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USE OF IZHIKEVICH NEURONS IN HOPFIELD MODELS

The Izhikevich chaotic neuron model represents a considerable advancement in computational neuroscience by offering a mathematical framework that closely mirrors the behavior of biological neurons, especially in generating chaotic or complex spiking patterns seen in the brain. This model has garnered attention due to its ability to replicate a wide range of real-life neural dynamics, including bursting and tonic spiking, which are fundamental to understanding the complexities of neural communication and processing. On the other hand, Hopfield networks, a type of recurrent neural network, have long been recognized for their ability to serve as content-addressable memory systems, storing and recalling information based on associative dynamics. Often described as spin glass systems, Hopfield networks operate by finding stable states or patterns within the neural network, emulating certain memory functions of the human brain. Recently, research into innovative activation functions has opened new possibilities for enhancing the capabilities of recurrent networks. The chaotic activation functions, in particular, present an intriguing area of exploration within Hopfield networks. This article investigates the effects of embedding these chaotic activation functions in Hopfield networks, examining how they influence the network's stability, adaptability, and efficiency. Through this exploration, we aim to reveal the impact of chaos on the network's dynamics, providing insights that could potentially lead to improved performance in applications requiring complex memory and associative processing. The study contributes to the growing field of neuromorphic engineering, with implications for both artificial intelligence and neuroscience.

K e y w o r d s : *self-concept of the individual, dichotomous orientation of the transformation of the self-concept of the individual, constructive transformation of the self-concept of the individual, destructive transformation of the self-concept of the individual.*

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Introduction

In the field of computational neuroscience, the ability to accurately model and simulate the complex behavior of neurons is crucial for understanding brain function. One of the most promising advances in this area is the development of Izhikevich chaotic neurons. These models, named after neuroscientist Eugene Izhikevich, offer a powerful tool for replicating the highly dynamic and unpredictable behavior observed in real biological neurons.

Chaotic neurons differ from traditional neuron models by incorporating chaotic dynamics–nonlinear patterns that are sensitive to initial conditions, much like real brain activity. In biological systems, such chaotic patterns are essential for various cognitive processes, from memory encoding to decision-making, and even in maintaining the brain's flexibility to adapt to new information. Traditional neuron models, which often rely on simpler, more linear systems, struggle to capture this complexity.

The Izhikevich model strikes a balance between biological realism and computational efficiency, allowing for the simulation of large neural networks without requiring the immense computational resources that more complex models demand. This balance is critical in making large-scale simulations of brain function more accessible, aiding researchers in studying everything from learning and memory to neurological disorders.

The introduction of chaotic dynamics into neuron models represents a paradigm shift in computational neuroscience. By better capturing the intricacies of real-life neural processes, Izhikevich chaotic neurons allow scientists to explore new hypotheses about brain behavior, including how the brain processes information, adapts to change, and possibly even how it malfunctions in the case of diseases like epilepsy or schizophrenia.

As this research continues to evolve, the implications extend far beyond academic inquiry. By understanding how chaotic processes contribute to brain function, future developments could influence the creation of more advanced artificial intelligence systems, as well as new treatments for neurological conditions. In this sense, Izhikevich chaotic neurons represent not just a step forward for neuroscience but a key to unlocking deeper understanding of the brain's underlying mechanisms.

Activation functions play a vital role in transmitting signals between neuron layers in deep learning models. While sigmoid and hyperbolic tangent functions were commonly utilized in the past, recent research has revealed the benefits of using chaos-based activation functions (Dubey, Singh, & Chaudhuri, 2022). These functions offer features that enable faster and more efficient convergence during neural network training. By broadening the search space for optimal parameters, they reduce the risk of the model becoming stuck in local minima and support a more stable learning process (Wang, Ren, & Wang, 2022; Kilicarslan, Adem, & Celik, 2021).

Improved prediction accuracy and reduced training time for deep models can be achieved as a result. Furthermore, introducing chaos into activation functions helps resolve the issue of training reproducibility, as these functions are generally less sensitive to initial conditions (Chen, & Wang, 2023). However, it should be recognized that chaos-based activation functions might require more sophisticated optimization techniques and could also increase computational demands.

The Hopfield model plays a pivotal role in the realm of neural network theory, offering a profound mathematical foundation for understanding complex systems of artificial neurons. At its core, the Hopfield neural network is a recurrent, fully connected network, meaning that every node (or neuron) in the network is connected to every other node, forming a dense network topology. This full connectivity allows the network to engage in more intricate and dynamic interactions, making it capable of solving a variety of problems.

One of the distinctive features of the Hopfield model is its symmetric connection matrix. The weights between the neurons in this network are not only interconnected but also symmetrical–meaning that the weight from neuron i to neuron j is the same as the weight from neuron j to neuron i. This symmetry is a crucial aspect of the Hopfield model, as it ensures that the network's dynamics can evolve in a manner that guarantees the convergence to stable equilibrium states. These equilibrium states, where the system eventually settles, are of great interest because they correspond to local minima of a function known as the network energy.

The network energy is typically represented as a negatively defined quadratic form, often visualized geometrically as a surface over an n-dimensional cube. The network evolves by updating the states of its neurons according to a set of rules that ultimately drive the system towards these minima. This energy function is a measure of how "far" the system is from its most stable configuration, and as the network processes its inputs, it reduces this energy until it finds a point where no further reduction is possible–this point is where the system reaches equilibrium.

