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PACKET QUANTIZATION OF THE ELECTROMAGNETIC FIELD USING ELEMENTARY WAVE PACKETS AS BASIS FUNCTIONS

The possibility of positional localization of a photon in space is shown. The proposed technique is based on the use of operators for the creation and destruction of photon packets and corresponding wave packets. As the latter, an orthonormal series of elementary wave packets (EWP), proposed in 2009 to describe, in particular, femtosecond optical pulses, was used. The electromagnetic structure of photons and the formula for their energy have been obtained. The latter depends on the ratio of the order of the EWP function n to the duration of the photon pulse. Therefore, the structure of a photon at a fixed energy can be different, in particular, at $n \gg 1$, it turns into an ordinary harmonic signal. The results were used to write down a modified formula for the thermal radiation of an absolutely black body – Planck's formula.

Keywords: *photon, spatial localization, elementary wave packets, Planck's formula.*

MSC 2020 classification: 81S08, 81V80.

Introduction

Significant practical improvements in devices for the generation and detection of single-photon optical signals led to the activation of the development of new areas of quantum optics: quantum computers, the latest information systems, quantum tomographs. At the same time, a modern experimental base was formed for the study of fundamental quantum phenomena: entangled quantum states, particle-wave properties of quantum objects, conditions for distinguishing classical and quantum properties, etc. A special place in this list is occupied by the carrier of the electromagnetic field – the photon. Its integral characteristics are well known and have been used in quantum optics for more than 100 years, starting with the works of M. Planck, A. Einstein, and P. Dirac. However, the modern stage requires the study and application of more detailed structural characteristics of photons as quantum objects limited in time and space. The thesis that any physical object must be local cannot be doubted (Babaei, & Mostafazadeh, 2017; Bykov, 2008; Bennet, 2016; Hodgson, 2023; Raimer, & Polakos, 2023). The use in Planck's formula for the energy of a frequency photon as a characteristic of a monochromatic (infinite) signal can be considered as an approximate limiting case. The problem is similar to that in classical physics: is the distribution of parameters of local objects correct on the basis of infinite harmonic functions? The answer is almost obvious: the basis functions must be defined in a limited domain (Ovechko, 2009; Ovechko, 2012). This work is devoted to the substantiation of such a series distribution for the field representation of photons.

1. The technique of packet quantization of the electromagnetic field

The quantum state of the electromagnetic field is considered coherent. Consider a one-dimensional signal with linear or circular polarization. Then operator of the electric field (Bykov, 2008)

$$E\ (\mathrm{z,t}) = \varphi(\mathrm{z,t})\ A\ + \mathrm{e.c.} = \varphi(\mathrm{z,t})\ A\ + \varphi(\mathrm{z,t})A^{+},$$

where $A\ = \int dk\, W(k)\hat{a}(k)$ – the packet photon annihilation operator; $A^{+} = \int dk\, W(k)\hat{a}^{+}(k)$ – the packet photon creation operator; W (k) – Fourier spectrum of the elementary wave packets U (t); φ (z, t) – the shape of the wave packet.

Commutator for A , A^{+} operators is [A , A^{+}]=1. Accordingly, these operators generate a system of stationary states |n>

$$|\mathrm{n}> = \frac{A^{+n}}{\sqrt{n!}} \mid 0 >, \quad A \mid 0 > = 0.$$

An arbitrary quantum state can be represented as

$$|\varphi> = \sum_{n=0}^{\infty} \varphi_n \mid \mathrm{n} >.$$

The problem of packet quantization now consists in the conditioned choice of the wave packet system. The author of the work (Bykov, 2008) proposed the functions of a parabolic cylinder. The latter are orthonormal, but are exponentially infinitely decreasing functions. We proposed as basic functions elementary wave packets (EWP) domain (Ovechko, 2009; Ovechko, 2012), which were proposed and used by us to describe pulsed electromagnetic fields, in particular femtosecond laser pulses (Ovechko, 2012; Mygashko, & Ovechko, 2018). Graphs of odd(a) and even(b) EWP-functions of lower orders are shown in Fig. 1.

