КОМП'ЮТЕРНІ НАУКИ ТА ІНФОРМАТИКА

UDC 519.9
DOI: https://doi.org/10.17721/1812-5409.2024/1.21

Volodymyr PICHKUR[1], DSc (Phys. & Math.), Prof.
ORCID ID: 0000-0002-5641-8145
e-mail: volodymyr.pichkur@knu.ua

Valentyn SOBCHUK[1], DSc, (Engin.), Prof.
ORCID ID: 0000-0002-4002-8206
e-mail: sobchuk@knu.ua

Dmytro CHERNIY[1], DSc, (Engin.), Assoc. Prof.
ORCID ID: 0000-0002-6378-8048
e-mail: dmytro.cherniy@knu.ua

Anton RYZHOV[1], PhD (Phys. & Math.), Assoc. Prof.
ORCID ID: 0000-0003-4099-0742
e-mail: ryzhov@knu.ua

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

FUNCTIONAL STABILITY OF PRODUCTION PROCESSES AS CONTROL PROBLEM OF DISCRETE SYSTEMS WITH CHANGE OF STATE VECTOR DIMENSION

The paper proposes an approach to mathematical modeling of technological processes of industrial enterprises for the organization of production in accordance with established standards with compliance with acceptable tolerances and requirements. For the first time, the authors consider the property of functional stability of production processes in two aspects: as a property of the system to maintain its functional state under conditions of change and as a property of the system to restore its functional state after the effects of external and internal factors (functional stability and functional resilience). We presents the mathematical model of production processes in the form of linear discrete control systems under the following condition: the state vector changes dimension. This condition shows that the parameters characterizing the state of the system can change at different stages of production processes due to technological features. This causes the state vector dimension to change. The authors give definition of functional stability of the process, prove theorems on conditions of functional stability and give solution of control design problem using generalized inverse matrices properties.

Keywords: ***discrete systems, functional stability, functional resilience, mathematical model, technological process, control design, generalized inversion.***

AMS 2020 classification: 37L15, 65F20, 93-10, 91B38.

Introduction

Rapid development of technologies in recent years is key to development in many social, economic, military, and information domains. Production processes require deep automation, integration of processes with modern complex information, information-communication or cyberphysical systems. This significantly increases their efficiency. The rapid dynamics of technological progress creates new challenges to information systems. These systems ensure the stability of production and technological processes to the influence of various factors of external and internal nature. The property of complex technical and information systems that ensures their effective functioning in a changing environment is called functional stability (Mashkov, & Kosenko, 2015). The first important results in this area were obtained by O. Mashkov and colleagues (Gostev, Mashkov, & Mashkov, 1995).

In the paper, we will distinguish between two views on this property. On the one hand, functional stability is the ability of the system to maintain its functional state under the influence of changing external and internal destabilizing factors (Maksymuk et al., 2020; Sobchuk, V., Olimpiyeva, Musienko, & Sobchuk, A., 2021). On the other hand, functional resilience is the ability of the system to restore its functional state after the impact of negative external and internal destabilizing factors (Pichkur, & Sobchuk, 2021; Rohret, Kraft, & Vella, 2013). The main difference between these two concepts is that functional stability characterizes the system's ability to maintain its functional state under conditions of change, functional resilience characterizes the system's ability to restore its functional state after negative impacts.

One can achieve functional stability in the system due to following factors:

● *reliability of system's components.* If the system components are reliable, they will function properly even under the influence of external and internal factors;

● *redundancy.* A system with redundant components can continue to function even if some of its components fail;

● *flexibility*. If the system is flexible, then it can adapt to changing conditions caused by the influence of factors of a different nature.

One can achieve functional resilience (Bellini, Coconea, & Nesi, 2020) in the system due to such factors as:

● *recoverability.* A system that has ability to self-recover will be able to recover its functional state after the influence of external and internal factors without outside help;

● *adaptivity.* If the system is adaptive, then it will be able to change its functional state in order to resist the negative unauthorized effects of various nature.

Information systems controlling the technological processes of production enterprises must be functionally stable in both senses. The functional stability is important. It ensures smooth operation of production processes under the influence of external and internal factors (Asrorov, Sobchuk, & Kurylko, 2019). For example, such factors are fluctuations in temperature, pressure, composition of raw materials, etc. The functional resilience of the systems is important because it allows them to adapt to the conditions of its operation, regenerate itself from the negative effects of various factors (e. g. power supply failures, equipment breakdowns, deviations from specified telemetry parameters, etc.)

