Вісник Київського національного університету імені Тараса Шевченка
Серія фізико-математичні науки

2021, 2

Bulletin of Taras Shevchenko National University of Kyiv
Series Physics &Mathematics

УДК 517.946

https://doi.org/10.17721/1812-5409.2021/2.14

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Необхідні умови для розв’язків рівнянь змішаного типу

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Necessary conditions for solutions for the mixed type equations

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Представлена робота присвячена дослідженню необхідних умов для лінійних рівнянь змішаного типу в обмеженій області на площині. Ці необхідні умови визначаються за допомогою інтегральних співвідношень і при цьому використовуються фундаментальні розв’язки таких рівнянь.

Ключові слова: змішаний тип рівняння, фундаментальний розв’язок, інтегральне співвідношення, необхідні умови.

The presented work consists of investigations of necessary conditions for linear mixed type equations in the limited area on a plane. These necessary conditions are determined by the help of integral relations and at the same time the fundamental solutions of such equations are used.

Key words: mixed type equation, fundamental solution, integral relation, necessary conditions.

Статтю представив д.ф.-м.н. Хусаінов Д.Я.

I. INTRODUCTION

The mathematical physics equations [1] - [3] and special derivative equations [4] - [7] mainly dealt with problems for three types of equations. They are equations of hyperbolic, parabolic, and elliptical types, that Cauchy problem and mixed problems were considered for the equations of hyperbolic and parabolic types, but for the equations of elliptical type Boundary value problem.

For the first time, Tricomi [8] considered an equation that was belonged to hyperbolic in one and elliptical in the leftover part of the area. Now this equation is called the Tricomi equation. This is a mixed type equation with a changeable coefficient.

Later, were also considered problems for mixed type equations with constant [9].

Finally, Hadamard considered such equation, that let this equation has both ellipticity and hyperbolicity characteristics at any point in the area, which is under consideration [10]. This is called a composite type equation.

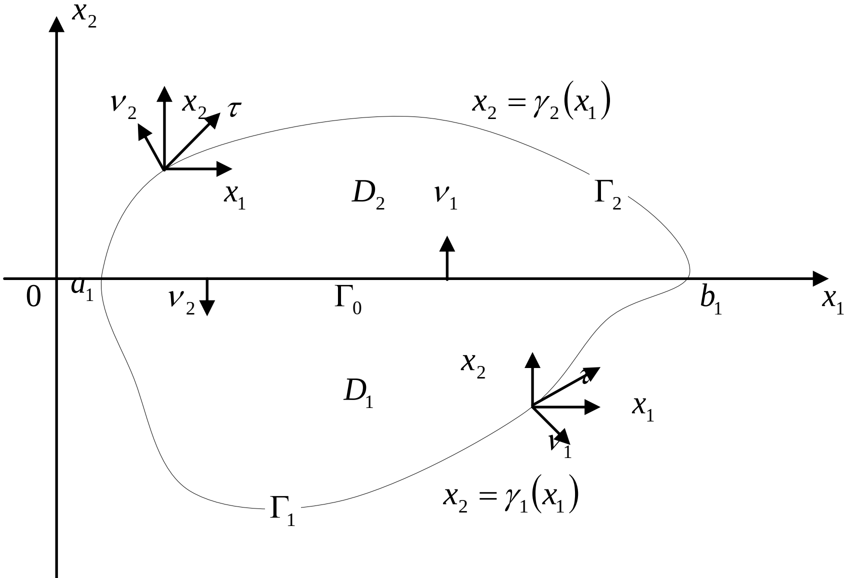
Later, Gellersted, Bitsadze and his students [11]-[20] dealt with these problems. Eventually, they dealt with the investigation of Boundary value problems for the mixed and composite type equations [17] - [20]. These problems are considered within local Boundary conditions. We consider nonlocal problems and global (containing integrals) problems within Boundary conditions [21] - [27].

II. PROBLEM STATEMENT

Suppose $D = D_1 \cup D_2 \subset R^2$ being limited flatness area in the right semi-flatness
$D_1 = \{x \in (x_1, x_2): 0 < a_1 < x_1 < b_1,\ \gamma_1(x_1) < x_2 < 0\}$,
limited area located in the fourth quarter,
$D_2 = \{x \in (x_1, x_2): 0 < a_1 < x_1 < b_1,\ 0 < x_2 < \gamma_2(x_1)\}$,
but, limited area in the first quarter.