The activation functions in Hopfield models are designed to emulate the behavior of neurons in the brain. Although linear and sigmoidal activation functions are commonly used, studies have demonstrated that chaotic activation functions can yield interesting results (Deng, Wong, & Lin, 2024a). Chaotic functions, for example, introduce an element of randomness into the problem-solving process, which can be particularly useful for tasks like optimization or pattern recognition, where variability and dynamic responses are essential (Deng, Wong, & Lin, 2024b).

Incorporating chaotic activation functions into Hopfield networks can enhance performance, especially when dealing with problems characterized by numerous local minima. Their complex, unpredictable nature helps the network navigate toward global optima more effectively (Magallon et al., 2024). However, this approach also presents challenges, such as increased complexity in training and higher computational costs, as these functions require more advanced optimization strategies and processing techniques.

Another advantage of using chaotic activation functions in Hopfield networks is their ability to create more intricate dynamic relationships between neurons. This can improve the network's ability to adapt to changes in input data or evolving environmental conditions. Such chaotic interactions also enable the model to better mimic complex dynamic processes, such as memory retention and bias learning (Cursino, & Dias, 2024; Krotov, 2023).

The use of chaotic activation functions in neural networks introduces a level of complexity that demands meticulous attention to both the fine-tuning of model parameters and the selection of the most appropriate training methods. Unlike traditional activation functions, chaotic functions exhibit non-linear and unpredictable behavior, which can create challenges during the training process. As such, any adjustments made to the model's parameters–such as learning rates, weight initialization, and network architecture – must be approached with great care. If these adjustments are inadequate or improperly balanced, the network may exhibit erratic or unstable behavior, potentially causing the learning process to stagnate or fail entirely. Furthermore, the chaotic nature of these functions means that the training dynamics may not converge in a straightforward manner, often requiring more advanced techniques like adaptive learning rates or specialized optimization algorithms to manage the inherent instability.

One of the key challenges with chaotic activation functions lies in their unpredictability, which can sometimes manifest as highly variable outputs that deviate significantly for minor changes in input. This unpredictability may not be conducive to tasks where high precision and consistent performance are paramount. For example, applications such as financial forecasting, medical diagnostics, or autonomous driving require stable and reliable decision-making processes, as even small fluctuations in the network's behavior could result in critical errors. In such contexts, the inherent variability of chaotic functions might compromise the model's ability to make consistently accurate predictions, which is crucial for applications that rely on a high degree of reliability and precision.

Looking ahead, the future of chaotic activation functions in neural networks holds promise. Advancements in computational power and optimization algorithms may address current limitations, paving the way for broader adoption. Hybrid approaches that combine chaos with traditional methods could offer a balance between precision and adaptability. Furthermore, the exploration of chaos theory in quantum computing and neuromorphic hardware could unlock new frontiers for neural network design.

There is a growing demand for more in-depth research on the application of chaotic activation functions within Hopfield networks, spurred by the increasing interest in neural networks and their potential roles in complex information processing tasks. Although some studies have already highlighted the promise of this approach, the subject is still not well represented in the scientific literature. Therefore, it is crucial to conduct more extensive experimental and theoretical investigations to explore the potential advantages and disadvantages of integrating chaotic activation functions into Hopfield networks (Liang et al., 2024; Kashyap et al., 2022).

1. Activation functions' comparison

Differentiable activation functions in neural networks provide several significant advantages over their nondifferentiable counterparts. Most importantly, they allow for the effective use of optimization algorithms like backpropagation during network training. The smooth nature of differentiable functions aids in finding optimal parameters through gradient search techniques, which streamlines the training process and improves the convergence rate.

It is also important to recognize that differentiable activation functions enable the application of regularization methods that help reduce overfitting and improve generalization capabilities. For example, techniques such as L1 or L2 regularization can be applied to model weights, helping to control their complexity and mitigate the risk of overfitting.

Additionally, differentiable activation functions create a smooth and continuous relationship between the network's inputs and outputs, allowing for gradual transitions between values and simplifying the optimization process. They also facilitate the use of more sophisticated optimization algorithms, such as adaptive gradient descent methods, which improve both the speed and efficiency of network convergence.

In (Lieberman-Aiden, 2009), it was proposed that a neuron's behavior can be described by a dynamical system, represented as a one-dimensional function that is both piecewise linear and piecewise continuous:

$$x(t + l) = F\big(x(t), h(t)\big). \tag{1}$$

This category of functions is defined by the function *K(h)*, which is influenced by the slope angle. This relationship must satisfy the following conditions: 1) symmetry, such that *K(-h) = K(h)*; 2) a monotonic decrease for *h* > 0 (and consequently, a monotonic increase for *h* < 0); and 3) *K*(0) = 2. The following function meets these criteria

$$K(h) = \frac{2}{1+|h|/\mu}. \quad (2)$$

The behavior of the formal neuron is defined by a ratio:

$$x(t+1) = \begin{cases} 1, & h(t) > 0 \\ -1, & h(t) < 0 \end{cases}. \quad (3)$$

It is well-known that a real neuron operates as a highly complex system with inertia (Freeman, 1987; Hirsch, 1989; Babloyantz, Nicolis, & Salazor, 1985; Hopfield, 1982). The characteristic time scales of its processes range from just a few milliseconds to several hundred milliseconds, or even longer (Skarda, & Freeman, 1987; Tsuda, 1992; Shepherd, 1990; Yao, & Freeman, 1990). In (Lieberman-Aiden, 2009), it was suggested that when the external input h is large, the neuron quickly reaches an excited state. However, when h is smaller, the transition to this state slows down. This behavior is captured by the chaotic Izhikevich activation function (1), where a decrease in h leads to an increase in *K(h)*, resulting in a longer transition time in (2). Despite this, the special point at *x* = 1 remains stable. Once *K(h)* surpasses 1, *x = 1* loses its stability, giving rise to chaotic behavior in the neuron.

