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SIX-SECTOR LORENZ MODEL FOR CRITICAL INFRASTRUCTURE RISK MANAGEMENT

The work is dedicated to the problem of developing mathematical methods for assessing the vulnerability of critical infrastructure (CI).

The main goal of the study is to create a mathematical model for forecasting crisis phenomena, ranking threat levels, and identifying weak links in CI that significantly affect the deformation of the security space. The model includes an economic block and a risk assessment block. The economic block is described by a six-sector Lorenz model with variable coefficients, which integrates similarly described economic sectors into a unified structure. Each sector is considered in terms of productivity, employment, and structural disruptions in sectors directly related to CI activities: energy, transport infrastructure, water supply networks and hydraulic structures, chemical industry, production, storage, and transportation of hazardous materials, as well as food and medical supplies, and information technology. The risk assessment block uses catastrophe theory methods to evaluate vulnerability of CI. Risk is assessed based on the proximity of the model parameters to their bifurcation values, upon reaching which the system transitions from a stationary state, characterized by acceptable vulnerability, to a state of increased vulnerability.

The main results of the work are related to the numerical study of the developed model. The conditions for the transition of interconnected economic sectors to a regime of deterministic chaos, due to changes in specific demand and supply for production system outputs and the number of jobs in them, as well as changes in the rate of elimination of structural disruptions, have been identified. It has been shown that there are parameter value ranges in the model where minor changes lead to significant transformations in the CI system and its security.

Further development of this work will focus on determining the dependence of the control parameters of the developed model on technological, environmental, economic, and social factors that characterize the functioning of CI.

K e y w o r d s : ***Lorentz model, mathematical modeling, critical infrastructure, deterministic chaos, risk assessment***.

AMS 2020 classification: 62H30, 34C60.

Introduction

The increase in the number of technogenic disasters, along with the scale of the damage they cause, has focused attention on the problem of a crisis in management systems. This crisis is associated with a narrowing of the forecasting horizon, manifesting in the fact that low-probability events occur much more frequently than predicted by traditional safety assessment methods. Due to globalization, the technosphere has transformed into a system characterized by the following features: 1) the presence of multiple development trajectories and "windows of vulnerability"; 2) a limited timeframe for decision-making; 3) the rapid pace of changes in order parameters that define human safety; 4) a strong dependence on initial conditions, making it so that small influences can lead to significant divergences in development trajectories (Wilkinson, Elani, & Eidinow, 2002).

The increasing role of minor impacts expands the scope of traditional risks and gives rise to fundamentally new ones. These are associated with events of enormous destructive power, prolonged in time, unfolding on a transnational scale, where their effects are felt in geographical zones far from the incident site (Renn, 2003). This has led to a sharp increase in the vulnerability of critical infrastructure (CI). Until recently, security policy in the field of CI protection was based on addressing issues related only to the crisis in management systems (Ermoliev, & von Vinterfeldt, 2012; Norkin et al., 2018; Pepelyaev, Golodnikov, & Golodnikova, 2022; Zgurovsky, 2024). However, in the context of military conflicts, it is necessary to approach the problem of CI protection from a new perspective. There is a need to significantly revise the old security paradigms, considering the role that CI has come to play in modern warfare as a target for attacks. It is essential to consider the likelihood of deliberate destruction of energy facilities, hydraulic structures, and hazardous enterprises in the nuclear and chemical industries. Therefore, the task of developing and implementing automated systems that enable forecasting and prevention of crisis phenomena, ranking threat levels, and identifying weak points that significantly influence the deformation of the security space has become highly relevant. The development of mathematical models and methods to address these tasks is the goal of this work.

In works (Sergienko, Yanenko, & Atoev, 1997; Atoyev et al., 2017; Atoyev, & Knopov, 2022), mathematical models for analyzing the vulnerability of CI have been developed based on the theory of smooth mappings, using examples from housing and communal services, potentially hazardous facilities, and environmentally polluting enterprises. The advantages of the proposed approach include the ability to: 1) forecast and manage complex risks; 2) rank different types of threats, identify weak links in CI, and quantitatively assess the proximity to critical parameter values indicating increase in risk; 3) calculating, using optimal control theory, the trajectories of changes in CI parameters to reduce the risks of man-made disasters and minimize the costs of their elimination.

