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І.М. Аскеров, проф., д.м.н.

Наближений метод розв'язання одної періодичної оптимальної регульованої крайової задачі

Ланкаранський державний університет,
пр-т Генерала Ази Асланова, 50, Ланкаран,
Азербайджанська Республіка, Az 4200
e-mail: idrakasgerov@gmail.com

I.M. Askerov, Dr.Sci.(Math.), accosiate professor

Approximate method of solving one periodic optimal regulated boundary value problem

Lankaran State University,
General Hazi Aslanov Ave., 50, Lankaran,
Azerbaijan Republic, Az 4200
e-mail: idrakasgerov@gmail.com

В представленій роботі розглянуто розв'язання одної періодичної оптимальної регульованої крайової задачі асимптотичним методом. Для розв'язання задачі з розширеним функціональним записом були знайдені крайові умови та рівняння Ейлера-Лагранжа. Підхід до розв'язання задачі в залежності від малого параметра шляхом відшукання системи нелінійних диференціальних рівнянь і розв'язку рівнянь Ейлера-Лагранжа, розв'язок загальної задачі при першому підході зводиться до розв'язку двох нелінійних алгебраїчних рівнянь.

Ключові слова: оптимальне регулювання, асимптотичний метод, періодичний стан.

In the present work we considered the solution of one periodic optimal regulated boundary value problem by the asymptotic method. For the solution of the problem with extended functional writing, boundary conditions and Euler-Lagrange equations were found. The approach to the solution of the problem depending on a small parameter by seeking a system of nonlinear differential equations and solving Euler-Lagrange equations, the solution of the general problem in the first approach comes down to solving two nonlinear algebraic equations.

Key words: optimal regulation, asymptotic method, periodic condition.

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I. INTRODUCTION

Many scientists have studied the asymptotic method of studying some regulated problem and a system of nonlinear differential equations. As examples, the article [1] shows the methods of asymptotic solution of nonlinear differential equations that describe some dances. But in [2], the solution of the system depending on a small parameter was studied after making certain changes to the system that describes the telegraph problem of hyperbolic equations with special derivatives, then differential equations with special derivatives depending on the small parameter were introduced into the system. Also in [6], the article have been investigated the issues of the optimal control of some nonlinear mechanical systems by the asymptotic method. The considered method is of great importance in solving applied problems [7].

In the problem under consideration when the equation of an object is represented by nonlinear differential equations depending on the small parametr, asymptotic solution of the optimal control problem is given. The problem was solved until the end and in the first approximation was found the solution, dependening on the small parametr.

II. A PROBLEM STATEMENT

Suppose that the equation of motion of an object is given in the following form with a system of nonlinear differential equations:

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$$\begin{aligned}
&\dot{y}(x,\ \varepsilon) = f^{1}(y(x),\varepsilon), \qquad 0 \le x \le l-0,\\
&\dot{y}(x,\ \varepsilon) = f^{2}(y(x),\varepsilon), \quad\ l+0 \le x \le 2l,\\
&y(0,\varepsilon) = u(\varepsilon).
\end{aligned} \tag{1}$$

(1) the system of equations communicate with each other at a point$l$ as follows:

$$y(l+0,\varepsilon) = \gamma^{T} y(l-0,\varepsilon) + \gamma_{1}^{T}(y(l-0,\varepsilon))\bar{y} \tag{2}$$

Suppose, in the (1) system, the functions $f^{1}(y(x),\varepsilon), \dfrac{\partial f^{1}}{\partial y}$ in part $0 \le x \le l-0$ and, functions $f^{2}(y(x),\varepsilon), \dfrac{\partial f^{2}}{\partial y}$ in part $l+0 \le x \le 2l$ do not intersect. In this work, the following condition is added as an additional:

$$y(l+0,\varepsilon) = y(2l,\ \varepsilon). \tag{3}$$

Here y - n dimensional vector, but u is n dimensional unknown vector (this u parametr may be accepted as a manager), $\bar{y}$-scalar, γ - $n \times n$ dimensional vector and T - symbol is a transponder.

