates (Fig.1b).

According to (9) the interface $x = x_s$ between the zones I and IIa moves with the speed

$$U_x = u_x^1\big|_{x=x_s} = H(1-H)^2 g\delta\rho\sin\alpha F^{-1}. \qquad (12)$$

The averaged values will be introduced in the form

$$\langle f\rangle = \frac{1}{x_s}\int_0^{x_s} f dx. \qquad (13)$$

Then after some estimations of terms at the conditions $R/L \ll 1, H_0 < 1$ the system (1)-(7) can be written in the form (the averaging signs are omitted):

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial y}Nu_y^1 = -kN^2\Delta +$$
$$\frac{NH(1-H)^2\delta\rho\, ng\sin(\theta)}{Fx_s},$$

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial y}Hu_y^1 = \frac{H^2(1-H)^2\delta\rho\, ng\sin(\theta)}{Fx_s},$$

$$
\begin{gathered}
\frac{\partial}{\partial y}\left[Hu_y^1+(1-H)u_y^2\right]=0,\\
H\frac{\partial p}{\partial y}+F\left(u_y^1-u_y^2\right)+\rho_s ng\,H\cos(\theta)=0,\\
(1-H)\frac{\partial p}{\partial y}-F\left(u_y^1-u_y^2\right)+\\
\rho_f ng(1-H)\cos(\theta)=0,
\end{gathered}
\tag{14}
$$

where $\Delta(x_s,y)=\frac{1}{x_s}\int_0^{x_s}\left(1+O\left(\frac{\delta}{R}\right)\right)xdx.$

With the non-dimensional variables

$$
Y=\frac{y}{L},\quad T=\frac{t}{T^*},\quad T^*=\frac{L}{u_0},\quad u_0=\frac{ng\delta\rho w_0^{2/3}}{\mu_f},
$$

$$
W=\frac{w}{w_0},\quad U_y^1=\frac{u_y^1}{u_0},\quad K=\frac{kL}{u_0w_0}
$$

where T^* is the characteristic time, w_0 is the volume of a single particle, the equations for the concentration and volume of the aggregates are the following

$$
\begin{gathered}
\frac{\partial W}{\partial T}+U_y^1\frac{\partial W}{\partial Y}=KH\Delta,\\
\frac{\partial H}{\partial T}+\frac{\partial}{\partial Y}HU_y^1=-\frac{H(1-H)^{4,5}W^{2/3}\sin(\theta)}{L},\\
U_y^1(H,W,Y)=-(1-H)^{4,5}W^{2/3}\cos(\theta).
\end{gathered}
\tag{15}
$$

The boundary conditions for the variables are

$$
C(0,Y)=C_0,\ W(0,Y)=1,\ U^1(T,L)=0. \tag{16}
$$

The system (15) is hyperbolic [6,7].

The characteristic equations are

$$
\text{(I)}\ \frac{dY}{dT}=(1-H)^{4,5}W^{2/3}\cos(\theta), \tag{17}
$$

$$
\text{(II)}\ \frac{dY}{dT}=-A(5.5H-1)\cos(\theta). \tag{18}
$$

where $A=(1-H)^{3,5}W^{2/3}$.

The conditions at the characteristics are

$$
\text{(I)}\ \frac{dW}{dT}=-KH\Delta, \tag{19}
$$

$$
\begin{gathered}
(II)\frac{2}{W}\left(\frac{dW}{dT}-KC\Delta\right)=\frac{13.5}{1-H}\left(\frac{dH}{dT}+\right.\\
\left.H(1-H)^{4.5}\cos(\theta)\right)+\frac{H(1-H)A\sin(\theta)}{L}
\end{gathered}
\tag{20}
$$

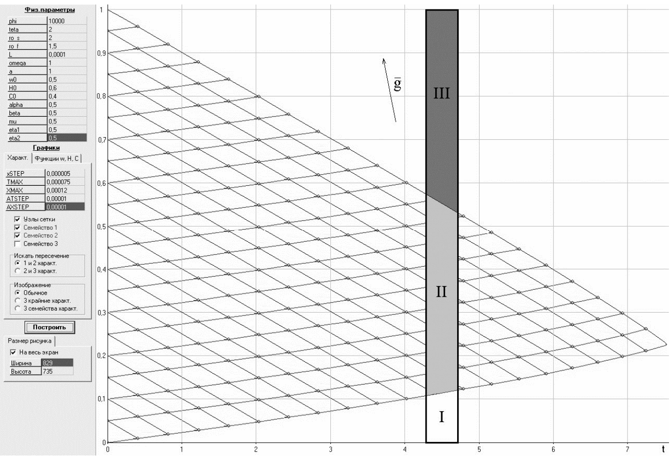
The characteristics of the family (I) have a positive slope, while the family (II) in the physiological range have negative slope. Solution of the one-dimensional problem (15)-(16) can be obtained on (17)-(20) by the method of characteristics.

Numerical results and discussion

Like in the case of the vertical tube [6,7], the families of characteristics (I) and (II) have positive and negative slopes accordingly. The family (I) corresponds to the interface between the zones II and III, while the family (II) describes the movement of the interface between the zones I and II moving with the corresponding velocity (12). Numerical computations on (17)-(20) have been carried out using the typical parameters for human blood [6,7]

$$
\mathrm{H_0}=0.35\div0.5\ \ \mu_\mathrm{f}=(1.1\div1.7)\cdot10^{-3}\,\mathrm{Pa\cdot s},
$$
$$
\mathrm{G=g},\ \rho_\mathrm{f}=1030\div1080\ \ \mathrm{kg/m^3},
$$
$$
\rho_\mathrm{s}=1050\div1150\,\mathrm{kg/m^3},\ L=5\,\mathrm{cm},\ R=2\,\mathrm{mm},
$$
$$
\mathrm{k}=10^{-5}\div10^{-2}\ \ \mathrm{m^{-3}s^{-1}}.
$$

The numerical procedure is described in details in [6,7]. The example of the snapshot of the software elaborated is given in Fig.2. The results of numerical computations are presented in Fig.3-4.



