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KINETOSTATIC ANALYSIS OF ASSUR DYADS

The core of computer-aided design systems for design and calculation of plane mechanisms is the modular mathematical methods of studying the dynamics of the mechanisms. The most common approach is Multibody Dynamics, where a mechanism is divided into links that are assumed to be rigid bodies. The number of links is equal to the number of analysis modules. Another approach takes into consideration the structure of a mechanism, which can allow a reduction in the number of modules that need analysis. This approach necessitates the development of a plane mechanism classification and a comprehensive set of mathematical methods for analyzing the kinematics, kinetostatics, and dynamics of all possible structural elements. Of all the classification systems, Artobolevsky's one is the most aligned with the stated needs. Class II plane mechanisms according to the pointed classification have been fully covered by modular kinematic analysis algorithms, but kinetostatic analysis remains limited to a graph-analytical approach. The paper presents a unified analytical method of kinetostatic analysis of Assur dyads, which, along with class I mechanisms, constitute the structural elements of the class II mechanisms. The method is based on the vector equations of kinetostatics, solved using vector algebra tools. A full kinematic analysis must be performed beforehand. Active forces, inertial forces and reactions in kinematic pairs are represented as vectors, decomposed in the local bases rigidly attached to the dyad's links. The bases must be determined during the position analysis. The presented method is appropriate to use in combination with kinematic analysis, which also relies on vector algebra methods. It can be algorithmized and integrated into computer-aided design systems for class II plane mechanisms.

Keywords: ***modular approach, kinetostatic analysis, class II plane mechanisms, Assur dyads.***

AMS 2020 classification: 70-10, 70B15, 70E99.

Introduction

Modern computer-aided design systems for plane mechanisms are based on mathematical methods of mechanism analysis, using a modular approach. The most common approach, multibody dynamics, involves decomposion of mechanisms into links that adhere to the principles of rigid body dynamics or analytical mechanics (Nikravesh, 1988; Roberson & Schwertassek, 1988). A classic example of this method is the library "Modelica. Mechanics. MultiBody" in the Modelica programming language (Otter, Elmqvist, & Mattsson, 2003). Another approach considers the structural properties of the mechanism (Brát, & Lederer, 1973; Hansen, 1996; Simionescu, 2016). The advantage of the last one is the reduction in the number of modules, which requires a well-developed classification of mechanisms and a complete set of mathematical methods of kinematic, kinetostatic, and dynamic analyses of any possible structural element.

The objects of the research are Assur dyads that form the II class of structural groups according to Artobolevsky's classification (Artobolevski, 1977). Assur dyads, together with class I mechanisms, form a set of structural elements of class II mechanisms in accordance with Artobolevsky's classification. This is the only class of mechanisms with completely algorithmized kinematic analysis. Projection approaches solving vector contour equations that describe the position of structural groups are given in (Artobolevski, 1977; Kinytskyi, 2002). The method of coordinate planes can be found in (Honchar, 2011). The vector solution of the Chace equation (Chace, 1963) enables the algorithmization of Assur dyads kinematics using vector algebra tools (Khoroshev, Duchenko, & Kykot, 2023; 2024).

There is still no general analytical approach to kinetostatic analysis of class II structural groups; grapho-analytical is the only known one (Kinytskyi, 2002; Honchar, 2011). There are some separate investigations devoted to analytical methods of kinetostatic analysis of specific plane mechanisms or kinematic chains. For instance, the paper (Durango, Calle, & Ruiz, 2010) presents an analytical method of kinetostatic analysis of the RRR-type dyad with consideration of frictional forces. Additionally, see (Uicker, Pennock, & Shigley, 2016; Jomartov, & Tuleshov, 2018; Abhery, 2022).

The aim of the research is to develop a unified approach of kinetostatic analysis of Assur dyads, which can be implemented in computer-aided design systems of class II mechanisms according to Artobolevsky.