For the problem under consideration, the question of optimal regulation will be in the following form. The problem consists of the fact that of the possible regulated ones to find such a regulated one $u(\varepsilon)$ that became a solution, which meets (2) and (3) the conditions (1) of the system and at the same time is the minimum number for the next functional.

$$\begin{aligned}
J = \frac{1}{2} y^{T}(l+0,\ \varepsilon)Ry(l+0,\varepsilon) + u^{T}(\varepsilon)\beta u(\varepsilon) +\\
+\int_{0}^{2l} y^{T}(x,\varepsilon)Q(x)y(x,\varepsilon)dx.
\end{aligned} \tag{4}$$

But here, R - $n \times n$ - dimensional symmetric matrix, and β -given nimber, but $Q(x)$ is a continuous symmetric matrix $n \times n$ - dimensional elements, considering to x.

III. SOLUTION OF THE PROBLEM

To solve the problem (1) - (4), let's write the corresponding extended functional:

$$\begin{aligned}
\bar{J} = J + \lambda^{T}(l+0)(\gamma^{T} y(l-0,\varepsilon) +\\
+\gamma_{1}^{T}(y(l-0,\varepsilon))\bar{y} - y(l+0,\varepsilon)) +\\
+\frac{\beta}{4l}\int_{0}^{2l} \lambda(x)(f(y(x),\varepsilon) - \dot{y}(x))dx +\\
+\delta(y(0,\varepsilon) - u(\varepsilon)).
\end{aligned} \tag{5}$$

Here from the extended functionality (5)

$$\begin{aligned}
\dot{\lambda}(x,\ \varepsilon) = -\left[\frac{\partial f(y(x),\varepsilon)}{\partial y(x,\ \varepsilon)}\right]^{T} \cdot \lambda(x,\varepsilon) -\\
-\frac{8l}{\beta} y(x,\ \varepsilon)Q(x)
\end{aligned} \tag{6}$$

we obtain the Euler-Lagrange equations and the following boundary conditions $\lambda(x,\varepsilon)$ corresponding to the attached multipliers:

$$\begin{aligned}
&\alpha y(l+0,\varepsilon) + \left(\frac{\beta}{4l} - 1\right)\lambda(l+0.\varepsilon) - \frac{\beta}{4l}\lambda(2l,\varepsilon) = 0,\\
&\gamma\lambda(l+0,\varepsilon) - \frac{\beta}{4l}\lambda(l-0.\varepsilon) +\\
&\qquad + \frac{\partial \gamma_{1}(y(l-0,\varepsilon))}{\partial y(l-0,\varepsilon)}\lambda(l+0,\varepsilon)\bar{y} = 0,\\
&\qquad u(\varepsilon) + \frac{\beta}{4l}\lambda(0,\varepsilon) + \delta = 0.
\end{aligned} \tag{7}$$

Here δ - constant number. In (1) system of equations functions f^{1} and f^{2} let`s have in general way with $f(y(x),\ \varepsilon)$. If we look at the issue at intervals $0 \le x \le l-0$ and while we look at $f^{1}(y(x),\ \varepsilon)$, $l+0 \le x \le 2l$ this interval, then we accept $f^{2}(y(x),\ \varepsilon)$. Now suppose that function $f(y(x),\ \varepsilon)$ has the following separation according to the small parametr.

$$f(y(x),\varepsilon)=f_0(y(x))+\varepsilon f_1(y(x))+\varepsilon^2 f_2(y(x))+\ldots$$

Here,

$$f_0(y(x))=f(y(x,\varepsilon))\Big|_{\varepsilon=0},$$

$$f_1(y(x))=\left.\frac{\partial f(y(x),\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=0},$$

$$f_2(y(x))=\left.\frac{\partial^2 f(y(x),\varepsilon)}{\partial\varepsilon^2}\right|_{\varepsilon=0}\quad\ldots$$

Therefore, we write the differences in the small parameter ε on the right-hand side of (1) the system and presenting them in (6) the equation, we look for solutions (1) and (6) to the equations in sequential order, as a result of solving the obtained system of equations from part $0\le x\le l-0$ and $l+0\le x\le 2l$, we obtain the following conditions:

