In this expression, we have to give values n:
If n = 0,

$$\left(y_1^{(I)}\right)^{[I]}=f_0^{\left(y_0^{(I)}\right)^{[I]}},$$

if we take it (5):

$$\left(y_1^{(I)}\right)^{[I]}=f_0^{\frac{\alpha_2-\alpha_1}{\alpha_1-\alpha_0}}, \qquad (9_1)$$

if n=1, we obtain from (8):

$$\left(y_2^{(I)}\right)^{[I]}=f_1^{\left(y_1^{(I)}\right)^{[I]}}=f_1^{f_0^{\frac{\alpha_2-\alpha_1}{\alpha_1-\alpha_0}}}, \qquad (9_2)$$

if we continue this process:

$$\left(y_n^{(I)}\right)^{[I]}=f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{f_0^{\frac{\alpha_2-\alpha_1}{\alpha_1-\alpha_0}}}}}} \equiv g_{\mathrm{n}}. \qquad (9_n)$$

Using the definition of the discrete multiplicative derivative to this equation we obtain:

$$\frac{y_{n+1}^{(I)}}{y_n^{(I)}}=g_{\mathrm{n}},\ \mathrm{n}\geq 0, \qquad (10)$$

Here by assigning values to n, obtained

$$\frac{y_1^{(I)}}{y_0^{(I)}}=g_0,$$
$$\frac{y_2^{(I)}}{y_1^{(I)}}=g_1,$$
$$\vdots$$
$$\frac{y_n^{(I)}}{y_{n-1}^{(I)}}=g_{n-1}.$$

Multyplying the expressions:

$$y_n^{(I)} = y_0^{(I)}\cdot\prod_{k=0}^{n-1} g_k =$$

$$=(\alpha_1-\alpha_0)\cdot\prod_{k=0}^{n-1} g_k \equiv h_n, \mathrm{n}\geq 0. \qquad (11)$$

Finally, using the definition of the discrete additive derivative equation (11), we have to assign values n:

$$y_1 - y_0 = h_0,$$
$$y_2 - y_1 = h_1,$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$y_n - y_{n-1} = h_{n-1}.$$

If we add on them side by side,
So we obtain:

$$y_n = y_0+\sum_{k=0}^{n-1} h_k = \alpha_0+\sum_{k=0}^{n-1} h_k,\ \mathrm{n}\geq 0. \qquad (12)$$

Theorem 2. If the $f_n > 0$, n ≥ 0 and α_k k= $\overline{0,2}$ are the numbers given $\alpha_0 \neq \alpha_1$, then there is the unique solution of Cauchy problem (1), (5) and this solution is in the form of (12), so that the h_s are given by (11), and by the g_k (9_n).

IV. THE BOUNDARY PROBLEM

Now let's see the boundary problem for (1) equation:

$$\left(y_0^{(I)}\right)^{[I]}=\frac{y_2-y_1}{y_1-y_0}=\alpha,\ y_0^{(I)}=\beta,\ y_m=\gamma, \qquad (13)$$

if n=0, then we obtain from (8):

$$\left(y_1^{(I)}\right)^{[I]}=f_0^{\left(y_0^{(I)}\right)^{[I]}}=f_0^{\alpha}. \qquad (14)$$

Considering obtained (14) into (9_2) we obtain:

$$\left(y_2^{(I)}\right)^{[I]}=f_1^{\left(y_1^{(I)}\right)^{[I]}}=f_1^{f_0^{\alpha}}. \qquad (15)$$

By continuing this process, let's take from (8):

$$\left(y_n^{(I)}\right)^{[I]}=f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{f_1^{f_0^{\alpha}}}}}} \equiv G_n, \qquad (16)$$

Here using the definition of the descrete multiplicative derivative , we obtain:

$$\frac{y_{n+1}^{(I)}}{y_n^{(I)}}=G_n, \mathrm{n}\geq 0. \qquad (17)$$

Obtained expression (10) we have solved in the solution of Cauchy problem, let's give values to n:

$$\frac{y_1^{(I)}}{y_0^{(I)}} = G_0,$$

$$\frac{y_2^{(I)}}{y_1^{(I)}} = G_1,$$

$$\vdots$$

$$\frac{y_n^{(I)}}{y_{n-1}^{(I)}} = G_{n-1}.$$

Multiplying these sum exprsessions side by side:

$$y_n^{(I)} = y_0^{(I)} \prod_{s=0}^{n-1} G_s = \beta \prod_{s=0}^{n-1} G_s \equiv H_n, \quad (18)$$

We obtain this equation.