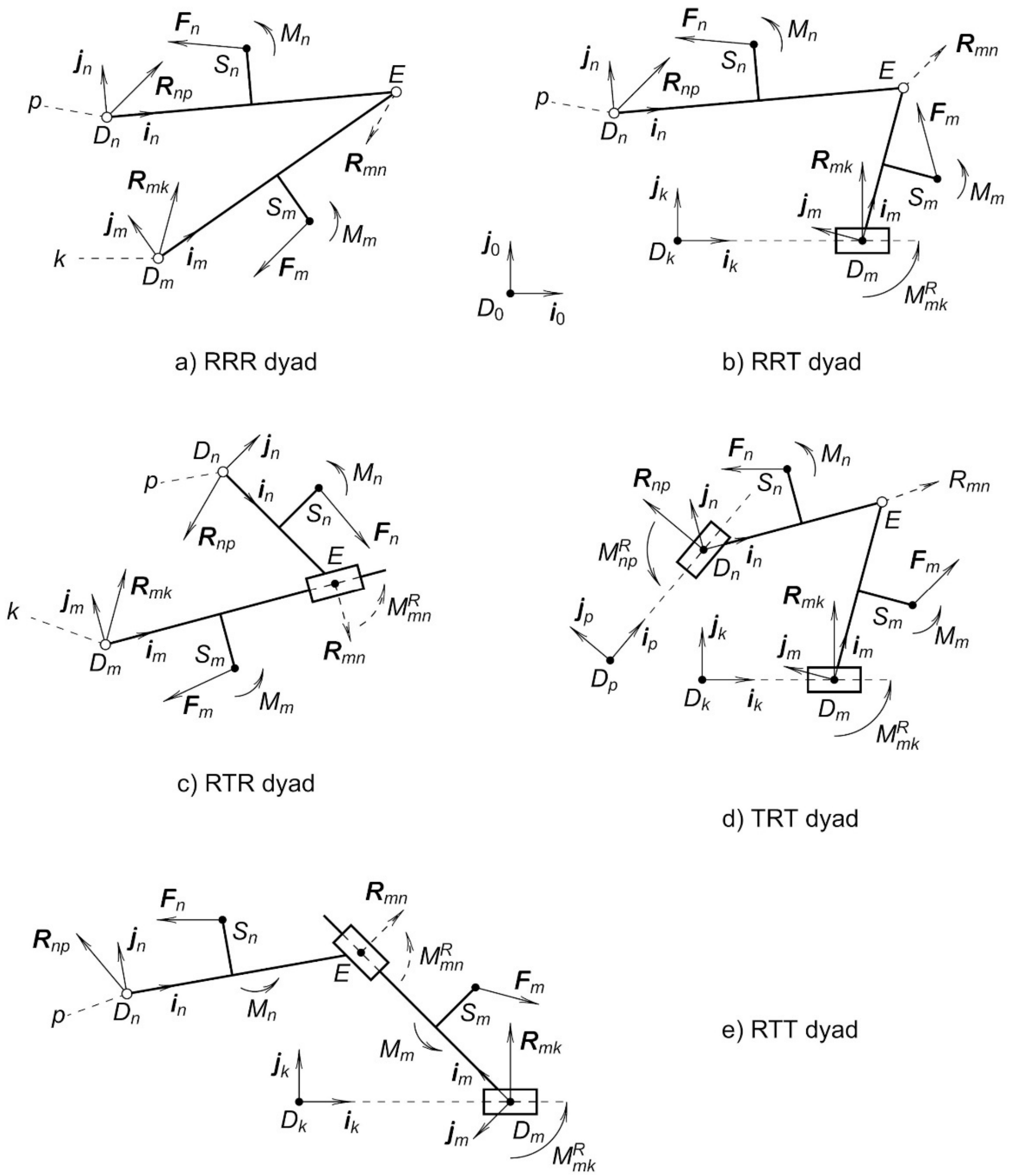
1. General statements

Consider a plane kinematic chain with zero degree of freedom consisting of two links D_mE, D_nE and three kinematic (rotational or translational) pairs D_m, E and D_n (Fig. 1). The kinematic pairs allow relative movements of the connected links in the plane of motion of the kinematic chain. Such a kinematic chain is called an Assur dyad and it is a class II structural group according to Artobolevsky's classification. Five types of dyads are distinguished (Artobolevski, 1977): RRR dyad – all pairs are rotational; RRT dyad – two pairs are rotational, one outer pair is translational; RTR dyad – two pairs are rotational, an inner one is translational; TRT dyad – two pairs are translational, one inner pair is rotational; RTT dyad – two pairs are translational, an outer one is rotational.

The numbers m and n are assigned to the links. Suppose that the considered kinematic chain is a part of a plane mechanism, kinematic pairs D_m and D_n are formed by the links of the group and external to the group links numbered k and p. Distances between kinematic pairs are $|D_mE| = l_m$ and $|D_nE| = l_n$. Points D_k and D_p are taken on links k and p. If pairs D_m and D_n are translational, points D_k and D_p should be chosen on the guidelines of the relative movement of the links connected in a kinematic pair. Assume $|D_kD_m| = l_k > 0$ and $|D_pD_n| = l_p > 0$.

The structural group is referred to a fixed rectangular coordinate system $D_0x_0y_0z_0$ with a right standard basis $\{\mathbf{i}_0;\ \mathbf{j}_0;\ \mathbf{k}_0\}$ in $\mathbb{R}^3$. Unit vectors $\mathbf{i}_0$ and $\mathbf{j}_0$ lie in the plane of the group. Consider local coordinate systems $D_mx_my_mz_m$, $D_nx_ny_nz_n$, $D_kx_ky_kz_k$ and $D_px_py_pz_p$ with bases $\{\mathbf{i}_m;\ \mathbf{j}_m = \mathbf{k}_m \times \mathbf{i}_m;\ \mathbf{k}_m = \mathbf{k}_0\}$, $\{\mathbf{i}_n;\ \mathbf{j}_n = \mathbf{k}_n \times \mathbf{i}_n;\ \mathbf{k}_n = \mathbf{k}_0\}$, $\{\mathbf{i}_k;\ \mathbf{j}_k = \mathbf{k}_k \times \mathbf{i}_k;\ \mathbf{k}_k = \mathbf{k}_0\}$, $\{\mathbf{i}_p;\ \mathbf{j}_p = \mathbf{k}_p \times \mathbf{i}_p;\ \mathbf{k}_p = \mathbf{k}_0\}$ connected with links m, n, k and p so, that unit vectors $\mathbf{i}_m$, $\mathbf{i}_n$, $\mathbf{i}_k$ and $\mathbf{i}_p$ lie on lines D_mE, D_nE, D_kD_m and D_pD_n correspondently. In Figure 1 unit vectors $\mathbf{k}_0$, $\mathbf{k}_m$, $\mathbf{k}_n$, $\mathbf{k}_k$ and $\mathbf{k}_p$ are indicated implicitly, the force diagrams of structural groups are presented in the planes of their motion.