The transition from order to chaos, as described in (3), is a phenomenon often observed in functions with discontinuities or sharp changes, referred to as a homoclinic explosion (Sparrow, 1982). Previous research has shown that the specific form of the function *K(h)* does not significantly affect the outcomes, as long as the system follows this chaotic behavior. The critical parameter to focus on is the value *h = p*, where *K(h)* equals 1, marking the point at which the shift to chaos occurs.

One drawback of the chaotic Izhikevich activation function mentioned earlier is its lack of differentiability for small values of h(t). Differentiability is a critical property in many machine learning algorithms, particularly those relying on gradient-based optimization methods, such as backpropagation. The absence of differentiability in an activation function can lead to challenges in effectively training neural networks, as gradients are essential for adjusting weights to minimize error during the learning process.

This problem can be addressed by replacing the linear function within the Izhikevich activation model with a parametric Bézier function (see Fig. 1). Bézier functions, widely used in computer graphics for curve modeling, provide a flexible and smooth representation of data. In the context of neural networks, this replacement ensures that the activation functions remain differentiable, even for small values of *h(t)*. The smoothness and control offered by Bézier functions can facilitate more stable gradient computations, enhancing the training efficiency and convergence of neural networks.

Moreover, the parametric nature of Bézier functions introduces additional parameters that can be fine-tuned during the training process. These parameters offer a higher degree of flexibility, allowing the activation function to adapt to specific learning requirements of different layers or architectures. Importantly, this substitution preserves the bijectivity condition of the equation x(p), where x represents the input vector, and p denotes the parameters of the Bézier function. Bijectivity is crucial in ensuring that every input maps uniquely to an output, maintaining the mathematical integrity and predictability of the activation function.

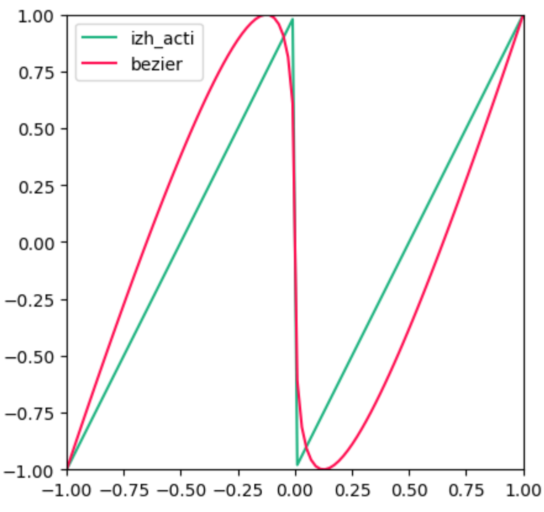

Fig. 1. The Izhikevich chaotic activation function and its altered form utilizing a parametric Bézier curve

The adaptation of parametric Bézier functions in place of traditional activation mechanisms exemplifies the innovative approaches being developed to address the inherent challenges in neural network training. This adjustment not only highlights the interplay between mathematical rigor and computational efficiency but also underscores the importance of interdisciplinary methods in advancing machine learning technologies. An experiment was conducted to evaluate the activation functions used in the Hopfield network, specifically the signum function, the chaotic Izhikevich activation function, and its differentiable variant. The input data comprised samples from normal distributions *N*(μ, σ²), representing two distinct classes. The parameters μ and σ² were chosen to cover a range of distribution overlap scenarios (see Fig. 2). Training images were created based on the training data for each binary classification category, and the models' performance was measured on the test data using Leave-One-Out validation, with the F1-score as the primary metric.