In works (Atoyev, 2005; Atoyev, & Knopov, 2023), multi-sector Lorenz models were developed to study the interconnections between different sectors of the economy and to determine the conditions for instability in such complex systems. These models integrate in a holistic structure various economic sectors, described in the same way. Each sector is considered in terms of productivity levels, the number of jobs, and structural disruptions. They allow for the calculation of the losses from instability in the functioning of such complex systems as CI. The measure of these losses is the level of structural disruptions.

To assess risks for CI, we will combine these two approaches. The model should include: 1) a block for evaluating the proximity of parameters characterizing CI to their bifurcation values, the attainment of which sharply increases the probability of its destruction; 2) an economic block for assessing the impact of CI destruction on the economy. Furthermore, the system of interconnections between these blocks should be considered.

2. Mathematical model of vulnerability assessment

The mathematical formalism of catastrophe theory (Poston, & Stewart, 1978) allows for the calculation of bifurcation values, curves, and surfaces of control parameters. The risk of the system transitioning from one steady state (normal) to another (catastrophe) will be assessed by the degree of proximity of the control parameter values to their bifurcation values.

The function $U(x)$, called the potential, will be introduced as follows:

$$U(x) = -\int F(x)dx$$

where $F(x)$ is a function describing the motion in a potential field $U(x)$.

In mechanics, quantum mechanics, and field theory, Hamiltonian systems are primarily considered. However, in "real life", we often deal with dissipative systems. Tor study the system behavior near local extrema function $U(x)$, the function $F(x)$ is expanded into a series in the neighborhood of stationary states, and the expansion is limited to a few small terms. To assess risk, we will use the "cusp" catastrophe, whose potential has the following form:

$$U(x) = -\frac{x^4}{4} - A_1\frac{x^2}{2} - A_2x.$$

The manifold of the catastrophe is defined by the equation

$$-\frac{\partial U(x)}{\partial \mathrm{x}} = \frac{\mathrm{dx}}{\mathrm{dt}} = x^3 + A_1x + A_2. \tag{1}$$

The polynomial (1), according to Sturm's theorem regarding the number of real roots of a polynomial (Andronov, Witt, & Khaikin, 1959), has at most three and at least one real root. The number of roots depends on the discriminant $\Delta = 4A_1^3 + 27A_2^2$. If the condition $\Delta < 0$ is satisfied, there are three real roots. The system has three steady states, two of which are stable. The first stable steady state represents the norm (an acceptable level of vulnerability of the system), while the second represents a crisis (a dangerous level of vulnerability). When $\Delta > 0$ there is one real root and two imaginary ones. The bifurcation curve has the following form:

$$\Delta(A_1, A_2) = 0. \tag{2}$$

The measure of the risk for the transition of the system from one steady state, corresponding to the norm, to another, corresponding to a crisis, is the degree of proximity of parameters A_1 and A_2 to their bifurcation values A_1^* and A_2^*. Since the parameter A_2 characterizes rapid impacts on the system, it was used to assess the risk in (Atoyev, Tomin, & Aksionova, 2007; Atoyev et al., 2017). The bifurcation value of this parameter is determined from equation (2)

$$A_2^* = \pm 2\tfrac{A_1}{3}(A_1/3)^{1/2}.$$

The risk value R can be assessed as follows:

$$R = 1 - (A_2^* - A_2)/A_2^*. \tag{3}$$

If $A_2 = 0$, then $R = 0$, and if $A_2 = A_2^*$, then $R = 1$. If $A_2 > A_2^*$, we will also consider $R = 1$. For the chosen acceptable level of risk R_{ac} , the corresponding value A_{2ac} is determined from (3), which will be used in further research to determine the strategy for ensuring the specified level of CI security.