The dyad is a statically determined mechanical system (Artobolevski, 1977), that is, it undergoes a kinetostatic analysis, the purpose of which is to determine the unknown forces, taking into account the inertia forces of the links. We will focus on the determination of unknown reactions in kinematic pairs.



a) RRR dyad

b) RRT dyad

c) RTR dyad

d) TRT dyad

e) RTT dyad

Fig. 1. Force diagrams of Assur dyads

Before compiling the equations of kinetostatics, it is assumed that a full kinematic analysis of the structural group was conducted in any way, so the positions, absolute velocities and accelerations of all points of the dyad, unit vectors $\mathbf{i}_m$, $\mathbf{i}_n$, absolute angular velocities ω_m , ω_n and angular accelerations ε_m , ε_n of all links of the group are known.

Assume γ is number of a link from the set $\{m,n\}$. The radius vector of the center of mass S_γ of the link γ, and its absolute acceleration $\mathbf{w}_{S\gamma}$ can be presented in a form

$$\mathbf{r}_{S\gamma} = \mathbf{r}_{D\gamma} + x_{S\gamma}\mathbf{i}_\gamma + y_{S\gamma}\mathbf{j}_\gamma, \quad \mathbf{w}_{S\gamma} = \mathbf{w}_{D\gamma} - \left(\omega_\gamma^2 x_{S\gamma} + \varepsilon_\gamma y_{S\gamma}\right)\mathbf{i}_\gamma - \left(\omega_\gamma^2 y_{S\gamma} - \varepsilon_\gamma x_{S\gamma}\right)\mathbf{j}_\gamma, \tag{1}$$

where $x_{S\gamma}$, $y_{S\gamma}$ are given coordinates of S_γ in coordinate systems $D_\gamma x_\gamma y_\gamma z_\gamma$; $\mathbf{r}_{D\gamma}$ and $\mathbf{w}_{D\gamma}$ are known radius vectors and absolute accelerations of points D_γ .

Using equation (1), for resultant vector $\mathbf{\Phi}_\gamma$ and resultant moment $\mathbf{M}_\gamma^\Phi$ of inertia forces of link γ in basis $\{\mathbf{i}_\gamma;\ \mathbf{j}_\gamma;\ \mathbf{k}_\gamma\}$ we write

$$\mathbf{\Phi}_\gamma = -m_\gamma\left(\left(\mathbf{w}_{D\gamma}\cdot\mathbf{i}_\gamma - \omega_\gamma^2 x_{S\gamma} - \varepsilon_\gamma y_{S\gamma}\right)\mathbf{i}_\gamma + \left(\mathbf{w}_{D\gamma}\cdot\mathbf{j}_\gamma - \omega_\gamma^2 y_{S\gamma} + \varepsilon_\gamma x_{S\gamma}\right)\mathbf{j}_\gamma\right), \quad \mathbf{M}_\gamma^\Phi = -J_\gamma\varepsilon_\gamma\mathbf{k}_\gamma. \tag{2}$$

Here m_γ and J_γ are the mass of link γ and its moment of inertia relative to the axis, passing through the center of mass of the link perpendicular to the plane of the structural group.

Assume that link γ undergoes a known active load in the plane of the group, which is given by resultant force $\mathbf{F}_\gamma$ at the center of mass S_γ of the link, and resultant moment $\mathbf{M}_\gamma$. Since the resultant vectors of active forces and inertial forces are known and applied to the same point, we will simplify the presentation by using symbols $\mathbf{F}_\gamma$ and $\mathbf{M}_\gamma$ to represent $\mathbf{F}_\gamma + \mathbf{\Phi}_\gamma$ and $\mathbf{M}_\gamma + \mathbf{M}_\gamma^\Phi$. It is obvious that $\mathbf{F}_\gamma \perp \mathbf{k}_\gamma$ and $\mathbf{M}_\gamma \parallel \mathbf{k}_\gamma$. The force diagrams presented in Figure 1 are made taking into account the proposed simplification. Moments are marked with their algebraic values.