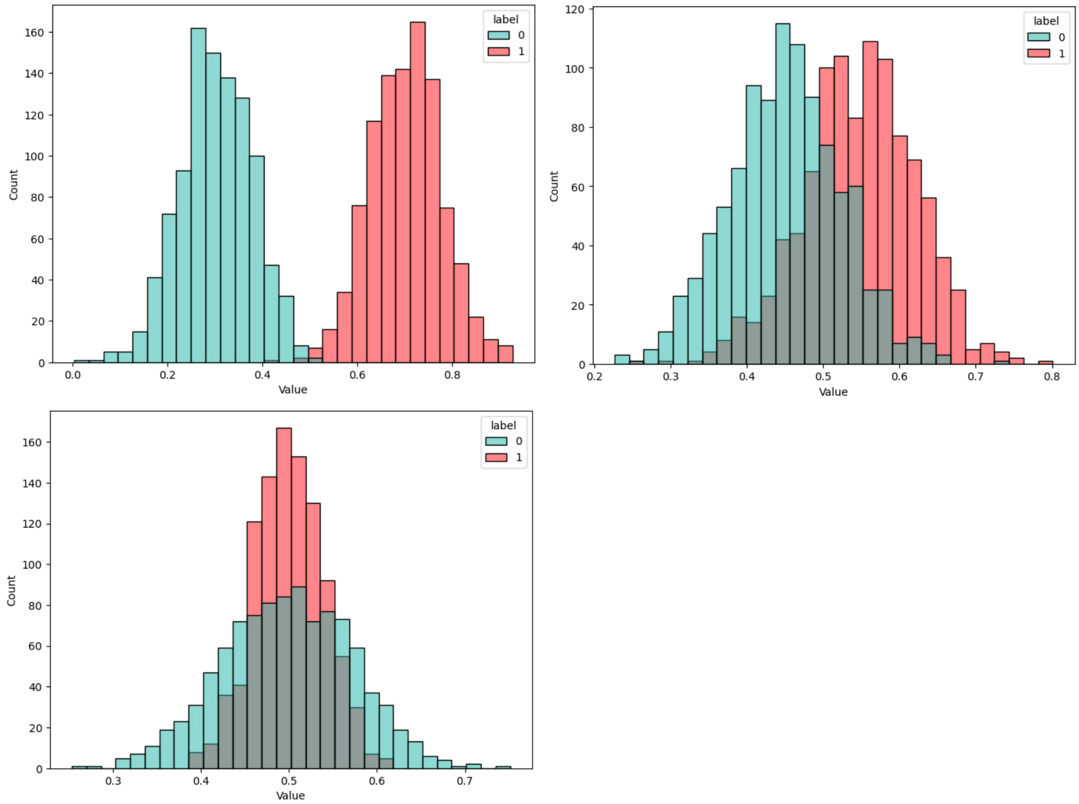

Fig. 2. Samples for binary classification derived from normal distributions with distinct variances and mean values

The null hypothesis posits that the selection of an activation function does not influence the outcomes of the model, whereas the alternative hypothesis asserts that chaotic activation functions provide enhanced results.

When analyzing cases where the distributions intersect, with parameters set as $\mu1=0.34$, $\mu2=0.66$, and $\sigma1^2=\sigma2^2=0.05$, an average F1-score of 0.79 was recorded after conducting 100 trials with the signum activation function. Through statistical power analysis, it is possible to ascertain the minimum number of experiments necessary to achieve a statistically significant increase in the mean F1-score. To reach statistical significance using a z-test for an improvement of at least 1 %, a minimum of 25,580 experiments per group would be required.

A total of 26,000 experiments were carried out for each of the activation functions under comparison (including signum, chaotic Izhikevich, and its differentiable variant), and the F1-scores were computed for each trial. The average F1-scores obtained were 0.86 for the signum function, 0.87 for the chaotic Izhikevich function, and 0.872 for the differentiable version. Following this, z-tests were conducted to analyze the results for each experimental pair (refer to Fig. 3).

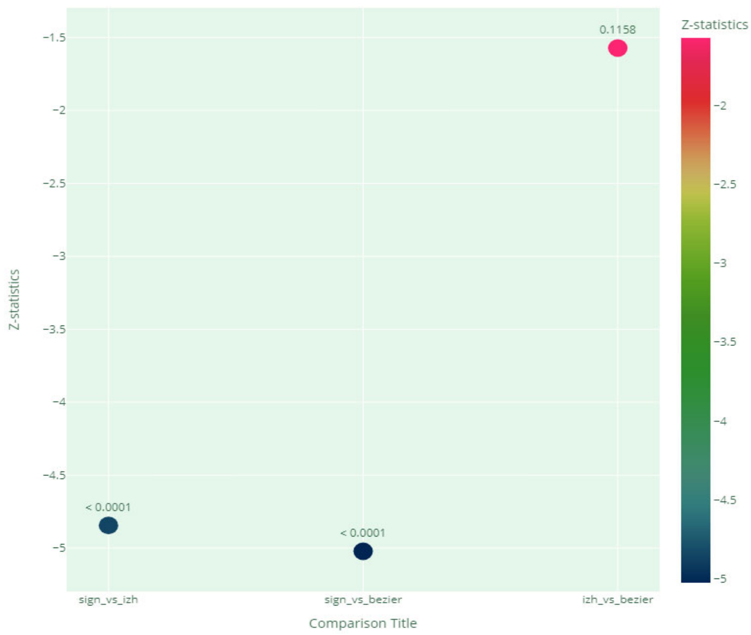

Fig. 3. Results of z-tests for experiments with activation functions

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The findings indicated that the samples utilizing chaotic activation functions were statistically significantly different from those using the signum function. However, no statistically significant difference was found between the two chaotic functions, even though the modified function yielded slightly better outcomes, as demonstrated by the z-statistics.

Additional experiments with different parameters of normal distributions were conducted, and F1-scores were determined for each sample, with paired z-tests performed (see Fig. 4).

