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CONTINUOUS TRANSFORMATIONS PRESERVING TAILS OF NEGABINARY REPRESENTATIONS OF NUMBERS FROM UNIT INTERVAL

For the negabinary representation of numbers from the $[0;1]$ interval, we consider continuous transformations of this interval (i.e., bijective mappings of the set onto itself) that preserve the tails of number representations. These transformations are constructed using left and right shift operators. It is proven that the set of such transformations, under the operation of composition, forms a non-commutative group, which contains a continuous subgroup of increasing transformations.

K e y w o r d s: ***Negabinary representation of numbers, cylinder, left and right shift operators, tail set, continuous transformations of the interval, group of interval transformations.***

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Introduction

The negabinary numeral system has long been used in electronics and programming (Rao, Rao, & Krishnamurthy, 1974; Grover et al., 1983; Cherri, & Kamal, 2004). The negabinary representation of numbers in the interval $[0;1]$:

$$x = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{\alpha_n}{(-2)^n} = \frac{2}{3} - \left(\frac{\alpha_1}{2} - \frac{\alpha_2}{2^2} + \frac{\alpha_3}{2^3} - \cdots\right) \equiv \Delta^{-2}_{\alpha_1\alpha_2\alpha_3\ldots}$$

is metrically equivalent to the classical binary representation (Pratsiovytyi, 2017). Moreover, it is topologically equivalent to the continued fraction A_2 representation of numbers (Pratsiovytyi, 2022). These are the examples of two-symbol encoding, which has multiple applications in number metric theory, probability theory, dynamical systems theory, fractal analysis, etc (Galambos, 2006; Schweiger, 1995; Albeverio, Pratsiovytyi, & Torbin, 2004). The efficiency of using the negabinary representation has been convincingly demonstrated multiple times in applications within the constructive theory of locally complex functions (Pratsiovytyi, 2022).

It is known that the group of continuous transformations of the unit interval (bijections of the interval onto itself) contains many interesting subgroups, including those related to different coding systems for real numbers and fractals (Pratsiovytyi, 2022). The cardinality of such subgroups remained mysterious for some time. In works (Osaulenko, 2016; Pratsiovytyi, Lysenko, & Maslova, 2020; Isaieva, & Pratsiovytyi, 2016; Pratsiovytyi, Chuikov, & Skrypnyk, 2019), only their infinitude was established. However, in (Pratsiovytyi, Lysenko, & Ratushniak, 2024), the continuity (cardinality of the continuum) of the group of transformations preserving the tails of Q_2-representations was proven. In this paper, we obtain analogous results for transformations that preserve the tails of the negabinary representation of numbers.

1. Negabinary representation of an integer

The negabinary representation of an integer x is (Pratsiovytyi, Goncharenko, & Lysenko, 2015; Kasatkin, 1982) its expression in the form

$$\alpha_1 \cdot (-2)^0 + \alpha_2 \cdot (-2)^1 + \cdots + \alpha_n \cdot (-2)^{n-1} + \cdots \equiv (\alpha_1\alpha_2\cdots\alpha_n\cdots)_{-2},$$

where α_i take values of 0 or 1. For any integer x, there exists a unique sequence $(\alpha_1, \ldots, \alpha_{2n})$ such that

$$x = 1 \cdot (-2)^{2n} + \alpha_{2n} \cdot (-2)^{2n-1} + \cdots + \alpha_1 \cdot (-2)^0.$$

In this representation, the negabinary encoding of any natural number always contains an odd number of digits.

Example. : $9_{10} = 11001_{-2}$, $-3_{10} = 1101_{-2}$

Let $A \equiv \{0,1\}$ be the alphabet of the binary system, and let $L \equiv A \times A \times \cdots$ denote the space of sequences of elements from this alphabet (the space of sequences consisting of 0 and 1). It is known (Pratsiovytyi, 2022) that for any $x \in [0,1]$, there exists a sequence $(\alpha_n) \in L$ such that

$$x = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{\alpha_n}{(-2)^n} = \frac{2}{3} - \left(\frac{\alpha_1}{2} - \frac{\alpha_2}{2^2} + \frac{\alpha_3}{2^3} - \cdots\right) \equiv \Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}. \tag{1}$$

The representation of x by the series (1) is called the negabinary representation, and the symbolic notation $\Delta^{-2}_{\alpha_1\alpha_2\alpha_3\ldots\alpha_n\ldots}$ is referred to as the negabinary encoding.