$0<x<l-0$

$$\begin{bmatrix} y^{(0)}(l-0)\\ \lambda^{(0)}(l-0)\end{bmatrix}=L_1(y^{(0)}(0),\lambda^{(0)}(0)),\qquad(8)$$

$l+0<x<2l$

$$\begin{bmatrix} y^{(0)}(2l)\\ \lambda^{(0)}(2l)\end{bmatrix}=L_2(y^{(0)}(l+0),\lambda^{(0)}(l+0)),\qquad(9)$$

and

$0<x<l-0$

$$\begin{bmatrix} y^{(1)}(l-0)\\ \lambda^{(1)}(l-0)\end{bmatrix}=H_1(y^{(1)}(0),\lambda^{(1)}(0)),\qquad(10)$$

$l+0<x<2l$

$$\begin{bmatrix} y^{(1)}(2l)\\ \lambda^{(1)}(2l)\end{bmatrix}=L_2(y^{(1)}(l+0),\lambda^{(1)}(l+0)),\ldots\qquad(11)$$

Considering all the above, under conditions (2), (3) and (7) as a result of condition (2) we obtain

$$\begin{aligned} y^{(0)}(l+0)&=\gamma^T y^{(0)}(l-0)+\\ &+\gamma_1^T(y^{(0)}(l-0))\bar{y}\end{aligned}\qquad(12)$$

$$\begin{aligned} y^{(1)}(l+0)&=\gamma^T y^{(1)}(l-0)+\\ &\left.+\frac{\partial\gamma_1^T(y^1(l-0),\varepsilon)\bar{y}}{\partial\varepsilon}\gamma_1^T(y(l-0,\varepsilon))\bar{y}\right|_{\varepsilon=0}\quad\ldots\end{aligned}\qquad(13)$$

Conditions from conditions (3),

$$y^{(0)}(l+0)=y^{(0)}(2l),\qquad(14)$$

$$y^{(0)}(l+0)=y^{(0)}(2l)\ldots.\qquad(15)$$

Conditions, finally, from (7) conditions

$$\alpha y^{(0)}(l+0)+\left(\frac{\beta}{4l}-1\right)\lambda^{(0)}(l+0)-\frac{\beta}{4l}\lambda^{(0)}(2l)=0,$$

$$\begin{aligned}&\gamma\lambda^{(0)}(l+0)-\frac{\beta}{4l}\lambda^{(0)}(l-0)+\\ &+\frac{\partial\gamma_1(y^{(0)}(l-0))}{\partial y^{(0)}(l-0)}\lambda^{(0)}(l+0)\bar{y}=0\\ &u^{(0)}+\frac{\beta}{4l}\lambda^{(0)}(0)+\delta_0=0,\end{aligned}\qquad(16)$$

and

$$\alpha y^{(1)}(l+0)+\left(\frac{\beta}{4l}-1\right)\lambda^{(1)}(l+0)-\frac{\beta}{4l}\lambda^{(1)}(2l)=0$$

$$\begin{aligned}&\lambda^{(1)}(l+0)-\frac{\beta}{4l}\lambda^{(1)}(l-0)=0\\ &u^{(1)}+\frac{\beta}{4l}\lambda^{(1)}(0)+\delta_1=0\ldots\end{aligned}\qquad(17)$$

we get these conditions.

V. CONCLUSIONS

At the end, solving the obtained two nonlinear algebraic equations from (8), (9), (12), (14), (16) and (10), (11), (13), (15), (17) conditions, as a result of the first system, $y^{(0)}(0)=u^{(0)}$, of the second system $y^{(1)}(0)=u^{(1)}$ finding (1) - (4) problems in the first approach

$$u(\varepsilon)\approx u^{(0)}+\varepsilon u^{(1)}+\ldots \tag{18}$$

using the expression we get an approximate solution.

On the other hand, in the end, in order not to get a system of nonlinear algebraic equations, this can be achieved by linearizing equations (1), (2) and (8) - (11)

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