Физ.параметры

phi	10000
teta	2
ro_s	2
ro_f	1,5
L	0,0001
omega	1
a	1
w0	0,5
H0	0,6
C0	0,4
alpha	0,5
beta	0,5
mu	0,5
eta1	0,5
eta2	0,5

Графики

xSTEP	0,000005
TMAX	0,000075
XMAX	0,00012
ATSTEP	0,00001
AXSTEP	0,00001

☑ Узлы сетки
☑ Семейство 1
☑ Семейство 2
☐ Семейство 3

Искать пересечение
● 1 и 2 характ.
○ 2 и 3 характ.

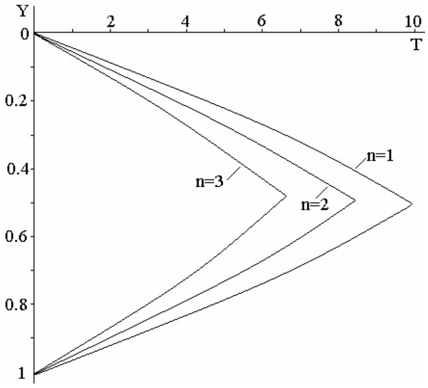
Изображение
● Обычное
○ 3 крайние характ.
○ 3 семейства характ.

Размер рисунка
☑ На весь экран

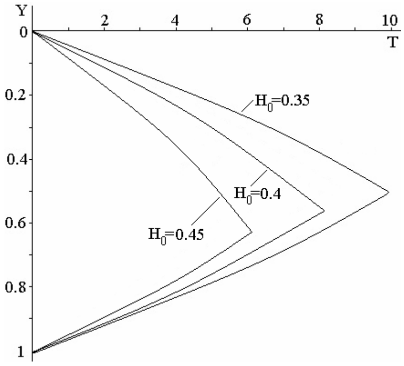
Ширина	829
Высота	735

Fig.2. An example of the interface of the software and the zone distribution in the inclined tube: I is the clear fluid zone, II is the zone of sedimenting aggregates, III is the compact zone.

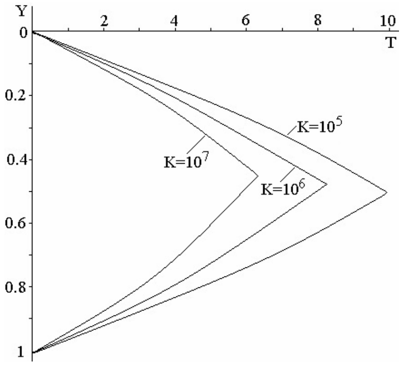
The increased external force uniformly accelerates particle sedimentation along the tube (Fig.3a). When the initial concentration increases, the changes are more noticeable for the I-II interface (Fig.3b). When particle aggregation rate increases, the changes are more noticeable at the II-III interface (Fig.3c). Small increase in the inclination angle significantly accelerates the zone I formation (Fig.3d) because of the decrease of the length the particles move before reaching the lower wall of the inclined tube. When the inclination angle becomes bigger than some critical angle θ^*, the sedimentation decelerates. In the case of the material parameters used in the computations presented in Fig.3d, the optimal angle is $\theta^* \sim 8^\circ$.



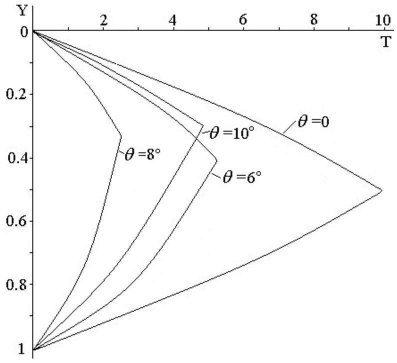
a



b



c



d

Fig.3. Locations of the characteristics at different external force (a), initial concentrations of the particles (b), aggregation rate (c), and inclination angle (d).

Before decision making upon the nanofluid ageing or blood aggregatability the test must be conducted for the same nanofluid in its basic state before being used for a long time or for the healthy native blood samples. The proposed approach will be tested experimentally in future works.

Conclusions

The Boycotte effect which is used in testing of some industrial suspensions is very attractive for usage in the medical diagnostics instead of conventional blood sedimentation test, for investigation of ageing of micro and nanofluids. The developed theory allows easy determination of the sedimentation curves as the moving interfaces I-II and II-III by the method of characteristics. It was shown, the sedimentation rate increased with increasing the particle concentration, their aggregation rate and external force, but with distinct regularities for the I-II and II-III interfaces. The corresponding dependence on the angle is more complex. Sedimentation is accelerated by small angles but at the angles exceeded some critical value the settling is decelerated and hampered. Based on the obtained results, a novel method of determination not only the aggregation ability of the particles but also the particle-specific and angle-dependent shear stress factor is proposed. The experimental validation of the proposed approach will be tested in our future experimental studies on blood and different types of nanofluids used in the microfluidic flow systems.

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