Consider the components of functional stability of information systems controlling the technological processes of production enterprises:

● *reliability.* The system must be equipped with reliable components. Such components are sensors, controllers, actuators, etc. This will ensure the proper functioning of the system, even in conditions of fluctuations in temperature, pressure, vibration, composition of the raw material structure, etc.;

● *redundancy.* System's redundancy means that there are additional components in the system that are not requited. These components can be used to increase its reliability and stability. Redundancy can be implemented in various ways: redundant components, parallel processing of components and redundant circuits. Redundancy increases probability that the system will be able to continue processing even if one or more components fail;

● *flexibility.* The system must be flexible. It must adapt to changed external conditions. For example, the system must be able to regulate temperature, pressure, vibration, etc. The system must be flexible to the composition or structure of raw materials or product quality requirements;

● *recoverability.* The system must have the ability to self-recover. It should recover from the negative effects of external and internal factors without outside help. For example, the system must be able to automatically restart after power failures, etc.;

● *adaptivity.* Adaptivity of the system is its ability to change its functional state in order to resist the negative effects of factors of various nature. For example, the system should be able to automatically change the operating mode in the event of an equipment failure, etc.

We can conclude that functional stability is an integral property of information systems controlling the technological processes of modern manufacturing enterprises. Functional stability ensures smooth operating of production processes and allows for quick recovery after abnormal situations (Barabash et al., 2023).

In the paper we consider mathematical model of production process in the form of a linear discrete control system with state dimension change. We define functional stability as a control problem, prove necessary and sufficient conditions of functional stability. Further we give solution of control design that implements the functionally stable production process.

1. Literature review. At present, there are many works related to the problems of stability and control of discrete systems. The monograph (Halanay, & Rasvan, 2000) studies the problems of stability theory for discrete systems in detail: the method of first approximation, the Lyapunov functions method, the problem of invariance, dissipativity conditions of discrete systems, stable oscillations problem etc. Stability conditions for systems of discrete equations are highlighted in (Michel, Ling, & Derong, 2008). Problems of the qualitative theory of discrete systems, stability and boundary-value problems are highlighted in (Agarwal, 2000). The work (Lasalle, 1986) describes the second Lyapunov method for discrete systems, stability according to the first approximation method, controllability, solutions in stabilization and state estimation problems. Quantitative and qualitative analysis to discrete systems is also given in (Galor, 2007). The article (Diblík, Dzhalladova, & Michalková, 2013) studies company's work efficiency using conditions of financial stability. In this paper the mathematical model is described by a system of difference equations with random coefficients.

The papers (Garashchenko, & Bashnyakov, 1999; Garashchenko, & Kutsenko, 2000; Garashchenko, & Pichkur, 2016; Pichkur, & Linder, 2021) offer practical stability conditions to discrete systems and inclusions. The results are based on the properties of optimal sets of practical stability. The paper (Valeyev, & Dzhalladova, 2002) considers optimal control of discrete systems with random coefficients. The technique of structural-parametric optimization based on the method of dynamic programming is highlighted in the paper (Strakhov, 2013). The article (Matvienko, 2007) gives the general solution to the terminal control of linear discrete systems problem.

The papers (Zamrii et al., 2022; Mashkov, 2020) offer methods ensuring the reliability of the telecommunication infrastructure of critical infrastructure enterprises. The researchers take into account the impact of the risks of unauthorized access to information systems and data transmission systems. The papers (Barabash et al., 2017; Yevseiev et al., 2021) describe principles, design and application methods of information infrastructure organization under the influence of external and internal destabilizing factors. In (Otenko, Podorozhna, & Otenko, 2021) the authors give techniques of information support to decision-making systems in industrial enterprises. One may see that the problem of functional stability of production processes is widely presented in the scientific literature. However, production processes control design under conditions of ensuring functional stability is open.

2. Statement of the problem and basic definitions. First of all, it should be noted that without automating the process of production control, it is impossible to organize the serial production of quality products in the conditions of modern enterprises. To solve similar problems at production enterprises, to ensure the stability of production processes due to real-time control of key production parameters, we offer a mathematical model that one can integrate into automated enterprise control system. In general, we understand a business process (technological process) as a sustainable, purposeful set of interrelated actions that, with the help of certain technologies and in an optimal time, transforms inputs (resources) into outputs (results). Results have value for internal and external consumers and ensure the growth of the financial and economic indicators of the enterprise.