$$\mathrm{Sod}(p) = \sqrt{\frac{2}{(n'+1)(n'+3)}}(-1)^{\frac{n'-1}{2}}(\mathrm{Sec}[\pi p]\mathrm{Sin}[\pi p(n'+1)] + (n'+1)\mathrm{Sin}[(n'+2)\pi p]),$$
$$n' = 1,3,5,\ldots \tag{1}$$

$$\mathrm{Sev}(p) = \sqrt{\frac{2}{n'(n'+2)}}\left(1 - (-1)^{\frac{n'}{2}}(Sec[\pi p]Cos[\pi p(n'+1)] + n'Cos[n+2][(n+2)\pi p])\right),$$
$$n' = 2,4,6,\ldots, p = t/\tau,\ -1/2 < p < 1/2. \tag{2}$$

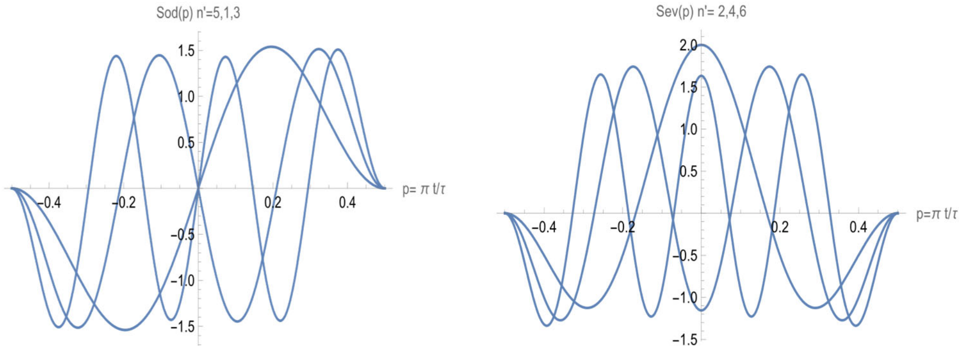
The shape of the electromagnetic quantum packet is

$$\mathbf{\Phi}(\mathrm{t},\rho,\varphi) = \mathrm{E_0}\mathrm{U}_n(\mathrm{p})\mathrm{R}(\rho,\varphi)\left(\mathbf{e}_x\mathrm{Cos}\left[\pi\,\Omega\frac{\mathrm{t}}{\tau} - \mathrm{F}\right] + \mathbf{e}_y\,\mathrm{Sin}\left[\pi\,\Omega\frac{\mathrm{t}}{\tau} + \mathrm{F}\right]\right),$$

$$E_0=(\varepsilon/\varepsilon_0\pi\rho_0^2 C\,\tau)^{1/2},\ U_n(p)=Sod(p), Sev(p),\ p=t/\tau,$$
$$R(\rho,\varphi)=(\pi\rho_0^2(1\text{-}40/9\pi^2))^{1/2}(Cos[\pi\rho/\rho0+Cos[2\pi\rho/\rho0]), \quad (3)$$

where ε – packet energy, $Sod(p), Sev(p)$ – odd and even pulses; $R(\rho, \varphi)$ – transverse distribution, $0 < \rho/\rho_0 < 1$; F = 0, π/4 – circular and linear polarization accordingly; $\Omega = 5, 6, 7, \dots$ for odd and even pulses accordingly.

The formula for the electromagnetic packet quantum of energy ε remains undefined in (3). The next section will be devoted to this issue.



a) b)

Fig. 1. EWP-functions: Sod(p)- odd(a) and Sev(p)-even(b), n'-functions order

2. Electromagnetic packet quantum energy

Consider the semiclassical formula for the probability of absorption of an electromagnetic pulse by a two-level quantum system (Debierre et al., 2015; Rag, & Gea-Banacloche, 2017). Let us take a number of elementary wave packets (1), (2) as a system of basic orthonormal functions.

$$P_{ge}(\tau) = \frac{|< e|d|g >|^2}{\hbar^2}\left|\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt\, e^{-i\omega_0 t}E(t)\right|^2,$$
$$E(t)=E_0\,U_n(t/\tau). \quad (4)$$

Separate consideration of even and odd pulses simplifies expression (4) for its further optimization in terms of duration τ. The replacement of variables in the integral (4) leads to the task of finding maxima separately for even and odd functions

$$P_{ge}(y) = \frac{|<e|dE_0|g>|^2}{(\hbar\omega_0)^2}\left|\int_{-1/2}^{1/2} dx\, y\, Sin[xy]Sod(x)\right|^2, \quad (5)$$
$$P_{ge}(y) = \frac{|<e|dE_0|g>|^2}{(\hbar\omega_0)^2}\left|\int_{-1/2}^{1/2} dx\, y\, Cos[xy]Sev(x)\right|^2, \quad (6)$$

where $x=t/\tau$, $y=\omega_0\tau$.