Here is drawn external normal with ν_1 and of the area D_1 to the border ∂D_1, external normal with ν_2 of the area D_2 to the border ∂D_2. If equation $x_2 = \gamma_1(x_1)$, $x_1 \in [a_1, b_1]$ the lower border of the area Γ_1, D_1.

If equation $x_2 = \gamma_2(x_1)$, $x_1 \in [a_1, b_1]$ Γ_2 but the upper border of the area D_2. The common border of the areas D_1 and D_2, but Γ_0, $x_2 = 0$, $x_1 \in [a_1, b_1]$, is a finite piece on the true axis. A complete description of this area was given in the picture1.



Picture 1.

Now, let to see the following equation in the D:

$$\frac{\partial^3 u_s(x)}{\partial x_1 \partial x_2^2} + i^s \frac{\partial^3 u_s(x)}{\partial x_1^2 \partial x_2} = 0,\ x = (x_1, x_2) \in D_s,\ s = 1,2, \quad (1)$$

here $i = \sqrt{-1},\ \partial D = \Gamma_1 \cup \Gamma_2$,
but it is the Lyapunov line, if $s = 1$ the resulting equation, it means

$$\frac{\partial^3 u_1(x)}{\partial x_1 \partial x_2^2} + i \frac{\partial^3 u_1(x)}{\partial x_1^2 \partial x_2} = \frac{\partial^2}{\partial x_1 \partial x_2}\left(\frac{\partial u_1(x)}{\partial x_2} + i \frac{\partial u_1(x)}{\partial x_1}\right) = 0,$$

$$x \in D_1, \quad (2)$$

equation, in D_1 is composite type (a mixed derivative of the Cauchy-Riemann, an elliptical type),
if $s = 2$,but

$$\frac{\partial^3 u_2(x)}{\partial x_1 \partial x_2^2} - \frac{\partial^3 u_2(x)}{\partial x_1^2 \partial x_2} = \frac{\partial^2}{\partial x_1 \partial x_2}\left(\frac{\partial u_2(x)}{\partial x_2} - \frac{\partial u_2(x)}{\partial x_1}\right) = 0,$$

$$x \in D_2, \quad (3)$$

is hyperbolic equation.

If we look at the classical works, the analogue of such an equation is derived from the Tricomy equation. Therefore, equation (1) is called a mixed and composite type equation.

III. FUNDAMENTAL SOLUTIONS

Given in the Cauchy-Riemann equation [3]:

$$U(x - \xi) = \frac{1}{2\pi} \cdot \frac{1}{x_2 - \xi_2 + i(x_1 - \xi_1)},\ x, \xi \in R^2, \quad (4)$$

for a fundamental solution of equation (2) using a fundamental solution

$$U_1(x-\xi)=-\frac{i}{2\pi}\left[x_2-\xi_2+i(x_1-\xi_1)\right]\cdot\left\{\ln\left[x_2-\xi_2+i(x_1-\xi_1)\right]-1\right\},$$

$$x,\xi\in R^2, \qquad (5)$$

we get the statement.

For equation (3) in the same way

$$U_2(x-\xi)=\int_{x_1-\xi_1}^{\frac{1}{2}\left[(x_2-\xi_2)+(x_1-\xi_1)\right]}\theta(t)\theta(x_2-\xi_2+x_1-\xi_1-t)dt,$$

$$x,\xi\in R^2, \qquad (6)$$

we have received a fundamental solution.

In the above statement $\theta(t)$ - this is Heaviside’s only function.

Theorem 1. The fundamental solution of the given equation (2) is in the form (5), and the fundamental solution of the equation (3) is in the form (6).

IV. MAIN APPROACHES

These approaches are obtained from the second Green formula by applying the Gauss-Ostrogradsky formula (part by part integration), after integration on the appropriate area multiplying equations (2) and (3) by their fundamental solutions (5) and (6). Then these equations (2), (3) and their fundamental solution multiplying to derivatives of (5), (6) by integrating along the appropriate areas and applying the Gauss-Ostrogradsky formula are obtained from the analog of the second Green formula. So that, in obtaining of these basic approaches part by part integration should be done in such a way that, integration on the area neither the solutions of equations nor should more than three derivatives of fundamental solutions be involved.