The production of products consists of stages. There are parameters and characteristics of raw materials, semi-completed products, or ultimately completed products at each of the stage. We denote parameters on i–th stage $x(i)$, $i = 0,1, \dots N$ (fig. 1).

Fig. 1. The structure of the technological production process

The number of indicators usually changes at each stages, so dimension of the vector $x(i)$ depends on stage number

$$x(i) \in R^{n_i}, \qquad i = 0,1, \dots N.$$

A vector

$$u(i) \in R^{m_i}$$

describes external influence on the production process (work effect, energy effect, chemical or other technological effects etc) at i-th stage, $i = 0,1, \dots N-1$. Quality of the products and intermediate quantity at each stage depends on whether the technology and value of the indicators are followed at each previous stage. We will assume that this requirement is fulfilled.

ISSN 1728-3817

Note that in real production conditions the course of the production process will ensure the repeatability and output satisfying the determined requirements and standards with certain deviations (tolerances) $\varepsilon > 0$ (fig. 2). Each production process or product has individual tolerances. That is, all linear, chemical, mechanical, geometric or other characteristics of the product must correspond to the reference sample without exceeding the thresholds of technological errors. In the opposite case, with significant deviations, the manufacturer considers the product to be defective. As a result, it becomes necessary to repeat the process. For this, additional resources should be allocated once again, changes should be made to the production plans. This will increase the processing time of the entire order.

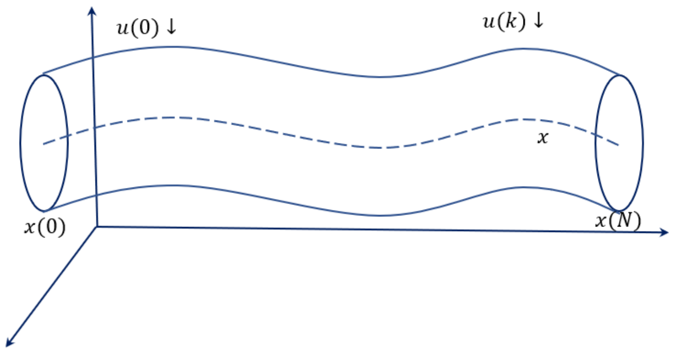

Fig. 2. Scheme of production process

We also denote by: $A(i)$ a matrix of the production process. Dimension of the matrix is $n_{i+1} \times n_i$, $i = 0,1,\dots,N-1$. The components of the matrix $A(i)$ show dependencies between product quality indicators on $i+1$-th stage and indicators on i-th stage; $C(i)$ a matrix of dimension $n_{i+1} \times m_i$, $i = 0,1,\dots,N-1$ that determines the structure of influence $u(i)$ on the production process.

Mathematical model of the production technological process can be written as follows

$$x(i+1) = A(i)x(i) + C(i)u(i),\ i = 0,1,\dots,N-1. \tag{1}$$

We represent (1) in the form

$$Ax = Cu. \tag{2}$$

We see that all stages of the technological production process can be described by system of algebraic equations (2), where

$$x = \big(x(0), x(1), \dots, x(N)\big)^T \in R^n, \qquad u = \big(u(0), u(1), \dots, u(N-1)\big)^T \in R^m,$$

$$C = \begin{pmatrix} C(0) & 0 & 0 & \cdots & 0 \\ 0 & C(1) & 0 & \cdots & 0 \\ 0 & 0 & C(2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & C(N-1) \end{pmatrix}$$

is a matrix of dimension $n \times m$,

$$A = \begin{pmatrix} -A(0) & E_1 & 0 & 0 & \cdots & 0 \\ 0 & -A(1) & E_2 & 0 & \cdots & 0 \\ 0 & 0 & -A(2) & E_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & -A(N-1) & E_N \end{pmatrix}$$

is a matrix of dimension $n \times n$. Here $n = n_0 + n_1 + \dots + n_N$, $m = m_0 + m_1 + \dots + m_{N-1}$, E_k is a unit matrix of dimension $n_k \times n_k$, $k = 1,2,\dots,N$. The matrices A and C determine topology of the production process. The vector u describes technological features of the production process.