Next, the algorithm for the study of probabilities (5), (6) is presented. Graphs of dependences of the numerical values of the integrals (5), (6) on the parameter $y = \omega_0\tau$ were constructed for each of the EWP functions within $1 < n < 12$ in Fig. 2.

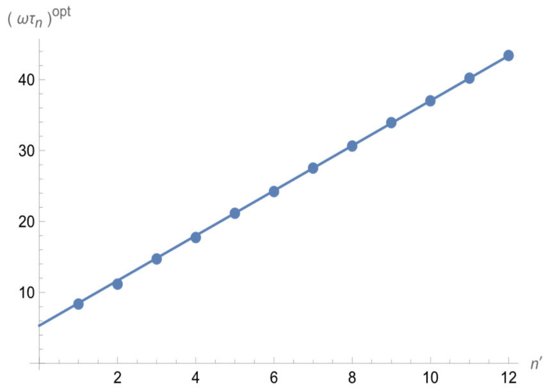


Fig. 2. Packet duration parameter $\omega_0\tau$ dependence with its order n'

Then, the location of the maxima $\omega_0\tau_n$ was found by the coordinates of the graphs, and the point dependence y (n) was constructed. The last one was approximated by a linear function

$$\omega_0\tau_n = a\,(n + b),$$

where $a \cong 3.17$, $b \cong 1.68$

The absorption process is one-photon. Therefore, energy $\hbar\omega_0$ is absorbed in one cycle, from were

$$\varepsilon=\hbar\omega_0=\frac{\hbar}{\tau_n}a(n+b). \quad (7)$$

The last formula determines the energy of a photon in the case of its packet representation. It depends on the duration of the packet τ_n and the order n of the EWP-function (its shape). If n>>b, then $\varepsilon = \frac{\hbar}{\tau_n}$ an, which corresponds to Planck's formula, if we put $\omega_0 = \frac{a\,n}{\tau_n}$. So $T_0 \cong \frac{2\tau_n}{n}$ It is possible to estimate the minimum pulse duration, for example, of a Ti: Sapphire laser (λ_0 = 805 nm). To do this, we substitute n = 1 in the formula (7).

Whence

$$\tau_n^{min} = \tau_1 = \frac{\hbar}{\varepsilon} a(1+b) \cong 3.6\, fs.$$

The shape of the laser pulses corresponds (Khelladi, 2021) to the even EWP – functions (n=2). From where $\tau_2 = \frac{\hbar}{\varepsilon}$ a(2+b) $\cong 4.9\, fs$. The duration estimate differs from the experimental value by only 25% (τ_{exp} = 6.5 fs) (Khelladi, 2021).

3. Thermal radiation

Let's use the formula for photon energy (7) and obtain Planck's modified formula for absolute black body (ABB) radiation. The probability of m photons excitation Pm for the equilibrium state of the field and matter at temperature T is equal to (Loudon, 2003)

$$P_m = e^{-m\,\epsilon_{ph}/kT}(1 - e^{-\epsilon_{ph}/kT}).$$

Then the average number of photons in the field mode is

$$<m> = \sum_m P_m = \frac{1}{-1 + e^{\epsilon_{ph}/kT}}.$$

The spectral radiation density by photon energy is equal to the product of the mode density by the average number of photons in the mode and the photon energy. Then

$$\partial 2\rho\ \partial V \partial \epsilon ph\ =\ \varepsilon ph 2\pi 2 C3\ \hbar 3\ \ \varepsilon ph - 1 + e\ \ \epsilon ph kT. \qquad (8)$$

In the last formula (8), we take into account $\tau_0 = h/\epsilon_{ph}$. Then

$$\frac{\partial\rho}{\partial\varepsilon_{ph}} = \frac{\partial\rho}{\partial\tau_0}\frac{1}{\left|\frac{\partial\varepsilon}{\partial\tau_0}\right|} = \frac{\partial\rho}{\partial\tau_0}\frac{\tau_0^2}{h}.$$

Finally

$$\frac{\partial^2\rho}{\partial V\partial\tau_0} = \frac{\partial I}{\partial V} = \frac{8\pi h}{\tau_0^5 C^3}\left(\frac{1}{-1 + e^{h/\tau_0 kT}}\right)\left[\frac{J}{s\cdot m^3}\right]. \qquad (9)$$

The resulting formula is analogous to Planck's low. The only variable is the duration τ_0. The results of the calculations are presented in Fig. 3.