Risk assessment for critical infrastructure requires reliable information about the objects being studied. The absence of such information, due to incomplete sampling or the stochastic nature of the research object, leads to uncertainty (Pepelyaev, Golodnikov, & Golodnikova, 2024). In this situation, the effectiveness of deterministic models decreases, making the issue of accounting for stochasticity particularly relevant (Wagenmakers et al., 2005).

There are several approaches to addressing this issue. In (Ebeling, & Engel-Herbert, 1982), the case of non-equilibrium stochastic systems approaching a stationary state was examined. It was shown that for gradient systems, the set of transitions (catastrophes) coincides under both deterministic and stochastic descriptions and consists of all points in the control space where the number and type of extremal points change. When the modeled process is stochastic and governed by the Langevin equation, equation (1) transforms into the following form:

$$-\frac{\partial U(x)}{\partial \mathrm{x}} = \frac{dx}{\mathrm{dt}} = x^3 + A_1x + A_2 + D^{1/2}\xi(t)$$

where D is a constant, $\xi(t)$ is an independent Gaussian δ-correlated fluctuating quantity. According to (Ebeling, & Engel-Herbert, 1982), the system is described using a single, globally stable stationary probability distribution $P^0(x, \boldsymbol{A})$, which is related to $U(x, \boldsymbol{A})$, as follows:

$$P^0(x, \boldsymbol{A}) = (1/n)\exp[2U(x, \boldsymbol{A})/\mathrm{D}].$$

3. Model for assessing the economic consequences of CI disruptions

For the economic block of the model, we will use a multi-sector Lorenz model. In this context, as in (Atoyev, & Knopov, 2023), we will consider the following assumptions: 1) there is competition for labor among different sectors of the economy, so the growth of the production function in one sector hinders job creation in others; 2) fluctuations in energy prices, changes in the balance of natural and material resources, changes due to the disruption of CI, and the introduction of new technologies that significantly alter productivity – all of these provoke random disturbances $w_{ij}(t)$ that become additional factors for increasing the level of structural disruptions; 3) processes in different sectors of the economy may proceed at different speeds, with time scaling across sectors carried out using parameters ε_i; 4) An increase in the vulnerability level of CI x leads to a decrease in the system's capacity to recover from structural disruptions; 5) control parameters A_{1i} and A_{2i} change with an increase in structural disruptions. The mathematical model that integrates the economic and security blocks has the following form:

$$\varepsilon_i\frac{dX_i}{dt} = \sigma_i(Y_i - X_i) + \delta_i\dot{w}_{ij},$$

$$\varepsilon_i\frac{dY_i}{dt} = [F_i(X_1, X_2, \dots X_6) - Z_i]X_i - Y_i + \delta_i\dot{w}_{ij}, \tag{4}$$

$$\varepsilon_i \frac{dZ_i}{dt} = X_i Y_i - b_i(1-k_i x) Z_i + \delta_i \dot{w}_{ij},$$
$$\varepsilon_x \frac{dx}{dt} = x^3 + \left[A_{1i} + \sum_{i=1}^{6} l_i Z_i\right] x + A_{2i} + \sum_{i=1}^{6} m_i Z_i + \delta_i \dot{w}_{ij},$$

where X_i are normalized levels of productivity, Y_i are normalized jobs, Z_i are normalized level of structural disturbances in the following sectors: energy facilities and networks (i=1); 2) transport service(i=2); hydraulic structures, water supply networks (i=3); chemical industry, production, transportation and storage of hazardous materials (i=4); food and medical supply facilities (i = 5); communication and information technology (i = 6); x is normalized level of vulnerability of CI; σ_i, r_i, b_i $(i = \overline{1,6})$ are parameters of the Lorenz model, which are functions of specific demands and supplies for the products of the respective production systems and the number of jobs occupied in them; $F_i = r_i(1 - \sum_{i=1}^{6} a_{ij} X_i)$, a_{ij} are parameters characterizing competition between different sectors of the economy in labor markets $(i \neq j)$; k_i, l_i, m_i are parameters characterizing the impact of critical infrastructure vulnerability on economic sectors, and the impact of structural disruptions on the dynamics of critical infrastructure vulnerability, respectively; w_{ij} are independent standard Wiener processes with parameters $E(w_{ij}(t) - w_{ij}(s)) = 0$; $E\,(w_{ij}(t) - w_{ij}(s))^2 = |t-s|$; δ_i are intensities of disturbance.