Let $\underline{x} = \big(\underline{x}(0), \underline{x}(1) \dots \underline{x}(N)\big)^T$ be *reference process*. The reference process fully corresponds the set of parameters $x(k)$, $k = 0,1,\dots,N$. The reference process describes the values of the parameters during ideal execution of the production process at each stage. Let us define $\varepsilon > 0$ as permissible deviations (tolerances) the systems state from the reference values.

Definition 1. *Given matrices A, C and a vector u. If there is a solution $x = \underline{x} + e$* system (2) *such that*

$$\|e\| \le \varepsilon,$$

then we will call such a technological process functionally stable.

3. Conditions of functional stability of production processes. In this section we present functional stability conditions of system (2) in definition 1 sense. Denote by A^+ generalized inverse matrix to matrix A, $Z(A^T) = E_n - AA^+$ projector onto matrix A^T kernel (Ben-Israel, & Greville, 2003). The following theorem is true.

Theorem 1. *Suppose that*

$$u^T Q u = 0, \tag{3}$$

where $Q = C^T Z(A^T) C$. If

$$\|A^+(Cu - A\underline{x})\| \le \varepsilon. \tag{4}$$

then the technological process described by equation (2) *is functionally stable.*

Proof. The matrix $Z(A^T)$ is a projector, hence $Z^2(A^T) = Z(A^T)$, $Z^T(A^T) = Z(A^T)$ (Ben-Israel, Greville, 2003). From (3) it follows that

$$u^T Q u = u^T C^T Z(A^T) C u = u^T C^T Z^T(A^T) Z(A^T) C u = 0.$$

It means that $\|Z(A^T)Cu\|^2 = 0$ and $Z(A^T)Cu = 0$. Substituting $Z(A^T) = E_n - AA^+$ in the last formula. we obtain

$$(C - AA^+C)u = 0.$$

As a consequence, $AA^+Cu = Cu$. Therefore

$$AA^+Cu - A\underline{x} = Cu - A\underline{x}.$$

From $A = AA^{+}A$ we have

$$AA^{+}Cu - A\underline{x} = AA^{+}Cu - AA^{+}A\underline{x} = AA^{+}(Cu - A\underline{x}).$$

Further

$$AA^{+}(Cu - A\underline{x}) = Cu - A\underline{x}.$$

This means that $Cu - A\underline{x} \in Im\ A$. There is $e \in R^{n}$ such that

$$Ae = Cu - A\underline{x}. \quad (5)$$

From (5) we get

$$A(e + \underline{x}) = Cu.$$

The vector $e + \underline{x}$ is a solution of (2). Since (5) holds and $e \in Im\ A$, we obtain

$$e \in \{A^{+}(Cu - A\underline{x}) + v\}, \quad (6)$$

where $v \in Ker\ (A)$ so that $Av = 0$. The vector

$$e = A^{+}(Cu - A\underline{x})$$

has the smallest norm on the set (6) at $v = 0$. Since (4) holds, we obtain $\|e\| \le \varepsilon$, $x = \underline{x} + e$. Therefore the technological process is functionally stable.

■

The inverse theorem to the theorem 1 is also true.

Theorem 2. *If the technological process is functionally stable, then*

$$u^{T}Qu = 0,$$

alongside with

$$\|A^{+}(Cu - A\underline{x})\| \le \varepsilon.$$

Proof. From definition 1 it follows that system (2) has a solution $x = \underline{x} + e$, where $\|e\| \le \varepsilon$, $Ax = Cu$. Consequently $A(\underline{x} + e) = Cu$ and

$$Ae = Cu - A\underline{x}. \quad (7)$$

From matrix image definition we obtain $Cu - A\underline{x} \in ImA$. Therefore, based on the fact that AA^{+} is a projector onto ImA (Yanai, Takeuchi, & Takane, 2011), we have

$$AA^{+}(Cu - A\underline{x}) = Cu - A\underline{x}.$$

Moreover

$$AA^{+}Cu - AA^{+}A\underline{x} = Cu - A\underline{x}.$$

But $AA^{+}A = A$ so that $AA^{+}Cu = Cu$. From here $(E - AA^{+})Cu = 0$. Therefore $Z(A^{T})Cu = 0$ and $\|Z(A^{T})Cu\|^{2} = 0$. Hence

$$u^{T}C^{T}Z^{T}(A^{T})Z(A^{T})C^{T}u = 0$$

and $u^{T}Qu = 0$. From (7) we have

$$e \in \{A^{+}(Cu - A\underline{x}) + Z(A)v, \qquad v \in R^{nN}\}.$$

Taking into account

$$\|A^{+}(Cu - A\underline{x})\| \le \varepsilon,$$

we obtain that one can found e at $v = 0$ such that $\|e\| \le \varepsilon$.