All kinematic pairs are assumed to be free of any kind of friction. If a kinematic pairs in point D_γ is rotational, then the reactive forces arising from the external link can be written in the form of a uniform force $\mathbf{R}_{\gamma\delta} = \left(\mathbf{R}_{\gamma\delta}\cdot\mathbf{i}_\alpha\right)\mathbf{i}_\alpha + \left(\mathbf{R}_{\gamma\delta}\cdot\mathbf{j}_\alpha\right)\mathbf{j}_\alpha$, applied to point D_γ . Here $\delta = k$ for $\gamma = m$ and $\delta = p$ for $\gamma = n$; $\{\mathbf{i}_\alpha;\ \mathbf{j}_\alpha;\ \mathbf{k}_\alpha\}$ is an arbitrary from the described above bases; $\left(\mathbf{R}_{\gamma\delta}\cdot\mathbf{i}_\alpha\right)$, $\left(\mathbf{R}_{\gamma\delta}\cdot\mathbf{j}_\alpha\right)$ are the components of reaction $\mathbf{R}_{\gamma\delta}$ in this basis. In case of translational pair in point D_γ , reactive forces from the external link can be presented in the form of a resultant vector $\mathbf{R}_{\gamma\delta} = \left(\mathbf{R}_{\gamma\delta}\cdot\mathbf{j}_\beta\right)\mathbf{j}_\beta$ and resultant moment $\mathbf{M}_{\gamma\delta}^R = \left(\mathbf{M}_{\gamma\delta}^R\cdot\mathbf{k}_\beta\right)\mathbf{k}_\beta$, where $\{\mathbf{i}_\beta;\ \mathbf{j}_\beta;\ \mathbf{k}_\beta\}$ is the basis, which is related to the guideline of the relative movement of the links in the kinematic pair. Similar considerations and notations are performed for the representation of reactive forces in the internal pair in point E . Assume $\mathbf{R}_{mn}$, and in case it is needed $\mathbf{M}_{mn}^R$, are the vectors describing reactive forces in point E , acting on link m from the side of link n . Unknown reactions will be determined if the relations for calculation of the vectors or their components in any known basis are obtained.

2. The kinetostatic analysis of RRR dyad

Given: D_γ and E are rotational pairs, $l_\gamma > 0$, $\{\mathbf{i}_\gamma;\ \mathbf{j}_\gamma;\ \mathbf{k}_\gamma\}$, $\mathbf{F}_\gamma$, $\mathbf{M}_\gamma$ $(\gamma \in \{m,n\})$. Find: $\mathbf{R}_{mk}$, $\mathbf{R}_{np}$ and $\mathbf{R}_{mn}$.

The force diagram is presented in Figure 1(a). Consider resolution of vectors $\mathbf{R}_{mk}$ and $\mathbf{R}_{mk}$ in given bases $\{\mathbf{i}_m;\ \mathbf{j}_m;\ \mathbf{k}_m\}$ and $\{\mathbf{i}_n;\ \mathbf{j}_n;\ \mathbf{k}_n\}$. The moment equation for link γ with respect to point E can be expressed in the form

$$\sum \mathbf{M}_{E\gamma} = \left(-l_\gamma\mathbf{i}_\gamma\right)\times\mathbf{R}_{\gamma\delta} + \left(\left(x_{S\gamma} - l_\gamma\right)\mathbf{i}_\gamma + y_{S\gamma}\mathbf{j}_\gamma\right)\times\mathbf{F}_\gamma + \mathbf{M}_\gamma = 0. \tag{3}$$

After elementary transformations, we obtain

$$\mathbf{R}_{\gamma\delta}\cdot\mathbf{j}_\gamma = l_\gamma^{-1}\left\{\left(x_{S\gamma} - l_\gamma\right)\left(\mathbf{F}_\gamma\cdot\mathbf{j}_\gamma\right) - y_{S\gamma}\left(\mathbf{F}_\gamma\cdot\mathbf{i}_\gamma\right) + \left(\mathbf{M}_\gamma\cdot\mathbf{k}_\gamma\right)\right\}. \tag{4}$$

Equilibrium equation

$$\sum \mathbf{F}_{(m,n)} = \mathbf{R}_{mk} + \mathbf{F}_m + \mathbf{F}_n + \mathbf{R}_{np} = 0 \tag{5}$$

for the whole structural group can be rewritten as

$$\left(\mathbf{R}_{mk}\cdot\mathbf{i}_m\right)\mathbf{i}_m + \left\{\left(\mathbf{R}_{mk}\cdot\mathbf{j}_m\right)\mathbf{j}_m + \mathbf{F}_m + \mathbf{F}_n + \left(\mathbf{R}_{np}\cdot\mathbf{j}_n\right)\mathbf{j}_n\right\} + \left(\mathbf{R}_{np}\cdot\mathbf{i}_n\right)\mathbf{i}_n = 0. \tag{6}$$

In the case $\mathbf{i}_m\cdot\mathbf{j}_n \neq 0$, scalar multiplication of the last equation by either vector $\mathbf{j}_m$ or vector $\mathbf{j}_n$ leads to

$$\begin{aligned} \mathbf{R}_{mk}\cdot\mathbf{i}_m &= -\left(\mathbf{i}_m\cdot\mathbf{j}_n\right)^{-1}\left\{\left(\mathbf{R}_{mk}\cdot\mathbf{j}_m\right)\left(\mathbf{j}_m\cdot\mathbf{j}_n\right) + \mathbf{F}_m\cdot\mathbf{j}_n + \mathbf{F}_n\cdot\mathbf{j}_n + \mathbf{R}_{np}\cdot\mathbf{j}_n\right\}, \\ \mathbf{R}_{np}\cdot\mathbf{i}_n &= -\left(\mathbf{i}_n\cdot\mathbf{j}_m\right)^{-1}\left\{\left(\mathbf{R}_{np}\cdot\mathbf{j}_n\right)\left(\mathbf{j}_n\cdot\mathbf{j}_m\right) + \mathbf{F}_n\cdot\mathbf{j}_m + \mathbf{F}_m\cdot\mathbf{j}_m + \mathbf{R}_{mk}\cdot\mathbf{j}_m\right\}. \end{aligned} \tag{7}$$