Two of the main approaches we have mentioned are the following:

$$\int_{\partial D_1}\left\{\left[\frac{\partial^2 u_1\; x}{\partial x_1\partial x_2}U_1\; x-\xi\; +u_1\; x\;\frac{\partial^2 U_1\; x-\xi}{\partial x_1\partial x_2}-\right.\right.$$

$$\left.-i\frac{\partial u_1\; x}{\partial x_1}\frac{\partial U_1\; x-\xi}{\partial x_1}\right]\cos\;\nu_1,x_2\; +$$

$$+\left[-\frac{\partial u_1\; x}{\partial x_2}\cdot\frac{\partial U_1\; x-\xi}{\partial x_2}+i\frac{\partial^2 u_1\; x}{\partial x_1\partial x_2}U_1\; x-\xi\; +\right.$$

$$\left.\left.+iu_1\; x\;\frac{\partial^2 U_{\,1}\; x-\xi}{\partial x_1\partial x_2}\right]\cos\;\nu_1,x_1\;\right\}dx=$$

$$=\begin{cases}u_1(\xi), & \xi\in D_1,\\ \dfrac{1}{2}u_1(\xi), & \xi\in\partial D_1,\end{cases} \qquad (7)$$

But now $u_2(x)$ paid by the function (3) equation and let us bring the first main approach obtained with the help of a fundamental solution (6).

$$\int_{\partial D_2}\left\{\left[\frac{\partial^2 u_2\; x}{\partial x_1\partial x_2}U_2\; x-\xi\; +u_2\; x\;\frac{\partial^2 U_2\; x-\xi}{\partial x_1\partial x_2}+\right.\right.$$

$$\left.+\frac{\partial u_2\; x}{\partial x_1}\frac{\partial U_2\; x-\xi}{\partial x_1}\right]\cos\;\nu_2,x_2\; +$$

$$+\left[-\frac{\partial u_2\; x}{\partial x_2}\cdot\frac{\partial U_2\; x-\xi}{\partial x_2}-\frac{\partial^2 u_2\; x}{\partial x_1\partial x_2}U_2\; x-\xi\; -\right.$$

$$\left.\left.-u_2\; x\;\frac{\partial^2 U_{\,2}\; x-\xi}{\partial x_1\partial x_2}\right]\cos\;\nu_2,x_1\;\right\}dx=$$

$$=\begin{cases}u_2(\xi), & \xi\in D_2,\\ \dfrac{1}{2}u_2(\xi), & \xi\in\partial D_2,\end{cases} \qquad (8)$$

Thus, each of the main approaches we have obtained consists of two parts. The first parts are the solutions of the equations (2), (3), their derivatives and the analytical statements for the linear combinations, but the second parts give the necessary conditions.

Theorem 2. In the direction of $D=D_1\cup D_2$ - x_2 convex limited flatness area, if $\partial D=\Gamma_1\cup\Gamma_2$- is the Lyapunov line, then arbitrary solution of equation (1) pays main approaches.

V. NECESSARY CONDITIONS

At first let us give the necessary conditions, obtained from (7).

$$u_1\;\xi_1,\gamma_1\;\xi_1\;\;=-\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u_1\;x_1,\gamma_1\;x_1\;\left[1-i\gamma_1'\;x_1\;\right]dx_1}{\gamma_1\;x_1\;-\gamma_1\;\xi_1\;+i\;x_1-\xi_1}+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u_1\;x_1,0}{-\gamma_1\;\xi_1\;+i\;x_1-\xi_1}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}[\gamma_1(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)]\times$$

$$\times(\ln[\gamma_1(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)]-1)\left.\frac{\partial^2 u_1(x)}{\partial x_1\partial x_2}\right|_{x_2=\gamma_1(x_1)}\times$$

$$\times[1-i\gamma_1'(x_1)]dx_1+\frac{i}{\pi}\int_{a_1}^{b_1}[\gamma_1(\xi_1)-i(x_1-\xi_1)]\times$$

$$\times(\ln[-\gamma_1(\xi_1)+i(x_1-\xi_1)]-1)\left.\frac{\partial^2 u_1(x)}{\partial x_1\partial x_2}\right|_{x_2=0}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}\ln[\gamma_1(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)]\left.\frac{\partial u_1(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}dx_1-$$