■

4. Adjustment of the production process to reference process. Assume that no solution of system (2) meets the conditions of functional stability. This means we cannot implement the process. In this case we stop the technological process to analyze the reasons that lead to functional instability. At the same time we solve the problem

$$I(e) = \|Ae - Cu - A\underline{x}\| \to \min_{e}. \quad (8)$$

The set

$$e \in \{A^{+}(Cu - A\underline{x}) + Z(A)v, \qquad v \in R^{nN}\},$$

gives the solution of this minimization problem (8), where the vector

$$e = A^{+}(Cu - A\underline{x})$$

has the smallest norm. The solution of the problem (8) describes the limit of deviations from the reference value. If the deviations increase, then the process must be stopped, because the products do not meet the specified standards. In practice, this means that if the deviations from the reference value satisfy (4), then the production process ensures the production of products according to the standard. If condition (4) is not fulfilled, but there exists a solution to problem (8), then in this case the system works in conditions close to the loss of functional stability. This means that the process ensures the production of products of inadequate quality, with defects. If problem (8) has no solutions, then the process should be stopped. To restart the system, it is necessary to return to the initial position at the level of functional tasks of the subsystems of production planning APS, MES, PLM-systems of the information automated control system and apply the algorithms of planning adjustment and restart the production plan.

Let the vector $\underline{x}$ describes reference indicators. The next important problem is as follows: is it possible to organize the production process in such a way as to comply with the reference indicators $\underline{x}$ due to the selection of the vector u? If we answer such a question, then we can offer a method that makes it possible to fulfill technological requirements at all stages of the production process. The following theorems give positive answer to the question.

Theorem 3. *Let matrices A and C determine the structure of the production process, and a vector $\underline{x}$ defines reference values of parameters x. Then reference values $\underline{x}$ will be implemented in the production process if and only if*

$$\underline{x}^{T}P\underline{x} = 0, \quad (9)$$

where $P = A^{T}Z(C^{T})A$, $Z(C^{T}) = E_{m} - CC^{+}$.

Proof. One can implement the reference values $\underline{x}$ if and only if (2) holds for some vector u. This condition is equivalent

$$A\underline{x} \in Im\ C. \quad (10)$$

where ImC is the image of the matrix C. Let denote by $Y(C) = CC^+$ orthogonal projector onto ImC. Then condition (10) is equivalent to equality

$$Y(C)A\underline{x} = A\underline{x}. \tag{11}$$

It means $CC^+A\underline{x} = A\underline{x}$ and

$$(E - CC^+)\underline{x} = 0. \tag{12}$$

Taking into account $Z(C^T) = E - CC^+$ we obtain $Z(C^T)A\underline{x} = 0$. Therefore $\|Z(C^T)A\underline{x}\|^2 = 0$, where

$$\underline{x}^T A^T Z^T(C^T) Z(C^T) A\underline{x} = \underline{x}^T A^T Z(C^T) A\underline{x} = 0. \tag{13}$$

It means that (3) holds.

On the other hand from (13) we get (12) and (11). Hence $A\underline{x} \in ImC$. It means that there is $u \in R^m$ such that $A\underline{x} = Cu$.

■

Consider the problem of adjusting the production process to reference indicators. It consists of minimizing the quality criterion

$$I_0(u) = \|x - \underline{x}\| \tag{14}$$

under conditions $Ax = Cu$, $u \in R^m$. Let us consider the modified problem

$$I(u) = \|A(x - \underline{x})\| \to \min_u, \tag{15}$$

where $Ax = Cu$, $u \in R^m$. There following statement is true.

Theorem 4. *Any vector*

$$u \in \{C^+A\underline{x} + Z(C)w : w \in R^m\}, \tag{16}$$

gives solution to (15)*, particularly*

$$u_* = C^+A\underline{x}$$

has the smallest Euclidean norm. The optimal value of the quality criterion (15) *is equal to*

$$I(u_*) = \|Z(C^T)A\underline{x}\|,$$

where $Z(C) = E_m - C^+C$, $Z(C^T) = E_n - CC^+$.