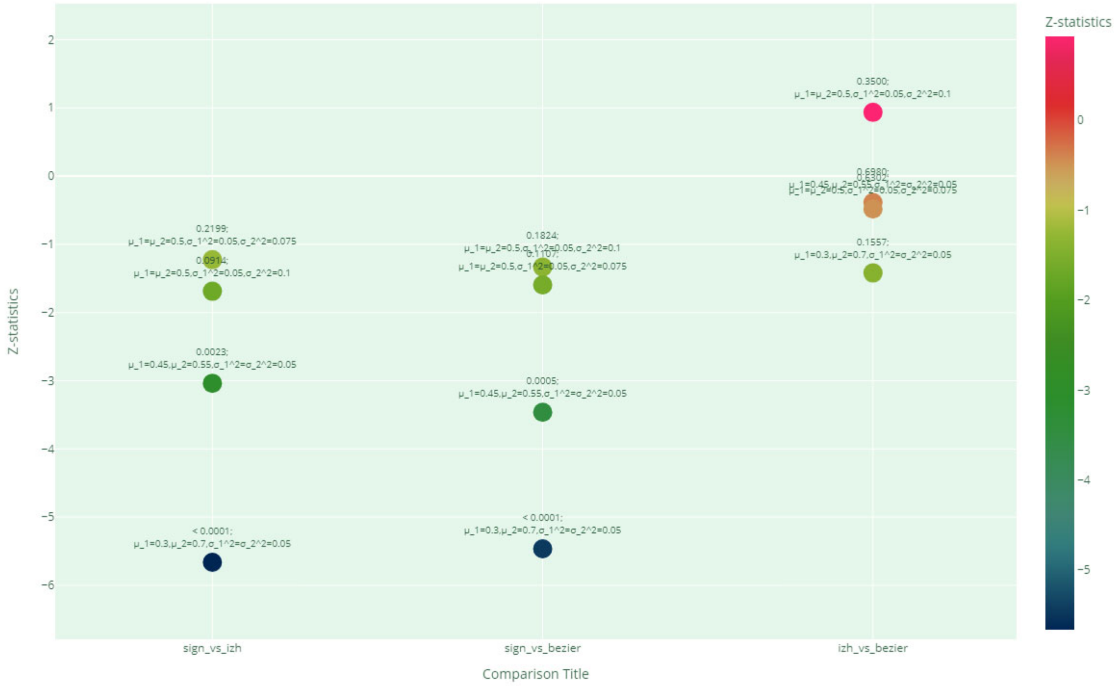

Fig. 4. Results of z-tests for experiments with activation functions for different distributions

In trials where the mean values varied, the previous trend persisted: chaotic functions consistently demonstrated statistically significant improvements over the signum function, with the differentiable modification exhibiting even greater performance. Conversely, in experiments where the means were identical, all functions delivered comparable results, revealing no statistically significant differences.

2. Experiment on real data

We will now investigate the efficacy of chaotic activation functions in an experiment utilizing real-world data.

In the 1960s, some of the earliest studies on malignant transformations emerged, focusing on X-chromatin content in somatic cells. These investigations underscored the instability of X-chromatin and its relationship with various functional changes in the body, as well as the general pathology of somatic cells. Significant alterations in the X-chromatin content of buccal epithelium and peripheral blood neutrophils were noted in individuals with tumors. Furthermore, it was shown that variations in the number of cells harboring X chromosomes are connected to abnormalities in the functional status of the heterozygous X chromosome (Radak, Lafta, & Fallahi, 2023).

Research emphasizing changes in buccal epithelial cells among cancer patients has garnered considerable interest. In the 1960s, H. Nieburgs (1968) and colleagues reported a notable redistribution of chromatin mass in somatic cells of 77 % of cancer patients, labeling these alterations as tumorigenic. These changes included the enlargement of epithelial cell nuclei and an increase in the size of "restricted" chromatin areas, which were surrounded by lighter regions (Boroday et al., 2016). Similar transformations were observed in cells from other organs, such as the liver and kidneys. In breast cancer patients, there was a reported increase in DNA content and the size of interphase nuclei in buccal epithelium. However, some researchers did not observe a significant difference in DNA content between patients and healthy individuals, as demonstrated in a study measuring the DNA content of buccal epithelial cells in men with bronchial epithelioma through cellular spectrophotometry (Ogden, Cowpe, & Green, 1990).

Another study analyzed a control group of 29 patients, 68 patients diagnosed with stage II breast cancer, and 33 patients with fibroadenomatosis, all confirmed through histological examination. The morphological dataset consisted of 20,256 images of interphase nuclei from buccal epithelium, with 6,752 nuclei scanned using three different formats: unfiltered, with a yellow filter, and with a purple filter.

The morphological samples were derived from smears of epithelial cells taken from the middle depth of the spinous layer of the oral mucosa, averaging 52 cells per preparation. The concentration of DNA-fuchsin in the epithelial cell nuclei was calculated by multiplying the optical density by the area. During the initial phase of the analysis, an image of chromatin distribution was generated as a 128 x 128-pixel matrix (Klyushin et al., 2021).

The model's input comprised three-channel (RGB) samples of fractal kernel sizes for each patient, exhibiting considerable variation in the number of elements across the samples. As a result, quantiles were calculated for each sample before training the neural network, where n signifies the number of elements in the smallest sample (refer to Fig. 5).