Remark 1. In (Pratsiovytyi, 2017) the relationship between the negabinary and the classic binary representation of the same number $x = \Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots} = \Delta^{2}_{\omega_1\omega_2\ldots\omega_n\ldots}$ is given by the formulas:

$$\alpha_n = \begin{cases} 1 - \omega_n & \text{if } n \text{ is odd}, \\ \omega_n & \text{if } n \text{ is even}. \end{cases}$$

Thus,

$$\Delta^{2}_{\omega_1\omega_2\omega_3\ldots} = \Delta^{-2}_{[1-\omega_1]\omega_2[1-\omega_3]\ldots}.$$

Lemma 1. *(Pratsiovytyi, Goncharenko, & Lysenko, 2015) Numbers x and x' that have the following negabinary representations:*

$$x = \Delta^{-2}_{c_1\ldots c_n 0(01)} \quad \text{and} \quad x' = \Delta^{-2}_{c_1\ldots c_n 1(10)}$$

are equal.

Numbers that have exactly two negabinary representations are called (Pratsiovytyi, Goncharenko, & Lysenko, 2015) negabinary-rational (NDR) numbers. The set of negabinary-rational numbers forms a dense subset of the rational numbers in $[0,1]$, but not every rational number is negabinary-rational. The set of rational numbers that are not negabinary-rational is also dense in $[0,1]$. Numbers from the interval $[0,1]$ that are not negabinary-rational are called negabinary-irrational. Each digit in the negabinary representation of a negabinary-irrational number is a correctly defined function of the represented number.

A cylinder set of rank n with base $\alpha_1\alpha_2\cdots\alpha_n$, corresponding to the negabinary representation of numbers in the interval $[0;1]$, is defined (Pratsiovytyi, 2017) as the set

$$\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n} = \left\{x = \Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\beta_1\beta_2\ldots}, (\beta_n) \in L\right\}.$$

The cylinder set is given by the interval

$$\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n} = [u - v; u + w],$$

where

$$u = \frac{2}{3} + \sum_{i=1}^{n} \frac{\alpha_i}{(-2)^i},$$

and

$$v = \begin{cases} \frac{1}{3\cdot 2^n}, & \text{if } n \text{ is odd,} \\ \frac{1}{3\cdot 2^{n-1}}, & \text{if } n \text{ is even;} \end{cases} \qquad w = \begin{cases} \frac{1}{3\cdot 2^{n-1}}, & \text{if } n \text{ is odd,} \\ \frac{1}{3\cdot 2^n}, & \text{if } n \text{ is even.} \end{cases}$$

2. Special functions of the negabinary representation of the fractional part of a real number

The left shift operator for negabinary digit sequences is defined as:

$$\eta(\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}) = \Delta^{-2}_{\alpha_2\alpha_3\ldots\alpha_n\ldots}$$

This operator is well-defined only when a convention is adopted to use one of the two representations of negabinary-rational (NDR) numbers (e.g., the representation with period (01)). The analytical expression of the left shift operator is:

$$\eta(x) = -2x + 2 - \alpha_1(x);$$

$$\eta(x) = \begin{cases} -2x + 2, & \text{if } x \in \Delta^{-2}_0, \\ -2x + 1, & \text{if } x \in \Delta^{-2}_1. \end{cases}$$

The operator is linear on each first-rank cylinder.

The right shift operator with parameter i, where $i \in \{0,1\}$, is defined as:

$$\tau_i(x = \Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}) = \Delta^{-2}_{i\alpha_1\alpha_2\ldots\alpha_n\ldots}.$$

The function has the following form:

$$\tau_i(x) = \begin{cases} -\frac{x}{2} + 1, & \text{if } i = 1, \\ -\frac{x}{2} + \frac{1}{2}, & \text{if } i = 0. \end{cases}$$

The n-multiple left shift operator for negabinary representation is defined as:

$$\eta^n(\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}) = \eta(\eta^{n-1}(\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots})) = \Delta^{-2}_{\alpha_{n+1}\alpha_{n+2}\ldots}.$$

The analytical expression is:

$$\eta^n(x) = (-2)^n \cdot x + a,$$

where

$$a = 2 - 2^{n-1}\cdot\alpha_1 + 2^{n-2}\cdot\alpha_2 + \cdots + (-1)^n\cdot\alpha_n.$$

The n-multiple right shift operator with the parameter set $(i_1, i_2, \ldots, i_n)$ is defined as:

$$\tau^n_{i_1i_2\ldots i_n}(\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}) = \tau_{i_1}(\tau^{n-1}_{i_2\ldots i_n}(\Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots})) = \Delta^{-2}_{i_1i_2\ldots i_n\alpha_1\alpha_2\ldots\alpha_n\ldots}.$$