If conditions (9) *of theorem 3 holds, then* (16) *defines the set of values of* u *according to the reference values* $\underline{x}$.

Proof. Substitute $Ax = Cu$ in (15) and we obtain the following problem

$$I(u) = \|Cu - A\underline{x}\| \to \min, \qquad u \in R^m.$$

An arbitrary vector u from

$$\{C^+A\underline{x} + Z(C)w, \qquad w \in R^m\}$$

solves such a problem, at the same time the vector $u_* = C^+A\underline{x}$ has the smallest Euclidean norm among the specified vectors. The optimal value of the functional (15) is equal to

$$I(u_*) = \|Cu_* - A\underline{x}\| = \|(CC^+ - E_m)A\underline{x}\| = \|Z(C^T)A\underline{x}\|.$$

From theorem 3 it follows that if (9) holds, then control (16) solves minimization problems (14).

■

Theorems 3 and 4 determine the constructive conditions of functional resilience of production processes. These statements show the ability of the system to restore its functional state in order to perform technological production tasks under the conditions of compliance with the necessary predetermined product parameters.

Discussion and conclusions

The paper deals with the problem of functional stability of production processes of modern manufacturing enterprises. The authors highlight for the first time the properties of the functional stability of production processes in two aspects: as the system's ability to maintain its functional state under conditions of change and as the system's ability to restore its functional state after the effects of external and internal destabilizing factors.

The authors offer mathematical model of the production technological process as a discrete control system with change of state dimension. The paper proves necessary and sufficient conditions of functional stability, solves the control design problem that implements the functionally stable production process. Theorems are constructive. They offer methods of control to complex technological processes of manufacturing enterprises that can be implemented in information systems.

Authors' contribution: Volodymyr Pichkur – formal analysis, application of formal methods for analysis and synthesis of empirical research data, writing – revision, editing and additions; Valentyn Sobchuk – conceptualization, methodology, design and creation of a mathematical model, validation of theoretical results, writing; Dmytro Cerniy – discussion of results, writing, revision, editing; Anton Ryzhov – participation in conceptual modeling of production processes, discussion of results.

Acknowledgments, funding sources. The research was partially supported by the research project No 24БФ038-01 within the framework of the comprehensive scientific program "Modern Mathematical Problems of Natural Science, Economics and Finance".

References

Agarwal, R. P. (2000). *Difference Equations and Inequalities: Theory, Methods, and Applications*. Marcel Dekker.
Albert, A. (1972). *Regression and the Moore-Penrose pseudoinverse*. Academic Press.
Asrorov, F. A., Sobchuk, V. V., & Kurylko O. B. (2019). Finding of bounded solutions to linear impulsive systems. *Eastern-European Journal of Enterprise Technologies*, 6(4), 14–20. https://doi.org/10.15587/1729-4061.2019.178635
Barabash, O., Kravchenko, Y., Mukhin, V., Kornaga, Y., & Leshchenko, O. (2017). Optimization of Parameters at SDN Technologie Networks. *International Journal of Intelligent Systems and Applications,* 9(9), 1–9. https://doi.org/10.5815/ijisa.2017.09.01
Barabash, O. V., Sobchuk, V. V., Musienko, A. P., Laptiev, O. A., Bohomia, V. O., & Kopytko, S. V. (2023). System Analysis and Method of Ensuring Functional Sustainability of the Information System of a Critical Infrastructure Object. In M. Zgurovsky, & N. Pankratova (Eds). *System Analysis and Artificial Intelligence. Studies in Computational Intelligence* (vol. 1107). Springer, Cham. https://doi.org/10.1007/978-3-031-37450-0_11
Bellini, E., Coconea, L., & Nesi, P. (2020). A Functional Resonance Analysis Method Driven Resilience Quantification for Socio-Technical Systems. *IEEE Systems Journal*, 14(1), 1234–1244. https://doi.org/10.1109/JSYST.2019.2905713
Ben-Israel, A., & Greville, T. N. E. (2003). *Generalized inverses: theory and applications*. Springer.
Diblík, J., Dzhalladova, I., & Michalková, M. (2013). Modeling of applied problems by stochastic systems and their analysis using the moment equations. *Adv Differ Equ*, 152. https://doi.org/10.1186/1687-1847-2013-152