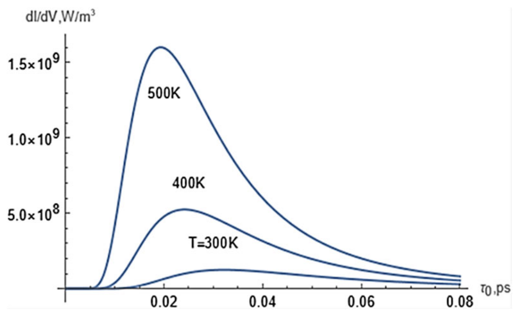


Fig. 3. Modified Plank's formula on the duration variables

Let's find the dependences corresponding to the displacement formulas of Wien and Stefan-Boltzmann. We will calculate maximum for the dependence (9).

$$\frac{\partial}{\partial\tau_0}\left(\frac{\partial I}{\partial V}\right) = 0.$$

From where

$$(T\,\tau_0)_{extr} = \frac{h/k}{5+W},$$

here W is the solution of the equation $We^W = \frac{-5}{e^5}$. Then

$$(T\cdot\ \tau_0)_{extr} = 9.664\cdot 10^{-12}[s\cdot K]. \qquad (10)$$

Equation (10) is modified Winn formula. (τ_0)extr = = 1.26 $\frac{\hbar}{kT}$ (10) can be compared with the coherence time of thermal radiation $\tau c = \frac{\hbar}{kT}$ (Mehta, & Wolf, 1967; Lerner, Cutler, & Miskovsky, 2015). They are identical within the definition. If (19) is multiplied by the light velocity C, then we obtain the Wien constant bW = 2.898 10^{10} [m · K]. For T = 300 K (τ_0)extr = 32.2 fs. To find the duration of the photon of the nth order, we must multiply τ_0 by (n+1.7)/2. Therefore, the higher the order of the photon n – the longer its duration (energy is constant). Integrating the formula (10) by τ_0 and multiplying by C/4 leads to the well-known Stefan – Boltzmann formula for the total intensity of the flux of electromagnetic radiation of an absolutely black body under conditions of thermal equilibrium.

Discussion and conclusions

The packet representation of photons using an orthonormal series of elementary wave packets (EWP) made it possible to describe the localized states of photons. A semiclassical solution for the probability of single-photon absorption in a two-level system was used to obtain the quantized formula for photon energy. The found formula of the energy depends on the duration of the quantum τ and its field distribution (of the order of n EWP – functions). The resulting formula was used to write

Planck's modified formula for blackbody radiation. From the formula, which is analogous to Winn's displacement formula, it follows that the duration of the quanta corresponding to the maximum of the Planck distribution coincides with the coherence time of thermal radiation at the corresponding temperature T.

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ПАКЕТНЕ КВАНТУВАННЯ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ З ВИКОРИСТАННЯМ ЯК БАЗИСНИХ ФУНКЦІЙ ЕЛЕМЕНТАРНИХ ХВИЛЬОВИХ ПАКЕТІВ

Показано можливість позиційної локалізації фотона у просторі. Запропонована методика базується на використанні операторів народження та знищення пакетів фотонів і відповідних хвильових пакетів. У ролі останніх було використано ортонормований ряд елементарних хвильових пакетів (ЕХП), запропонований у 2009 р. для опису, зокрема, фемтосекундних оптичних імпульсів. Отримано електромагнітну структуру фотонів і формулу для їхньої енергії, що залежить від відношення порядку EWP-функції n до тривалості імпульсу фотона τ. З огляду на це структура фотона за фіксованої енергії може бути різною, наприклад, за $n \gg 1$ вона переходить до звичайного гармонічного сигналу. Результати було використано для запису модифікованої формули теплового випромінювання абсолютно чорного тіла – формули Планка.

Ключові слова: ***фотон, просторова локалізація, елементарні хвильові пакети, формула Планка.***

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