$$-\frac{i}{\pi}\int_{a_1}^{b_1}\ln[-\gamma_1(\xi_1)+i(x_1-\xi_1)]\left.\frac{\partial u_1(x)}{\partial x_1}\right|_{x_2=0}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}\ln[\gamma_1(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)]\times$$

$$\times\left.\frac{\partial u_1(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\gamma_1'(x_1)\,dx_1, \qquad (9)$$

$$u_1(\xi_1,0)=-\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u_1(x_1,\gamma_1(x_1))}{\gamma_1(x_1)+i(x_1-\xi_1)}\times$$

$$\times[1-i\gamma_1'(x_1)]dx_1-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u_1(x_1,0)}{x_1-\xi_1}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}[\gamma_1(x_1)+i(x_1-\xi_1)]\times$$

$$\times\{\ln[\gamma_1(x_1)+i(x_1-\xi_1)]-1\}\left.\frac{\partial^2 u_1(x)}{\partial x_1\partial x_2}\right|_{x_2=\gamma_1(x_1)}\times$$

$$\times[1-i\gamma_1'(x_1)]dx_1+\frac{1}{\pi}\int_{a_1}^{b_1}(x_1-\xi_1)\times$$

$$\times(\ln[i(x_1-\xi_1)]-1)\left.\frac{\partial^2 u_1(x)}{\partial x_1\partial x_2}\right|_{x_2=0}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}\ln[\gamma_1(x_1)+i(x_1-\xi_1)]\left.\frac{\partial u_1(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}dx_1-$$

$$-\frac{i}{\pi}\int_{a_1}^{b_1}\ln[i(x_1-\xi_1)]\left.\frac{\partial u_1(x)}{\partial x_1}\right|_{x_2=0}dx_1+$$

$$+\frac{i}{\pi}\int_{a_1}^{b_1}\ln[\gamma_1(x_1)+i(x_1-\xi_1)]\times$$

$$\times\left.\frac{\partial u_1(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\gamma_1'(x_1)\,dx_1. \qquad (10)$$

In the same way in all approaches necessary conditions are separated.

Finally, as can be seen from obtained main approaches each of these statements give two necessary conditions, the first of which are conditions from obtained by integrating of equations (2) and (3), but the remaining second parts become the same as in ordinary differential equations.

These necessary conditions are the following:

$$\frac{\partial}{\partial\xi_1}\left.\left(\frac{\partial u_1(\xi)}{\partial\xi_2}+i\frac{\partial u_1(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=\gamma_1(\xi_1)}= \qquad (11)$$

$$=\frac{\partial}{\partial\xi_1}\left.\left(\frac{\partial u_1(\xi)}{\partial\xi_2}+i\frac{\partial u_1(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=0}=0,$$

$$\frac{\partial}{\partial\xi_2}\left.\left(\frac{\partial u_1(\xi)}{\partial\xi_2}+i\frac{\partial u_1(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=\gamma_1(\xi_1)}= \qquad (12)$$

$$=\frac{\partial}{\partial\xi_2}\left.\left(\frac{\partial u_1(\xi)}{\partial\xi_2}+i\frac{\partial u_1(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=0}=0,$$

$$\frac{\partial}{\partial\xi_1}\left.\left(\frac{\partial u_2(\xi)}{\partial\xi_2}-\frac{\partial u_2(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=0}= \qquad (13)$$

$$=\frac{\partial}{\partial\xi_1}\left.\left(\frac{\partial u_2(\xi)}{\partial\xi_2}-\frac{\partial u_2(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=\gamma_2(\xi_1)}=0,$$

and

$$\frac{\partial}{\partial\xi_2}\left.\left(\frac{\partial u_2(\xi)}{\partial\xi_2}-\frac{\partial u_2(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=0}= \qquad (14)$$

$$=\frac{\partial}{\partial\xi_2}\left.\left(\frac{\partial u_2(\xi)}{\partial\xi_2}-\frac{\partial u_2(\xi)}{\partial\xi_1}\right)\right|_{\xi_2=\gamma_2(\xi_1)}=0,$$

The main purpose of this work is to determine all possible linearly independent necessary conditions for a given equation (1).

Theorem 3. An arbitrary solution of equation (1) within the conditions of Theorem 2 pays all the necessary conditions.

These necessary conditions are made up of a linear combination of the necessary conditions given by the main approaches.

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Надійшла до редколегії 19.05.2019