Galor, O. (2007). *Discrete Dynamical Systems*. Springer. https://doi.org/10.1007/3-540-36776-4
Garashchenko, F. G., & Bashnyakov, A. N. (1999). Analysis of a convergence for iterative procedures on the basis of methods of practical stability. *Journal of Automation and Information Sciences*, 31(7–9), 6–13.
Garashchenko, F. G., & Kutsenko, I. A. (2000). Practical stability of discrete processes, estimates and their optimization. *Journal of Automation and Information Sciences*, 32(5), 40–49.
Garashchenko, F. G., & Pichkur, V. V. (2016). On Properties of Maximal Set of External Practical Stability of Discrete Systems. *Journal of Automation and Information Sciences*, 48(3), 46–53. https://doi.org/10.1615/JAutomatInfScien.v48.i3.50
Gostev, V. I., Mashkov, O. A., & Mashkov, V. A. (1995). Self-diagnostic of modular systems in random performance of elementary tests. *Cybernetics and Computing Technology, Discrete Control Systems*, 99(3), 104–112.
Halanay, A., & Rasvan, V. (2000). *Stability and Stable Oscillations in Discrete Time Systems*. CRC Press. https://doi.org/10.1201/9781482283280
Maksymuk, O. A., Sobchuk, V. V., Salanda, I. P., & Sachuk, Yu. V. (2020). A system of indicators and criteria for evaluation of the level of functional stability of information heterogenic networks. *Mathematical Modeling and Computing*, 7(2), 285–292. https://doi.org/10.23939/mmc2020.02.285
Mashkov, O, Chumakevych, V., Ptashnyk, V., & Puleko, I. (2020, June 2–3). Qualitative Evaluation of the Process of Functionally Stable Recovery Control of the Aircraft in Emergencies with an Algorithm Based on Solving Inverse Dynamic Problems. *Proceedings of the 2nd International Workshop on Modern Machine Learning Technologies and Data Science,* I, 384–394. http://ceur-ws.org/Vol-2631
Mashkov, O., & Kosenko, V. (2015). Ensuring of functional stability of difficult dynamic systems as one of urgent scientific tasks of modern theory of automatic control. *Informatics Control Measurement in Economy and Environment Protection: Informatyka Automatyka Pomiary w gospodarce i ochronie środowiska. Kwartalnik Naukowo-Techniczny,* 3, 39–42. https://doi.org/10.5604/20830157.1166550
Matvienko V. T. (2007). Control of trajectories set by linear dynamic systems with discrete argument. *Journal of Automation and Information Sciences*, 39(11), 4–10.
Michel, A. N., Ling, H., & Derong L. (2008). *Stability of Dynamical Systems: Continuous, Discontinuous, and Discrete Systems*. Birkhäuser.
Lasalle, J. P. (1986). *The Stability and Control of Discrete Process*. Springer-Verlag, New York. http://dx.doi.org/10.1007/978-1-4612-1076-4
Otenko, I., Podorozhna, M., & Otenko, V. (2021, September 13–19). Information Support for Making Strategic Decisions on the Development of an Industrial Enterprise. 2021 *CEUR Workshop Proceedings* 3200, 281–285. https://ceur-ws.org/Vol-3200/paper41.pdf
Pichkur, V. V., & Linder, Y. M. (2021). Practical Stability of Discrete Systems: Maximum Sets of Initial Conditions Concept. In V. A. Sadovnichiy, M. Z. Zgurovsky (Eds.). *Contemporary Approaches and Methods in Fundamental Mathematics and Mechanics. Understanding Complex Systems*. Springer, Cham. https://doi.org/10.1007/978-3-030-50302-4_17
Pichkur, V. V., & Sobchuk, V. V. (2021). Mathematical Model and Control Design of a Functionally Stable Technological Process. *Journal Of Optimization, Differential Equations And Their Applications (JODEA)*, 29(1), 32–41. http://dx.doi.org/10.15421/142102.
Rohret, D., Kraft, M., & Vella, M. (2013). Functional Resilience, Functional Resonance and Threat Anticipation for Rapidly Developed Systems. *Journal of Information Warfare*, 12(2), 50–62. https://www.jstor.org/stable/26486855
Sobchuk, V. V., Olimpiyeva, Y. I., Musienko, A. P., & Sobchuk, A. V. (2021, December 2–3). Ensuring the properties of functional stability of manufacturing processes based on the application of neural networks. *CEUR Workshop Proceedings*, 2845, 106–116. https://ceur-ws.org/Vol-2845/Paper_11.pdf
Strakhov, E. M. (2013). Dynamic programming in structural and parametric optimization. *International Journal of Pure and Applied Mathematics*, 82(3), 503–512.
Valeyev, K. G., & Dzhalladova, I. A. (2002). Optimization of Nonlinear Systems of Stochastic Difference Equations. *Ukrainian Mathematical Journal*, 54(1), 3–14. https://umj.imath.kiev.ua/index.php/umj/article/view/4035
Yanai, H., Takeuchi, K., & Takane, Y. (2011). Projection Matrices. In *Projection Matrices, Generalized Inverse Matrices, and Singular Value Decomposition. Statistics for Social and Behavioral Sciences,* 25–54. Springer. https://doi.org/10.1007/978-1-4419-9887-3_2
Yevseiev, S., Ponomarenko, V., Laptiev, O., Milov, O., Korol, O., Milevskyi, S. et. al. (Eds.). (2021). *Synergy of building cybersecurity systems*. PC Technology Center. http://doi.org/10.15587/978-617-7319-31-2
Zamrii, I. V., Haidur, H. I., Sobchuk, A. V., Hryshanovych, T. A., Zinchenko, K. I. & Polovinkin, I. P. (2022, December 15–17). The Method of Increasing the Efficiency of Signal Processing Due to the Use of Harmonic Operators. *IEEE 4th International Conference on Advanced Trends in Information Theory (ATIT)* (pp. 138–141). Kyiv. https://doi.org/10.1109/ATIT58178.2022.10024212