The analytical expression is:

$$\tau^n_{i_1 i_2 \dots i_n}(x) = \frac{x}{(-2)^n} + a,$$

where

$$a = \frac{2}{3} - \frac{2}{3}\left(\frac{1}{-2}\right)^n - \frac{i_1}{2} + \frac{i_2}{4} + \dots + \frac{i_n}{(-2)^n}.$$

3. Continuous transformations of the interval $[0;1]$ that preserve tails

Recall that a transformation of a set is a bijective mapping (i.e., both injective and surjective) of that set onto itself. A continuous transformation of the interval $[0;1]$ is a continuous increasing or decreasing function such that either $f(0)=0$, $f(1)=1$, or $f(0)=1$, $f(1)=0$.

The negabinary representations of numbers $\Delta^{-2}_{\alpha_1\alpha_2\cdots\alpha_n\cdots}$ and $\Delta^{-2}_{\beta_1\beta_2\cdots\beta_n\cdots}$ have the same tails if there exist natural numbers n and m such that $\alpha_{n+j} = \beta_{m+j}$ for all $j \in \mathbb{N}$.

A function f preserves the tails of the negabinary representation of numbers if there exist natural numbers n and m such that $a_{n+j}(x) = a_{m+j}(f(x))$ for all $j \in \mathbb{N}$.

Theorem 1. *The set G of all continuous transformations of the interval $[0;1]$ that preserve the tails of negabinary representations of numbers, under the operation of composition, forms a continuous non-commutative group, whose continuous subgroup consists of increasing transformations.*

Proof. It is known that all continuous transformations of the interval $[0;1]$ form a group under the operation of composition, where the identity transformation is the neutral element, and the inverse transformation serves as the symmetric (inverse) element for a given transformation. Clearly, the set G is closed under the operation of composition, and if $f \in G$, then $f^{-1} \in G$. Therefore, G forms a group by the subgroup criterion.

To prove that the group G is non-commutative, consider the transformations $f_1(x)$ and $f_2(x)$, defined by the following expressions:

$$f_1(x) = \begin{cases} \tau_1(x), \text{ when } 0 \leqslant x \leqslant \frac{2}{3} = \Delta^{-2}_{(0)}, \\ \eta(x), \text{ when } \frac{2}{3} \leqslant x \leqslant 1. \end{cases}$$

$$f_2(x) = \begin{cases} \tau_{10}(x), \text{ when } 0 \leqslant x \leqslant \frac{4}{5} = \Delta^{-2}_{(0110)}, \\ \eta^2(x), \text{ when } \frac{4}{5} \leqslant x \leqslant 1. \end{cases}$$