Expressions (4) and (7) allow the calculation of all non-zero components of reactions $\mathbf{R}_{mk}$ and $\mathbf{R}_{np}$ in bases $\{\mathbf{i}_m;\ \mathbf{j}_m;\ \mathbf{k}_m\}$ and $\{\mathbf{i}_n;\ \mathbf{j}_n;\ \mathbf{k}_n\}$ respectively. From equilibrium equation for link m , we obtain the expression for determination of reaction $\mathbf{R}_{mn}$ in the inner pair in point E

$$\mathbf{R}_{mn} = -(\mathbf{R}_{mk} + \mathbf{F}_m). \tag{8}$$

3. The kinetostatic analysis of TRT dyad

Given: D_m and D_n are translational pairs, E is a rotational pair, $l_\gamma \geq 0$, $\{\mathbf{i}_\gamma;\ \mathbf{j}_\gamma;\ \mathbf{k}_\gamma\}$, $\mathbf{F}_\gamma$, $\mathbf{M}_\gamma$ ($\gamma \in \{m,n\}$, if $l_\gamma = 0$, then $\mathbf{i}_\gamma = \mathbf{i}_\delta$), $\{\mathbf{i}_k;\ \mathbf{j}_k;\ \mathbf{k}_k\}$, $\{\mathbf{i}_p;\ \mathbf{j}_p;\ \mathbf{k}_p\}$. Find: $\mathbf{R}_{mk}$, $\mathbf{M}^R_{mk}$, $\mathbf{R}_{np}$, $\mathbf{M}^R_{np}$ and $\mathbf{R}_{mn}$.

The force diagram is presented in Figure 1(d). Since kinematic pairs in D_γ are translational, then $\mathbf{R}_{\gamma\delta} \parallel \mathbf{j}_\delta$. Equilibrium equation (5) for the whole structural group can be expressed as

$$(\mathbf{R}_{mk}\cdot\mathbf{j}_k)\mathbf{j}_k + \{\mathbf{F}_m + \mathbf{F}_n\} + (\mathbf{R}_{np}\cdot\mathbf{j}_p)\mathbf{j}_p = 0. \tag{9}$$

If $\mathbf{i}_k\cdot\mathbf{j}_p \neq 0$, then, after scalar multiplication of equation (9) by either vector $\mathbf{i}_k$ or vector $\mathbf{i}_p$, we obtain

$$\mathbf{R}_{mk}\cdot\mathbf{j}_k = -(\mathbf{j}_k\cdot\mathbf{i}_p)^{-1}\{\mathbf{F}_m\cdot\mathbf{i}_p + \mathbf{F}_n\cdot\mathbf{i}_p\},\quad \mathbf{R}_{np}\cdot\mathbf{j}_p = -(\mathbf{j}_p\cdot\mathbf{i}_k)^{-1}\{\mathbf{F}_m\cdot\mathbf{i}_k + \mathbf{F}_n\cdot\mathbf{i}_k\}. \tag{10}$$

Reactions $\mathbf{R}_{\gamma\delta}$ are therefore determined. To obtain relations for reactive moments $\mathbf{M}^R_{\gamma\delta}$ moment equation for link γ with respect to point E can be written. Finally, we get

$$\mathbf{M}^R_{\gamma\delta} = l_\gamma(\mathbf{i}_\gamma\times\mathbf{R}_{\gamma\delta}) + ((l_\gamma - x_{S\gamma})\mathbf{i}_\gamma - y_{S\gamma}\mathbf{j}_\gamma)\times\mathbf{F}_\gamma - \mathbf{M}_\gamma. \tag{11}$$

Reaction $\mathbf{R}_{mn}$, as in the previous case, can be calculated using formula (8).

4. The kinetostatic analysis of RRT dyad

Given: D_n and E are rotational pairs, D_m is a translational pair, $l_n > 0$, $l_m \geq 0$, $\{\mathbf{i}_\gamma;\ \mathbf{j}_\gamma;\ \mathbf{k}_\gamma\}$, $\mathbf{F}_\gamma$, $\mathbf{M}_\gamma$ $(\gamma\in\{m,n\})$, $\{\mathbf{i}_k;\ \mathbf{j}_k;\ \mathbf{k}_k\}$. If $l_m = 0$, then $\mathbf{i}_m = \mathbf{i}_k$. Find: $\mathbf{R}_{mk}$, $\mathbf{M}^R_{mk}$, $\mathbf{R}_{np}$ and $\mathbf{R}_{mn}$.

The force diagram is presented in Figure 1(b). The component $\mathbf{R}_{np}\cdot\mathbf{j}_n$ can be calculated by formula (4). Equilibrium equation (5) for the whole structural group is presented as

$$(\mathbf{R}_{mk}\cdot\mathbf{j}_k)\mathbf{j}_k + \{\mathbf{F}_m + \mathbf{F}_n + (\mathbf{R}_{np}\cdot\mathbf{j}_n)\mathbf{j}_n\} + (\mathbf{R}_{np}\cdot\mathbf{i}_n)\mathbf{i}_n = 0 \tag{12}$$

and, after scalar multiplication by either vector $\mathbf{i}_k$ or $\mathbf{j}_n$, for $\mathbf{i}_n\cdot\mathbf{i}_k \neq 0$, it transforms into

$$\begin{aligned}\mathbf{R}_{mk}\cdot\mathbf{j}_k &= -(\mathbf{j}_k\cdot\mathbf{j}_n)^{-1}\{\mathbf{F}_m\cdot\mathbf{j}_n + \mathbf{F}_n\cdot\mathbf{j}_n + \mathbf{R}_{np}\cdot\mathbf{j}_n\},\\ \mathbf{R}_{np}\cdot\mathbf{i}_n &= -(\mathbf{i}_n\cdot\mathbf{i}_k)^{-1}\{(\mathbf{R}_{np}\cdot\mathbf{j}_n)(\mathbf{j}_n\cdot\mathbf{i}_k) + \mathbf{F}_n\cdot\mathbf{i}_k + \mathbf{F}_m\cdot\mathbf{i}_k\}.\end{aligned} \tag{13}$$