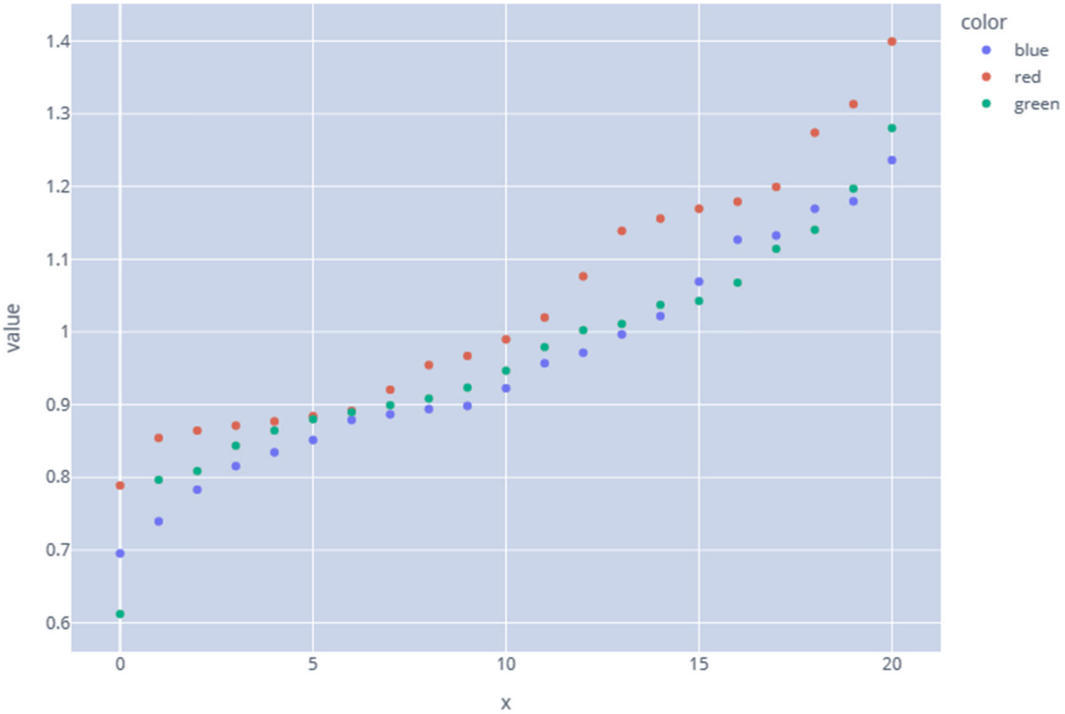

Fig. 5. An example of consolidation of training samples using quantiles

A total of 144 experiments were meticulously conducted, carefully divided into four distinct categories to facilitate a comprehensive analysis: cancer patients compared to healthy controls, cancer patients versus both healthy controls and fibroadenomatosis patients, cancer patients against fibroadenomatosis patients, and fibroadenomatosis patients compared to healthy controls. Each category represented a unique challenge in distinguishing between the various groups, and the experiments were designed to test multiple aspects of the data and model behavior under different conditions.

Training images were generated from the initial training data for each binary classification category, ensuring that the models were exposed to a balanced and varied set of input data. The performance of each model was rigorously evaluated using Leave-One-Out Cross-Validation (LOO-CV) on the test data. This method enabled the analysis of each individual data point's influence on the model's predictions. Key performance metrics, including precision, recall, and F1-score, were utilized to assess how well the models differentiated between the various groups, with a specific focus on achieving high accuracy and minimizing false positives and false negatives.

In the analysis comparing 'cancer patients and fibroadenomatosis patients,' it was observed that the differentiable modification introduced in the models yielded significantly better outcomes. Specifically, the use of the median as an aggregation function outperformed the arithmetic mean, consistently providing a more reliable aggregation of data, especially in the most successful experimental trials. The highest accuracy was recorded when utilizing input data in RGB format, offering the most comprehensive color representation for distinguishing between the patient groups. Additionally, input data formatted with single-channel inputs for red (R), green (G), and blue (B) channels also demonstrated promising results, further emphasizing the importance of selecting the right input format for model performance.

In the comparison between 'cancer patients and healthy controls,' the differentiable modification displayed comparable results to those achieved by the chaotic Izhikevich function, known for its non-linear and complex behavior. In most experiments, both chaotic functions outperformed the traditional signum function, showcasing the potential of chaotic dynamics in improving model predictions. Interestingly, the median and arithmetic mean showed nearly identical performance in related tests, with the best accuracy achieved when the input data was in the B (blue) and RGB formats, suggesting that these color channels were particularly effective in discriminating between cancer patients and healthy controls.

When examining the more complex comparison of 'cancer patients versus healthy controls and fibroadenomatosis patients,' all three activation functions–differentiable, chaotic, and signum–exhibited similar levels of effectiveness. This lack of a clear winner among the activation functions suggests that, in this case, the task of differentiating between the three categories posed a greater challenge for the models, regardless of the function used. Similarly, the aggregation techniques and input data formats did not show any clear advantage, with no single combination yielding significantly better results across all experiments.

In the comparison of 'fibroadenomatosis patients compared to healthy controls,' chaotic functions consistently outperformed the signum function across the majority of experiments. This trend aligns with previous findings and further supports the hypothesis that chaotic systems can enhance model performance in certain types of medical data classification. Moreover, in line with previous analyses, the median was found to be more effective than the arithmetic mean in aggregating training images, leading to better F1-scores in most experiments. The highest accuracy in this category was achieved with input data in both RGB and B formats, reinforcing the consistent success of these input channels across different classification tasks.

The most impressive results were seen in the experiments comparing 'cancer patients to fibroadenomatosis patients,' where the top five trials achieved an average F1-score of 0.84, reflecting a high level of model effectiveness in this binary comparison. This stands in contrast to the lowest performance, observed in the category comparing 'cancer patients to healthy controls and fibroadenomatosis patients,' where the best five experiments yielded an average F1-score of only 0.57. This significant variation in performance can largely be attributed to the complexity introduced by the second class – healthy controls and fibroadenomatosis patients. The combined dataset for these two groups proved to be especially difficult for the model to differentiate from cancer patients, likely due to shared features or overlapping characteristics between healthy and fibroadenomatosis patients that were not easily separable.

Furthermore, the aggregation method once again played a crucial role in model performance. The median, as an aggregation technique, demonstrated greater effectiveness in most experiments, resulting in higher F1-scores compared to the arithmetic mean. This finding underscores the importance of selecting the right aggregation function to ensure that the training data is represented in a way that maximizes the model's ability to distinguish between patient groups. The input formats that consistently emerged as the most successful across the various experimental groups were RGB and B, both of which regularly appeared among the top five trials in each category, suggesting that these formats offer the most robust and informative data for the classification task at hand.

In summary, the results from these 144 experiments highlight the intricate dynamics of using machine learning models to distinguish between different patient categories based on complex medical data. The findings suggest that differentiable modifications, chaotic functions, and thoughtful data aggregation methods can significantly enhance model performance, especially when carefully paired with the most appropriate input data formats. These insights provide valuable guidance for future research and model optimization in the field of medical data classification.