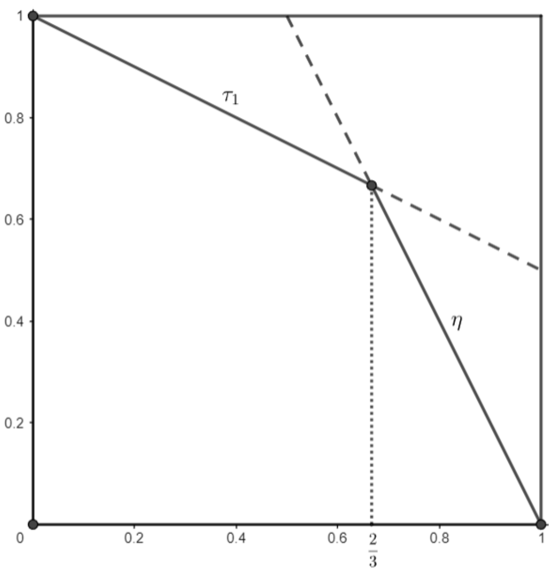

Fig. 1. $f_1(x)$

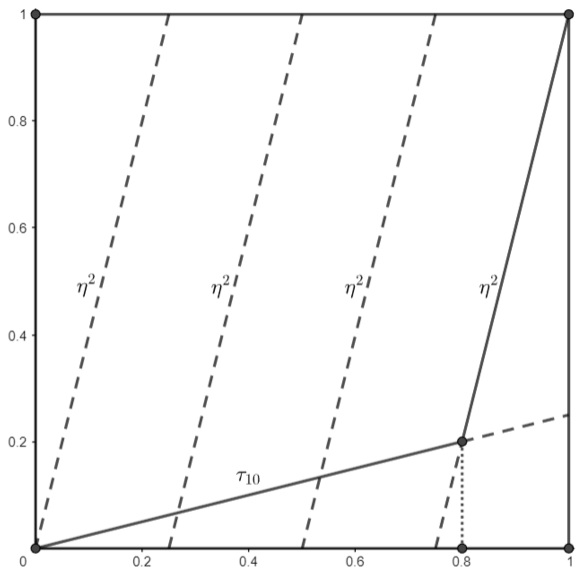

Fig. 2. $f_2(x)$

It is easy to see that $f_1(x)$ is a continuous strictly decreasing transformation, since τ_0 is a continuous and decreasing function, $\eta(x)$ is continuous and decreasing on the cylinder Δ_0^{-2}, and $\frac{2}{3}$ is a solution of the equation $\tau_0(x) = \eta(x)$. Similarly, $f_2(x)$ is a continuous strictly increasing transformation, since τ_{10} is continuous and increasing, $\eta^2(x)$ is continuous and increasing on the cylinder Δ^{-2}_{01}, and $\frac{4}{5}$ is a solution of the equation $\tau_{10}(x) = \eta^2(x)$.

Let $x_0 = \Delta^{-2}_{0111(0)} = \frac{41}{48}$ then:

$$f_1(f_2(x_0)) = f_1(\Delta^{-2}_{11(0)}) = f_1(\frac{5}{12}) = \Delta^{-2}_{111(0)} = \frac{7}{24},$$

$$f_2(f_1(x_0) = f_2(\Delta^{-2}_{111(0)}) = f_2(\frac{7}{24}) = \Delta^{-2}_{10111(0)} = \frac{7}{96}.$$

Thus, $f_1(f_2(x_0)) \neq f_2(f_1(x_0))$.

Let us prove that the group is infinite. Consider an infinite sequence of functions f_3:

$$f_3(x) = \begin{cases} \tau_{\underbrace{1010\ldots10}_{2k}}, \text{ when } 0 \leqslant x \leqslant x_k, \\ \eta^{2k}(x), \text{ when } x_k \leqslant x \leqslant 1, \end{cases}$$

where $x_k = \Delta^{-2}_{\underbrace{0101\ldots01}_{2k}\underbrace{1010\ldots10}_{2k}}$, depending on a natural parameter k.

Each function in this sequence is continuous, strictly increasing, and preserves the tails of the negabinary representation of numbers. Therefore, the group G is infinite. $\square$

Lemma 2. *The equation $\eta^n(x) = \tau_{i_1\ldots i_n}(x)$ has 2^n solutions $\Delta^{-2}_{(\alpha_1\ldots\alpha_n i_1\ldots i_n)}$, where $\alpha_j \in \{0,1\}$, $j = \overline{1,n}$, and the equation $\eta^n(x) = \tau_{i_1\ldots i_n}(x) = x$ has exactly one solution $\Delta^{-2}_{(i_1 i_2\ldots i_n)}$.*

Proof. Let us consider the equation $\eta^n(x) = \tau_{i_1\ldots i_n}(x)$. A number $x = \Delta^{-2}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}$ is a solution of this equation if and only if the following system holds:

$$\begin{cases} i_1 = \alpha_{n+1} = \alpha_{3n+1} = \ldots \\ i_2 = \alpha_{n+2} = \alpha_{3n+2} = \ldots \\ \ldots \\ i_n = \alpha_{2n} = \alpha_{4n} = \ldots \\ \alpha_1 = \alpha_{2n+1} = \alpha_{4n+1} = \ldots \\ \alpha_2 = \alpha_{2n+2} = \alpha_{4n+2} = \ldots \\ \ldots \\ \alpha_n = \alpha_{3n} = \alpha_{5n} = \ldots \end{cases}$$

The solutions to this system are the numbers $\Delta^{-2}_{(\alpha_1\ldots\alpha_n i_1\ldots i_n)}$, where $(\alpha_1, \alpha_2, \ldots, \alpha_n) \in A^n$. Therefore, in each cylinder of rank n, there is exactly one point that belongs to the set of solutions of the equation $\eta^n(x) = \tau_{i_1\ldots i_n}(x)$. Among all the solutions of the given equation, only one point $x = \Delta^{-2}_{(i_1 i_2\ldots i_n)}$ solves the equation $\tau_{i_1\ldots i_n}(x) = x$. $\square$

Introduce a sequence of points:

$$\{0; \Delta^{-2}_{(11)}; \Delta^{-2}_{(00)}; \Delta^{-2}_{(0111)}; \Delta^{-2}_{(0100)}; \cdots \Delta^{-2}_{(01\ldots0111)}; \Delta^{-2}_{(01\ldots0100)}; \cdots; 1\}$$

and denote them by $w_0, w_1, \ldots, w_j, \ldots$.