Thus the relations for components of reactions $\mathbf{R}_{mk}$ and $\mathbf{R}_{np}$ in bases $\{\mathbf{i}_k;\ \mathbf{j}_k;\ \mathbf{k}_k\}$ and $\{\mathbf{i}_n;\ \mathbf{j}_n;\ \mathbf{k}_n\}$ are founded. Vectors $\mathbf{R}_{mn}$ and $\mathbf{M}^R_{mk}$ are calculated using formulas (8) and (11) respectively.

5. The kinetostatic analysis of RTR dyad

Given: D_m and D_n are rotational pairs, E is a translational pair, $l_n \geq 0$, $l_m > 0$, $\{\mathbf{i}_\gamma;\ \mathbf{j}_\gamma;\ \mathbf{k}_\gamma\}$, $\mathbf{F}_\gamma$, $\mathbf{M}_\gamma$ $(\gamma\in\{m,n\})$. If $l_n = 0$, then $\mathbf{i}_n$ equals to $\mathbf{i}_m$. Find: $\mathbf{R}_{mk}$, $\mathbf{R}_{np}$, $\mathbf{R}_{mn}$ and $\mathbf{M}^R_{mn}$.

The force diagram is presented in Figure 1(c). Assume that points D_m and D_n do not coincide and introduce new basis $\{\mathbf{i}_\beta;\ \mathbf{j}_\beta = \mathbf{k}_\beta\times\mathbf{i}_\beta;\ \mathbf{k}_\beta = \mathbf{k}_0\}$ with unit vector $\mathbf{i}_\beta = |D_mD_n|^{-1}(l_m\mathbf{i}_m - l_n\mathbf{i}_n)$ lying on the guideline D_mD_n. Here $|D_mD_n| = \sqrt{l_m^2 + l_n^2 - 2l_ml_n(\mathbf{i}_m\cdot\mathbf{i}_n)}$.

The moment equation for the whole dyad with respect to point D_m is

$$\begin{aligned}\sum\mathbf{M}_{Dm(m,n)} &= |D_mD_n|\mathbf{i}_\beta\times\mathbf{R}_{np} + (|D_mD_n|\mathbf{i}_\beta + x_{Sn}\mathbf{i}_n + y_{Sn}\mathbf{j}_n)\times\mathbf{F}_n + \mathbf{M}_n +\\ &+ (x_{Sm}\mathbf{i}_m + y_{Sm}\mathbf{j}_m)\times\mathbf{F}_m + \mathbf{M}_m = 0.\end{aligned} \tag{14}$$

After elementary transformations, for component $\mathbf{R}_{np}\cdot\mathbf{j}_\beta$ it is obtained

$$\mathbf{R}_{np}\cdot\mathbf{j}_\beta = -(\mathbf{F}_n\cdot\mathbf{j}_\beta) - |D_mD_n|^{-1}\sum_{\gamma=m,n}\{x_{S\gamma}(\mathbf{F}_\gamma\cdot\mathbf{j}_\gamma) - y_{S\gamma}(\mathbf{F}_\gamma\cdot\mathbf{i}_\gamma) + (\mathbf{M}_\gamma\cdot\mathbf{k}_\beta)\}. \tag{15}$$

Equilibrium equation for link n can be presented in the form

$$(\mathbf{R}_{np}\cdot\mathbf{i}_\beta)\mathbf{i}_\beta + \{(\mathbf{R}_{np}\cdot\mathbf{j}_\beta)\mathbf{j}_\beta + \mathbf{F}_n\} - (\mathbf{R}_{mn}\cdot\mathbf{j}_m)\mathbf{j}_m = 0, \tag{16}$$

which for $\mathbf{i}_\beta\cdot\mathbf{i}_m \neq 0$ implies

$$\begin{aligned}\mathbf{R}_{np}\cdot\mathbf{i}_\beta &= -(\mathbf{i}_\beta\cdot\mathbf{i}_m)^{-1}\{(\mathbf{R}_{np}\cdot\mathbf{j}_\beta)(\mathbf{j}_\beta\cdot\mathbf{i}_m) + \mathbf{F}_n\cdot\mathbf{i}_m\},\\ \mathbf{R}_{mn}\cdot\mathbf{j}_m &= (\mathbf{j}_m\cdot\mathbf{j}_\beta)^{-1}\{(\mathbf{R}_{np}\cdot\mathbf{j}_\beta) + \mathbf{F}_n\cdot\mathbf{j}_\beta\}.\end{aligned} \tag{17}$$