Discussion and conclusions

Incorporating chaos into activation functions is becoming an increasingly important strategy for improving the efficiency, adaptability, and overall performance of deep learning models, particularly within the context of artificial neural networks (ANNs). Chaotic systems, by their very nature, exhibit complex, sensitive dependence on initial conditions, and by embedding such behavior into activation functions, models can simulate a more dynamic, unpredictable response to varying inputs. This adaptability is particularly beneficial in deep learning, where the capacity to handle diverse and fluctuating data patterns can significantly improve model performance. Traditional activation functions like ReLU or sigmoid, while effective in many scenarios, often struggle to account for the nonlinearities and complexities inherent in real-world datasets, leading to limited model flexibility and suboptimal generalization.

By introducing chaos into activation functions, the feature distribution space within the model is greatly expanded. This means that the neural network has access to a broader spectrum of representational possibilities, allowing it to adjust more effectively to changing or uncertain conditions in the data. Such a mechanism allows the network to better explore and map the relationships within the data, as chaotic functions inherently offer a rich diversity of behaviors, ranging from periodic to highly irregular patterns. This increased diversity enables models to capture more nuanced features and detect intricate patterns that might otherwise be missed by more conventional activation functions. Additionally, chaotic activation functions contribute to making the model more resilient to noise and fluctuations in the training data, as the chaotic behavior introduces a natural robustness to small, random perturbations. This becomes especially important in real-world applications, where data can often be incomplete, noisy, or otherwise unreliable.

Another significant advantage of chaotic activation functions is their ability to improve the model's generalization capabilities, especially when exposed to new, unseen data. When deep learning models are trained, they often face the challenge of overfitting to the training set, which can result in poor performance on out-of-sample data. The unpredictability introduced by chaotic functions helps to mitigate this risk by promoting more flexible learning, allowing the model to explore a wider range of potential solutions during training. This ability to generalize better means that chaotic activation functions enhance the model's ability to adapt to new scenarios, making it more versatile and better equipped to handle a broad variety of machine learning problems, from time series forecasting to pattern recognition.

Moreover, differentiable activation functions in neural networks provide several significant advantages over undifferentiable ones. Primarily, they enable the effective use of optimization algorithms, such as error backpropagation, for network training. The smooth properties of differentiable functions allow for the identification of optimal parameters via gradient descent, thereby streamlining the training process and improving the convergence rate.

This article presents evidence and highlights the advantages of chaotic activation functions within the Hopfield model, demonstrated through experiments with artificial samples generated from normal distributions, as well as real data related to breast cancer prediction. This area still requires further research and experimentation, and the development and implementation of new chaotic activation functions could be a promising direction for advancing deep learning. Such enhancements could benefit not only Hopfield models but also other algorithms and neural network architectures, potentially resulting in improved performance across a range of tasks.

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References

Babloyantz, A., Nicolis, S., & Salazor, M. (1985). Evidence of chaotic dynamics of brain activity during the sleep cycle. *Physics Letters A.*, 111, 152–156. https://doi.org/10.1016/0375-9601(85)90444-X

Boroday, N., Chekhun, V., Golubeva, E., & Klyushin, D. (2016). In vitro and in vivo densitometric analysis of DNA content and chromatin texture in tumor cell nuclei under the influence of a nano composite and magnetic field. *Advances in Cancer Research & Treatment*, 1–11. https://doi.org/10.5171/2016.706183

Chen, X., & Wang, Y. (2023). A Chaotic Neuron and its Ability to Prevent Overfitting. *Frontiers in Computing and Intelligent Systems*, 5, 53–61. https://doi.org/10.54097/fcis.v5i1.11673

Cursino, C., & Dias, L. (2024). Hybrid Hopfield Neural Network. *SN Computer Science*, 5, 25–74. https://doi.org/10.1007/s42979-023-02575-6

Deng, Q., Wong, C., & Lin, H. (2024a). Chaotic dynamical system of Hopfield neural network influenced by neuron activation threshold and its image encryption. *Nonlinear Dynamics*, 112, 1–18. https://doi.org/10.1007/s11071-024-09384-3

Deng, Q., Wong, C., & Lin, H. (2024b). Memristive Hopfield neural network dynamics with heterogeneous activation functions and its application. *Chaos Solitons & Fractals*, 178, 101–124. https://doi.org/10.1016/j.chaos.2023.114387

Dubey, S. R., Singh, S. K., & Chaudhuri B. B. (2022). Activation Functions in Deep Learning: A comprehensive Survey and Benchmark. *Neurocomputing*, 503, 24–27. https://doi.org/10.1016/j.neucom.2022.06.111

Freeman, W. (1987). Simulation of Chaotic EEG Patterns with a Dynamic Model of the Olfactory System. *Biological Cybernetics*, 56, 139–150. https://doi.org/10.1007/BF00317988

Hirsch, M. (1989). Convergent Activation Dynamics in Continuous Time Neural Networks. *Neural Networks*, 2, 331–351. https://doi.org/10.1016/0893-6080(89)90018-X

Hopfield, J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proc Natl Acad Sci U S A.*, 79 (Apr), 2554–2558. https://doi.org/10.1073/pnas.79.8.2554

Kashyap, D., Dsouza, N., Shi, L., & Beymer, D. (2022). Hopfield Encoding Networks. https://doi.org/10.13140/RG.2.2.17209.65123