Отримано редакцією журналу / Received: 10.04.24
Прорецензовано / Revised: 25.04.24
Схвалено до друку / Accepted: 20.05.24

Володимир ПІЧКУР[1], д-р фіз.-мат. наук, проф.
ORCID ID: 0000-0002-5641-8145
e-mail: volodymyr.pichkur@knu.ua

Валентин СОБЧУК[1], д-р техн. наук, проф.
ORCID ID: 0000-0002-4002-8206
e-mail: sobchuk@knu.ua

Дмитро ЧЕРНІЙ[1], д-р техн. наук, доц.
ORCID ID: 0000-0002-6378-8048
e-mail: dmytro.cherniy@knu.ua

Антон РИЖОВ[1], канд. фіз.-мат. наук, доц.
ORCID ID: 0000-0003-4099-0742
e-mail: ryzhov@knu.ua

[1]Київський національний університет імені Тараса Шевченка, Київ, Україна

ФУНКЦІОНАЛЬНА СТІЙКІСТЬ ВИРОБНИЧИХ ПРОЦЕСІВ ЯК ЗАДАЧА КЕРУВАННЯ ДИСКРЕТНИМИ СИСТЕМАМИ ЗІ ЗМІНОЮ РОЗМІРНОСТІ ВЕКТОРА СТАНУ

Запропоновано підхід до математичного моделювання технологічних процесів промислових підприємств для організації виробництва відповідно до встановлених стандартів з дотримання прийнятних допусків і вимог. Автори вперше розглядають властивість функціональної стійкості виробничих процесів у двох аспектах: як властивість системи зберігати свій функціональний стан в умовах змін і як властивість системи відновлювати свій функціональний стан після впливу зовнішніх і внутрішніх факторів (Functional Stability, Functional Resilience). Наведено математичну модель виробничих процесів у вигляді лінійних дискретних систем керування за умови, що вектор стану змінює розмірність. Ця умова показує, що параметри, які характеризують стан системи, можуть змінюватися на різних етапах виробничих процесів унаслідок технологічних особливостей. Це приводить до зміни розмірності вектора стану. Автори дають означення функціональної стійкості процесу, обґрунтовують теореми про умови функціональної стійкості та розв'язують задачу конструювання відповідного керування з використанням властивостей узагальнених обернених матриць

К л ю ч о в і с л о в а : ***дискретні системи, функціональна стійкість, математична модель, технологічний процес, конструювання керування, псевдообертання.***

Автори заявляють про відсутність конфлікту інтересів. Спонсори не брали участі в розробленні дослідження; у зборі, аналізі чи інтерпретації даних; у написанні рукопису; в рішенні про публікацію результатів.
The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses or interpretation of data; in the writing of the manuscript; in the decision to publish the results.