Relations for components of vectors $\mathbf{R}_{mn}$ and $\mathbf{R}_{np}$ in basis $\{\mathbf{i}_m;\ \mathbf{j}_m;\ \mathbf{k}_m\}$ and $\{\mathbf{i}_\beta;\ \mathbf{j}_\beta;\ \mathbf{k}_\beta\}$ are therefore found. Applying equilibrium and moment equation with respect to D_m for link m, force $\mathbf{R}_{mk}$ and moment $\mathbf{M}^R_{mn}$ are determined as

$$\mathbf{R}_{mk} = -(\mathbf{R}_{mn} + \mathbf{F}_m), \tag{18}$$

$$\mathbf{M}_{mn}^{R} = -l_m\left(\mathbf{R}_{mn}\cdot\mathbf{j}_m\right)\mathbf{k}_m - \left(x_{Sm}\mathbf{i}_m + y_{Sm}\mathbf{j}_m\right)\times\mathbf{F}_m - \mathbf{M}_m.$$

6. The kinetostatic analysis of RTT dyad

Given: D_m and E are translational pairs, D_n is rotational pair, $l_n \geq 0$, $l_m > 0$, $\{\mathbf{i}_\gamma; \mathbf{j}_\gamma; \mathbf{k}_\gamma\}$, $\mathbf{F}_\gamma$, $\mathbf{M}_\gamma$ $\left(\gamma\in\{m,n\}\right)$. If $l_n = 0$, then $\mathbf{i}_n$ equals to $\mathbf{i}_m$. Find: $\mathbf{R}_{mk}$, $\mathbf{M}_{mk}^{R}$, $\mathbf{R}_{np}$, $\mathbf{R}_{mn}$ and $\mathbf{M}_{mn}^{R}$.

The force diagram is presented in Figure 1(e). Equilibrium equation for link m can be written as

$$\left(\mathbf{R}_{mk}\cdot\mathbf{j}_k\right)\mathbf{j}_k + \mathbf{F}_m + \left(\mathbf{R}_{mn}\cdot\mathbf{j}_m\right)\mathbf{j}_m = 0, \tag{19}$$

and, after scalar multiplication by either vector $\mathbf{i}_k$ or $\mathbf{i}_m$, for $\mathbf{j}_k\cdot\mathbf{i}_m \neq 0$ leads to

$$\left(\mathbf{R}_{mk}\cdot\mathbf{j}_k\right) = -\left(\mathbf{j}_k\cdot\mathbf{i}_m\right)^{-1}\left(\mathbf{F}_m\cdot\mathbf{i}_m\right), \quad \left(\mathbf{R}_{mn}\cdot\mathbf{j}_m\right) = -\left(\mathbf{j}_m\cdot\mathbf{i}_k\right)^{-1}\left(\mathbf{F}_m\cdot\mathbf{i}_k\right). \tag{20}$$

Forces $\mathbf{R}_{mn}$ and $\mathbf{R}_{mk}$ can therefore be calculated. From equilibrium equation (5) for the whole structural group we find

$$\mathbf{R}_{np} = -\left(\mathbf{R}_{mk} + \mathbf{F}_m + \mathbf{F}_n\right). \tag{21}$$

Using the moment equations with respect to point E for link n and then for link m, for moments $\mathbf{M}_{mn}^{R}$ and $\mathbf{M}_{mk}^{R}$ we obtain

$$\begin{gathered}\mathbf{M}_{mn}^{R} = -\mathbf{M}_{nm}^{R} = -l_n\mathbf{i}_n\times\mathbf{R}_{np} + \left(\left(x_{Sn} - l_n\right)\mathbf{i}_n + y_{Sn}\mathbf{j}_n\right)\times\mathbf{F}_n + \mathbf{M}_n, \\ \mathbf{M}_{mk}^{R} = l_m\mathbf{i}_m\times\mathbf{R}_{mk} - \left(\left(x_{Sm} - l_m\right)\mathbf{i}_m + y_{Sm}\mathbf{j}_m\right)\times\mathbf{F}_m - \mathbf{M}_m - \mathbf{M}_{mn}^{R}.\end{gathered} \tag{22}$$

Discussion and conclusions

A unified approach to the kinetostatic analysis of Assur dyads is presented, which allows to conduct the kinetostatic analysis of any class II plane mechanism according to Artobolevsky's classification. The method is based on the solution of the vector equations of kinetostatics by vector algebra tools. Representations of active forces, inertial forces, and reactions in kinematic pairs are introduced in the form of decompositions in the local bases. These are rigidly attached to the dyad's links determined within the position analysis. The suggested method is appropriate to apply in combination with positional and kinematic analysis of Assur dyads, which are also based on vector algebra methods (Khoroshev, Duchenko, & Kykot, 2023; 2024). The obtained calculation schemes are convenient for algorithmization and can be an integral part of computer-aided design systems for class II plane mechanisms according to Artobolevsky's classification.

Authors' contribution: Kostiantyn Khoroshev – conceptualization, formal analysis, methodology, writing – original draft; Kyryl Duchenko – formal analysis, writing – original draft; Serhii Kykot – formal analysis.

References

Abhery, K. (2022). An analytical parametric method for force analysis of planar mechanisms with sliding frictions. *International Journal of Mechanical Engineering Education*, 50(3), 667–691. https://doi.org/10.1177/03064190211035186

Artobolevski, I. (1977). *Théorie Des Mécanismes Et Des Machines*. Éditions Mir.