Finally at (18) using the definition of the discrete additive derivative equation we obtain:

$$y_{n+1} - y_n = H_n, n \geq 0. \qquad (19)$$

But here

$$y_n = y_0 + \sum_{k=0}^{n-1} H_k, \qquad (20)$$

We acquire this expression. By using (13) the last of the given border conditions, Let's make the arbitrary y_0 constant in (20) :

$$\gamma = y_m = y_0 + \sum_{k=0}^{m-1} H_k,$$

I mean,

$$y_0 = \gamma - \sum_{k=0}^{m-1} H_k. \qquad (21)$$

Then from (20) (1), (3) for the solution of boundry problems:

$$y_n = \gamma - \sum_{k=n}^{m-1} H_k. \qquad (22)$$

We obtain this expression.

Theorem 3. If $f_n > 0$, n ≥ 0 is the sequence given, α > 0, β ≠ 0 and γ are the constants, then (1), (13) are the solution of the boundary problem (22). Here, H_k (18) is expressed in the form of G_k (16).

V. CONCLUSIONS

Thus, we recieved the solution of the problems Caughy and boundary for third complied discrete additivo-multipicativo-powerative derivative equation.

Список використаних джерел

1. Vorobiev N.N. Fibonacci numbers. Popular lectures on mathematics, ed. 6. / N.N. Vorobiev. – Moscow:"Science", 1984. – 144 p.

2. Makarachev Y.H. Algebra. A textbook for VIII class of secondary school, / Y.H. Makarachev, H.G. Mindyuk, V.M. Monakhov, K.S. Mudrovich, S.B. Suvarova. – Baku: "Maarif", 1980. – 250 p.

3. Gelfond A.O. Numerically of finite differences / A.O. Gelfond. – Moscow: "Nauka", 1967. – 376 p.

4. Aliev N. Discrete additive analysis / Aliev N., Bagirov G., İzadi F.A. – Tarbiat Moallem University publishers, Tabriz, İran, 1993. – P.144.

References

1. VOROBIEV, N.N. (1984) *Fibonacci numbers, Popular lectures on mathematics, ed. 6,* Moscow, "Science", 144 p.

2. MAKARACHEV, Y.H., MINDYUK, H.G, MONAKHOV, V.M., MUDROVICH, K.S, SUVAROVA, S.B. (1980) *Algebra, A textbook for VIII class of secondary school,* Baku, "Maarif", 250 p.

3. GELFOND, A.O. (1967) *Numerically of finite differences,* Moscow, "Nauka", 376 p.

4. ALIEV, N., BAGIROV, G., İZADI, F.A. (1993) *Discrete additive analysis,* Tarbiat Moallem University publishers, Tabriz, İran, 144 p.

5. İzadi F.A. Discrete Calculus By Analaogy / F.A.İzadi, N.Aliyev, G.Bagirov. – December 3, Iran, 2009. – 154 p.

6. Gantmakher F.R. Theory of Matrices / F.R. Gantmakher – Moscow, "Nauka", 1967. – 576 p.

7. Hassani O.H. Analytic Approach to Solve Specific Linear and Nonlinear Differential Equations / O.H. Hassani, N. Aliev // Jnt. Math. Forum Journal for Theory and Applications. – 2008. – 33-36, Vol. 3. – P. 1623–1631.

8. Aliyev N.A. Boundary problem for the second composite discrete multiplicative derivative equation / N.A.Aliyev, T.S.Mamiyeva // BU's News, Physics-Mathematics Series, Baku. – 2017. – No.1. – P.15–19.

9. Aliyev N.A. Prollekes for the equation of third-order additive multiplicatives / N.A.Aliyev, T.S.Mamiyeva // Materials from the republican scientific conference "Funcional analysis and its applications", Baku. – 2016. – P.17–18.

10. Mammadzada A. Discrete properties of new derivatives / A. Mammadzada // Materials of the Republican Scientific Conference of Young Researchers on "Modernizing Azerbaijan: New Rise Stage", Lankaran. – 2017. – P. 29–30.

11. Hall J. Perfect numerical methods for solving ordinary differential equations/ J. Hall, J.Watt. – «Mir» Moscow, 1979. – 312 p.

12. Mammadov Y.J. Approximate calculation methods / Y.J. Mammadov. – Baku: "Maarif", 1986. – 264 p.

13. Vazov V. Difference methods for the solution of partial differential equations / V. Vazov, J. Forsyte. – IL, Moscow, 1963. – 488 p.

14. Aliyev N. Solution of problems for the second composite discrete poverative derivative equation / N. Aliyev, A. Mammadzada // Scientific News, Natural Sciences. – 2018. – №1. – P. 55–58.