Kilicarslan, S., Adem, K., & Celik M. (2021). An overview of the activation functions used in deep learning algorithms. *Journal of New Results in Science*, 10, 75–88. https://doi.org/10.54187/jnrs.1011739

Klyushin, D., Golubeva, K., Boroday, N., & Shervarly, D. (2021). Breast Cancer Diagnosis Using Machine Learning and Fractal Analysis of Malignancy-Associated Changes in Buccal Epithelium. *Artificial Intelligence, Machine Learning, and Data Science Technologies Future Impact and Well- Being for Society*, 5, 1–18. https://doi.org/10.1201/9781003153405-1

Krotov, D. (2023). A new frontier for Hopfield networks. *Nature Reviews Physics*, 5, 52–70. https://doi.org/10.1038/s42254-023-00595-y

Liang, X., Yang, Y., Wang, R., & Chen, J. (2024). Synchronization of delayed stochastic reaction-diffusion Hopfield neural networks via sliding mode control. *Nonlinear Analysis Modelling and Control*, 29, 1–19. https://doi.org/10.15388/namc.2024.29.34884

Lieberman-Aiden, E. (2009). Comprehensive mapping of long-range interactions reveals folding principles of the human Genome. *Science*, 326, 289–193. https://doi.org/10.1126/science.1181369

Magallon, D., Garcia, J., Huerta, G., & Jaimes, R. (2024). Real-time neural identification using a recurrent wavelet first-order neural network of a chaotic system implemented in an FPAA. *Integration*, 96, 384–393. https://doi.org/10.1016/j.vlsi.2023.102134

Nieburgs, H. (1968). Recent progress in the interpretation of malignancy associated changes (MAC). *Acta Cytologica*, 12, 445–453. https://doi.org/10.1155/238921

Ogden, G., Cowpe, J., & Green, M. (1990). The effect of distant malignancy upon quantitative cytologic assessment of normal oral mucosa. *Cancer*, 65, 477–480. https://doi.org/10.1002/1097-0142(19900201)65:3<477::AID-CNCR2820650317>3.0.CO;2-G

Radak, M., Lafta, H., & Fallahi, H. (2023). Machine learning and deep learning techniques for breast cancer diagnosis and classification: a comprehensive review of medical imaging studies. *Journal of Cancer Research and Clinical Oncology*, 49, 10473–10491. https://doi.org/10.1007/s00432-023-04956-z

Shepherd, G. (1990). *The synaptic organization of the brain*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780195159561.001.1

Skarda, C., & Freeman, W. (1987). How brains make chaos in order to make sense of the world. *Behavioral and Brain Sciences*, 10, 161–195. https://doi.org/10.1017/S0140525X00047336

Sparrow, C. (1982). The Lorenz Equations. Bifurcations, Chaos, and Strange Attractors. *Applied Mathematical Sciences*, 41, 26–50. https://doi.org/10.1007/978-1-4612-5767-7

Tsuda, I. (1992). Dynamic link of memory – Chaotic memory map in nonequilibrium neural networks. *Neural Networks*, 5, 857–893. https://doi.org/10.1016/S0893-6080(05)80029-2

Wang, X., Ren H., & Wang A. (2022). Smish: A Novel Activation Function for Deep Learning Methods. *Electronics*, 11, 540–545. https://doi.org/10.3390/electronics11040540

Yao, Y., & Freeman, W. (1990). Model of Biological Pattern Recognition with Spatially Chaotic Dynamics. *Neural Networks*, 3, 153–170. https://doi.org/10.1016/0893-6080(90)90086-Z

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ВИКОРИСТАННЯ НЕЙРОНІВ ІЖИКЕВИЧА В МОДЕЛЯХ ГОПФІЛДА

Побудова моделі хаотичного нейрона Іжикевича є значним проривом у галузі обчислювальної нейронауки, оскільки надає цьому явищу математичну основу, що наближено відтворює поведінку біологічних нейронів, зокрема складні хаотичні спайкові патерни, які спостерігаються в мозку. Ця модель привернула увагу вчених завдяки своїй здатності відтворювати широкий спектр реальних нейронних динамік, включно зі спалахами та тонічними спайками, що є фундаментальним для розуміння складності нейронної комунікації й оброблення інформації. З іншого боку, мережі Хопфілда, один із типів рекурентних нейронних мереж, давно визнані за їхню здатність функціонувати як системи пам'яті з доступом за змістом, зберігаючи та викликаючи інформацію на основі асоціативної динаміки. Часто описані як спінові скляні системи, мережі Хопфілда функціонують завдяки знаходженню стабільних станів або патернів у межах нейронної мережі, імітуючи певні функції пам'яті людського мозку. Нещодавні дослідження в галузі інноваційних активаційних функцій відкрили нові можливості для вдосконалення характеристик рекурентних мереж. Зокрема, хаотичні активаційні функції є цікавим напрямком для досліджень у мережах Хопфілда. У пропонованій статті проаналізовано вплив упровадження цих хаотичних активаційних функцій у мережі Хопфілда, зокрема те, як їхня дія відображається на стабільності, адаптивності й ефективності роботи мережі. Мета роботи – розкрити вплив хаосу на динаміку мережі, надаючи нові уявлення, які потенційно можуть підвищити продуктивність у застосуваннях, що потребують складної пам'яті й асоціативної обробки. Це дослідження робить вагомий внесок у розвиток нейроморфної інженерії та має важливе значення для штучного інтелекту й нейронауки.

Ключові слова: ***нейрон Іжикевича, мережа Гопфілда, хаотичні активаційні функції, нейронні мережі.***

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