Brát, V., & Lederer, P. (1973). KIDYAN: Computer-aided kinematic and dynamic analysis of planar mechanisms. *Mechanism and Machine Theory*, 8(4), 457–467. https://doi.org/10.1016/0094-114X(73)90020-7

Chace, M. A. (1963). Vector analysis of linkages. *Journal of Manufacturing Science and Engineering, Transactions of the ASME*, 85(3), 289–297. https://doi.org/10.1115/1.3669867

Durango, S., Calle, G., & Ruiz, O. (2010). Analytical method for the kinetostatic analysis of the second-class RRR assur group allowing for friction in the kinematic pairs. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 32(3), 200–207. https://doi.org/10.1590/S1678-58782010000300002

Hansen, M. R. (1996). A general method for analysis of planar mechanisms using a modular approach. *Mechanism and Machine Theory*, 31(8), 1155–1166. https://doi.org/10.1016/0094-114X(96)84606-4

Honchar, M. O. (2011). *Theory of mechanisms and machines*. Vydavnychyi dim Vinnychenko [in Ukrainian]. [Гончар, М. О. (2011). Теорія механізмів і машин. Видавничий дім Винниченка].

Jomartov, A., & Tuleshov, A. (2018). Vector method for kinetostatic analysis of planar linkages. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 40(2), 56. https://doi.org/10.1007/s40430-018-1022-y

Khoroshev, K., Duchenko, K., & Kykot, S. (2024). The direct vector approach to the position analysis of Assur dyads. *Mechanism and Machine Theory*, 195, 105608. https://doi.org/10.1016/j.mechmachtheory.2024.105608

Khoroshev, K. G., Duchenko, K. O., & Kykot, S. V. (2023). The vector algebra approach to the kinematic analysis of the structural groups of the 2nd class by Artobolevsky. *Bulletin of Taras Shevchenko National University of Kyiv. Physical and Mathematical Sciences*, 2, 160–163. https://doi.org/10.17721/1812-5409.2023/2.28

Kinytskyi, Ya. T. (2002). *Theory of mechanisms and machines*. Naukova dumka [in Ukrainian]. [Кіницький, Я. Т. (2002). Теорія механізмів і машин. Наукова думка].

Nikravesh, P. E. (1988). *Computer-Aided Analysis of Mechanical Systems*. Prentice Hall.

Otter, M., Elmqvist, H., & Mattsson, S. E. (2003, November 3-4). The New Modelica MultiBody Library. In P. Fritzson (Ed.), Proceedings of the 3rd International Modelica Conference (pp. 311–330). Linköpings universitet. https://modelica.org/events/Conference2003/papers/h37_Otter_multibody.pdf

Roberson, R. E., & Schwertassek, R. (1988). *Dynamics of Multibody Systems*. Springer Verlag.

Simionescu, P. A. (2016, August 21–24). MEKIN2D: Suite for planar mechanism kinematics. *Proceedings of the ASME Design Engineering Technical Conference*, 5B-2016. Charlotte. https://doi.org/10.1115/DETC2016-59086

Uicker, J. J., Pennock, G. R., & Shigley, J. E. (2016). *Theory of Machines and Mechanisms* (5th ed.). Oxford University PressCovey.

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КІНЕТОСТАТИЧНИЙ АНАЛІЗ ДІАД АССУРА

Ядром комп'ютерних систем автоматизованого проєктування та розрахунку плоских механізмів є математичні методи модульного характеру дослідження динаміки механізмів. Найпоширеніший підхід – динаміка багатокомпонентних механічних систем, у якому механізм розбивають на ланки, що являють собою абсолютно тверді тіла. Кількість ланок дорівнює кількості модулів аналізу. Інший підхідураховує структуру механізму, що може сприяти зменшенню кількості модулів, які потребують аналізу. Однак для такого підходу необхідний розвиток системи класифікації механізмів і повного набору математичних методів кінематичного, кінетостатичного та динамічного аналізу всіх можливих структурних елементів. Найрозвиненішою в цьому сенсі є класифікація за Артоболевським. Сукупність плоских механізмів, що формують найпростіший II клас за Артоболевським, повністю забезпечена алгоритмізованими підходами модульного кінематичного аналізу. Ці методи аналітичні та підлягають алгоритмізації. Кінетостатичний аналіз для цього класу досі представлений тільки графо-аналітичним підходом. У пропонованій роботі подано уніфікований аналітичний метод кінетостатичного аналізу діад Ассура, що разом з механізмами I класу формують множину структурних елементів механізмів II класу за Артоболевським. В основу методу покладено векторні рівняння кінетостатики, що розв'язуються за допомогою математичного апарату векторної алгебри. Уведено представлення активних сил, сил інерції та реакцій у кінематичних парах у вигляді векторів, що розкладені за відомими з аналізу положень локальними базисами, жорстко зв'язаними з ланками структурних груп. Запропонований підхід рекомендуємо використовувати в поєднанні з кінематичним аналізом, що також базується на інструментарії векторної алгебри, а також він може бути алгоритмізований й утворювати невід'ємну частину систем автоматизованого проєктування плоских механізмів II класу за Артоболевським.

К л ю ч о в і с л о в а : ***модульний підхід, кінетостатика, плоскі механізми, діади Ассура.***

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