5. İZADI, F.A., ALIYEV, N., BAGIROV, G. (2009) *Discrete Calculus By Analaogy,* December 3, Iran, 154 p.

6. GANTMAKHER, F.R. (1967) *Theory of Matrices*, Moscow, "Nauka", 576 p.

7. HASSANI, O.H., ALIEV, N. (2008) "Analytic Approach to Solve Specific Linear and Nonlinear Differential Equations", *Jnt. Math. Forum Journal for Theory and Applications*, 33-36, Vol. 3.,p. 1623–1631.

8. ALIYEV, N.A., MAMIYEVA, T.S. (2017) "Boundary problem for the second composite discrete multiplicative derivative equation", *BU's News, Physics-Mathematics Series*, No.1, p.15-19.

9. ALIYEV, N.A., MAMIYEVA, T.S. (2016) "Prollekes for the equation of third-order additive multiplicatives", *Materials from the republican scientific conference "Funcional analysis and its applications"*, Baku, p. 17-18.

10. MAMMADZADA, A. (2017) "Discrete properties of new derivatives", *Materials of the Republican Scientific Conference of Young Researchers on "Modernizing Azerbaijan: New Rise Stage"*, Lankaran, p. 29-30.

11. HALL, J., WATT, J. (1979) *Perfect numerical methods for solving ordinary differential equations*, Moscow, «Mir», 312 p.

12. MAMMADOV, Y.J. (1986) *Approximate calculation methods*, Baku, "Maarif", p.264 .

13. VAZOV, V., FORSYTE, J. (1963) *Difference methods for the solution of partial differential equations*, IL, Moscow, 1963, 488 p.

14. ALIYEV, N., MAMMADZADA, A. (2018) "Solution of problems for the second composite discrete poverative derivative equation", *Scientific News, Natural Sciences.* Lankaran, №1, p.55-58.

Надійшла до редколегії 19.08.2018

УДК 519.95

Гусейнзаде Ш.С., к.т.н., доц.

Модель автоматизації адаптивного керування на мережах Петрі

Сумгаїтський державний університет
AZ5021, Азербайджанська Республіка,
м. Сумгаїт, пос. Джорат, I квл., д. 31

e-mail: shahla.huseynzade@gmail.com

Huseynzade Sh.S., Ph.D. (Tech.), Assoc. Prof.

Automation Model of the Adaptive Control on Petri nets

Sumgayit State University
AZ5021, Azerbaijan Republic, city Sumgayit, Jorat settlement, 1 quarter, 31

e-mail: shahla.huseynzade@gmail.com

Пропонується модель автоматизації адаптивного нечіткого керування водними насосами на основі мереж Петрі (МП). На базі критеріїв роботи водних насосів відповідно рівням води у насосних колодязях сформовані множини позицій та переходів МП. При описі необхідної поведінки системи відношеннями між позиціями і переходами МП із застосуванням логіки «Якщо... То...» розроблено алгоритм адаптації. На основі алгоритму визначені матриці вхідних та вихідних інцидентностей. Розроблено граф-модель МП. Візуалізація моделі реалізована у системі CPN Tools. Проведено комп'ютерні експерименти симуляції і аналізу мережи.

Ключові слова: мережі Петрі, алгоритм адаптації, керування водними насосами, матриця інцидентностей, симуляція мережі.

A model is proposed for automation of adaptive fuzzy control of water pumps based on Petri nets (PN). Based on the criteria for the operation of water pumps in accordance with the water levels in the pump well, many of the positions and transitions of the PN are formed. An adaptation algorithm is developed describing the necessary behavior of the system by the relationship between positions and transitions of the PN using the logic "If ... Then ..." . Based on the algorithm, input and output incidence matrices are defined. A graph-model of the PN was developed. Visualization of the model is realized in the CPN Tools system. Computer simulations and network analysis were conducted.

Key Words: Petri nets, adaptation algorithm, water pumps control, incidence matrix, network simulation.

Статтю представив д.ф.-м.н., проф. Хусаінов Д.Я.

I. INTRODUCTION

The water pump installations are placed on the way of drains and in places of wastewater treatment, work on their automation is carried out. However, changes in the rainwater runoff process make full automation difficult. When the amount of rainwater runoff increases, sudden water flows appear.

To prevent such phenomena, on the one hand, it is planned to control the rainwater runoff due to water-permeable bridges and soil accumulating rainwater, and on the other hand- to expand the sewage pipes, increase the number and throughput of pumps. However, increasing the productivity of the equipment generates new problems: due to the imbalance between the inflowing stream and the throughput, the frequency of turning the pumps on and off increases and there appears a need to control the opening of the shutter in the case of water flows exceeding the pump capacity. A control system of decision-making and control is required which would not depend on the individual qualities of the maintenance staff and would ensure load reduction.