The first point of this sequence is 0, the second point will be $\frac{1}{3}$, the third point will be $\frac{2}{3}$, and the next following points will also be the solutions of the equations $\tau_{\underbrace{0101\ldots0111}_{2n}}(x) = x = \eta^{2n}(x) = w_{2n}$, $\tau_{\underbrace{0101\ldots0100}_{2n}}(x) = x = \eta^{2n}(x) = w_{2n+1}$, where $n \in \mathbb{N}$.

Define the functions $f_T(x)$, $f_{00_n}(x)$, $f_{11_n}(x)$ on the interval $[w_{2n-1}; w_{2n}]$ as follows:

$$f_T(x) = x;$$

$$f_{00_n}(x) = \begin{cases} \eta^{2n}(x), \text{ when } w_{2n-1} \leqslant x \leqslant t_{00}, \\ \tau_{\underbrace{01\ldots0100}_{2n}}(x), \text{ when } t_{00} \leqslant x \leqslant w_{2n}; \end{cases}$$

$$f_{11_n}(x) = \begin{cases} \tau_{\underbrace{01\ldots0111}_{2n}}(x), \text{ when } w_{2n-1} \leqslant x \leqslant t_{11}, \\ \eta^{2n}(x), \text{ when } t_{11} \leqslant x \leqslant w_{2n}; \end{cases}$$

where t_{00}, t_{11} are the intersection points of the $2n$-multiple left shift and right shift operators, which have the following representations:

$$t_{00} \equiv \Delta^{-2}_{(\underbrace{01\ldots0111}_{2n}\underbrace{01\ldots0100}_{2n})}, t_{11} \equiv \Delta^{-2}_{(\underbrace{01\ldots0100}_{2n}\underbrace{01\ldots0111}_{2n})}.$$

Define a sequence (a_n), where $a_i \in \{0,1,2\}$. Then, define function $f_{a_n}(x)$, that is a piecewise-defined function and on each interval $[w_{2n-1}; w_{2n}]$ is defined as follows:

$$f_{a_n}(x) = \begin{cases} f_{00_n}(x), \text{ if the } n\text{–th element of the sequence } (a_n) \text{ is equal to 0}, \\ f_{11_n}(x), \text{ if the } n\text{–th element of the sequence } (a_n) \text{ is equal to 1}, \\ f_T(x), \text{ if the } n\text{–th element of the sequence } (a_n) \text{ is equal to 2}. \end{cases}$$

The set of all numbers in the interval $[0;1]$ can be put into a one-to-one correspondence with the constructed class of transformations V by the following rule: by writing a number c from the interval $[0;1]$ in the ternary numeral system,

$c = \sum_{n=1}^{\infty} c_n 3^{-n} \equiv \Delta^3_{c_1 c_2 \cdots c_n \cdots}$, we obtain its ternary code (the sequence c_n), which belongs to the alphabet $\{0, 1, 2\}$. This corresponds to the transformation $f_{a_n}(x) \in V$. Therefore, V is a continuous set.

Discussion and conclusions

The structure of the set G and its invariants require separate investigation. The question remains open whether there exist continuous transformations of the interval $[0;1]$, different from piecewise linear ones, that preserve the tails of the negabinary representation of numbers.

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НЕПЕРЕРВНІ ПЕРЕТВОРЕННЯ ВІДРІЗКА, ЩО ЗБЕРІГАЮТЬ ХВОСТИ НЕГА-ДВІЙКОВОГО ЗОБРАЖЕННЯ ЧИСЕЛ З ВІДРІЗКА [0;1]

Для нега-двійкового зображення чисел відрізка** $[0;1]$ **розглянуто неперервні перетворення цього відрізка (тобто бієктивні відображення множини на себе), які зберігають хвости зображення чисел. Вони конструюються за допомогою операторів ліво- та правостороннього зсувів. Доведено, що множина таких перетворень відносно операції "композиція" утворює некомутативну групу, що має континуальну підгрупу зростаючих перетворень.

Ключові слова: ***нега-двійкове зображення чисел, циліндр, оператори ліво- та правостороннього зсувів, хвостова множина, неперервні перетворення відрізка, група перетворень відрізка.***

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