The considered system as well as the majority of systems of a manufacturing industry is strongly nonlinear, and, therefore, parameters of system can experience come uncertainties. Thus, the management of such nonlinear installations is of practical importance. Automation of pumping installations is an important problem due to its extensive applications. In solving such problems, fuzzy logic provides a powerful platform that allows engineers to apply human reasoning to the control algorithm [1].

Previously, many methods of intellectual control were proposed to cope with the problem of controlling non-linear processes, such as neural networks, genetic algorithms and fuzzy logic. Many researchers have proposed various fuzzy logic controllers to solve problems [2]. In our research we seek to present yet another alternative method as Petri nets.

II. THE MAIN ASPECTS OF APPLYING THE PN TO THE MODELING OF ADAPTIVE FUZZY CONTROL OF WATER PUMPS

The construction of the models of the systems in the form of PN is based on a variety of events and conditions, as well as on some obvious cause-effect relationships. In Petri nets, sequences of events are displayed by transitions [3]. The fulfillment of any condition is associated with the appearance of a label in the position corresponding to this condition. To improve the quality of development, verification is used that the model satisfies the requirements formulated in advance [4]. The PN is the integration of a graph and a discrete dynamic system, it can serve both as a static and dynamic model of the object represented with it. Of no small importance is the known advantage of these networks, the convenience of their programming on the computer.

Definition 1. The Petri net is formally represented as a set of the form $N = (P, T, F, H, \mu^0)$, where $P = \{p_1,\ p_2, \dots ,\ p_n\}$, $n > 0$ is a finite non-empty set of positions (otherwise states or places); $T = \{t_1,\ t_2, \dots ,\ t_m\}$, $m > 0$ is a finite non-empty set of transitions (events); $F: P \times T \to \{0,1,2, \dots\}$ and $H: P \times T \to \{0,1,2, \dots\}$ are respectively, the functions of input and output incidents, and the view $\mu^0: P \to \{0,1,2, \dots\}$ initial marking (marking each position) [5].

The graphical representation of the Petri nets is a bipartite oriented graph with two types of vertices. Vertices $p \in P$ are represented by circles, and vertices $t \in T$ –rectangles. The arcs correspond to the functions of incidence of positions and transitions.

Below we consider the features of the water pump operation and the adaptive fuzzy control method, which can be used for efficient automation. The control rules are designed based on the performance criteria of water pumps in the pump well, which are used by a qualified worker [6]:

1. If the current water level in the pump well is high and it is expected to increase, another pump is turned on.
2. If the water level in the pump well is low and its lower position is presumed, one of the pumps is turned off.
3. If a sudden increase in rainwater runoff is expected, the pump is turned on urgently and a low water level in the pump well is supported.
4. If, with increasing water flow, a conclusion is drawn that there is no danger, the operation of the pumps is maintained in the same regime even when the water level rises.
5. In the case of a large influx of rainwater, the water level in the pump well is maintained at a lower elevation for safety reasons.

Evaluation of the water level in the pump well and the trend of its change are different depending on the situation with the inflow of water. To construct a model for effective automation of adaptive fuzzy control of water pumps based on PN, at first structural elements of the PN are described: All states associated with the water levels in the pump well, taking into account the situation with the influx of rainwater, are accepted as PN positions. Actions related to incoming information in the system about the situation with water inflow are accepted as PN transitions. The links between states and actions are accepted as arcs of PN.

To describe non complete knowledge of the behavior of the system, the linguistic variable "water level" and the terms of the linguistic variable are used. The terms of a variable correspond to its fuzzy values and are denoted by an expression characterizing one of the states of the parameter of the system. The values "slightly increased", "increased", "highly increased", "slightly reduced", "reduced", "highly reduced", "normal" correspond to the terms of the variable "water level", which will describe the behavior of the system and the desired response of the technical object. The term determines the degree of belonging of the data received from the sensor to the specified area of fuzzy values.

III. FORMATION OF STRUCTURAL ELEMENTS AND INCIDENCE MATRICES PN

Assume that there are 3 pumps in the pump well. On the basis of the criteria for the operation of water pumps in accordance with the water levels in the pump well, which are used by a qualified worker, many of the positions and transitions of the PN are described:

• Positions:

p1 ─ The registration sensor has input information; p2 ─ the water level in the pumping well is slightly increased; p3 ─ the water level in the pump well is increased; p4 ─ the water level in the pumping well is highly increased; p5 ─ the water