According to the results obtained in (Atoyev, 2005), the parameters σ_i, r_i, b_i are related to the characteristics that determine the functioning of economic sectors as follows

$$\sigma_i = (\alpha_{1i}\beta_{2i})/(\alpha_{2i}\gamma_{2i}), \quad r_i = (\beta_{1i}\gamma_{1i})/(\beta_{2i}\gamma_{2i}), \quad b_i = \zeta_i/(\alpha_{2i}\gamma_{2i}), \tag{5}$$

where α_{1i} and α_{2i} are parameters that characterize adaptive capabilities; β_{1i} are demands for the activity of the *i*-th manufacturing system, normalized to the unit of measurement of the material production system, which is considered to be a job in the corresponding industry Y_i; β_{2i} are supplies normalized to the unit of function of the *i*-th manufacturing system X_i, γ_{1i} are demands for increasing the number of jobs, normalized to the unit of X_i; γ_{2i} are supplies for increasing the number of jobs, normalized to the unit of Y_i; ζ_1 are the specific rates of disruption elimination.

The investigation of the classic Lorenz model (Lorenz, 1963) shows that increasing the parameter r is crucial for the onset of turbulence. The operating modes change at the following bifurcation values of r: $r = 13.926$, $r = 24.06$, $r = 24.74$. When $r > 24.74$, the system transitions to a regime of metastable chaos – a strange attractor emerges (Kaplan, & Yorke, 1979). In model (4) the transition of parameters r_i to the range of values corresponding to the regime of metastable chaos is achieved by increasing the specific demand for the products of the manufacturing system and the number of jobs employed (parameters β_{1i} and γ_{1i}) or by reducing their specific supply (parameters β_{2i} and γ_{2i}).

Let's consider how the rate of disruption elimination affects the level of security of CI. As follows from (5), a decrease in the rates of disruption elimination ζ_i, leads to decline in parameters b_i. Rates ζ_i change significantly faster than the adaptive capabilities α_{2i} and the supply of increasing the number of jobs γ_{2i}. Within short time intervals α_{2i} and γ_{2i} can be consider constants. In these conditions, we will associate the decrease in b_i only with the reduction in ζ_i.

Figure 1 shows how the dynamics of vulnerability x, the discriminant Δ, and the proximity of the current values of the parameters A_2 to their bifurcation values A_2^* change with the decrease of the parameters b_i of model (2). For simplicity in modeling, it was assumed that the coefficients for different sectors are the same.

Figure. 2 shows how the level of the total structural disruption of the CI $(\sum \int Z_i(t)dt)$ changes with the variation of the parameters r_i. The results of the research show that there are regions of parameters r_i. in which the level of structured disruption increases sharply. For the energy sector (i=1), when the demand-to-supply ratio reaches a level corresponding to the values of parameter r_1 in the range (22,24), the level of total disruption increases from 3856.4 to 8077.6. A further increase in r_1 initially reduces this level to 3925.9 at $r_i = 26$, but then the level of structural disturbances begins rise again.

In Figure 3, it is shown how the dynamics of disturbances Z_i change with a decrease in parameters b_i, as well as the projections of the phase space of system (2) onto the planes $Z_i - X_i$ and $Z_i - Y_i$, using one of the sectors of the economy related to the energy system as an example ($i = 1 = E$). The simulation results presented in Fig. 1 and Fig. 2 were obtained with the following values for the model parameters: $A_{1i} = 2$, $A_{2i} = 1$, $k_i = 0.5$, $l_i = m_i = 0.015$, $r_i = 24$, $\varepsilon_i = \varepsilon_x = 0.2$.

4. Dependence on small changes in parameters

The formation of a complex and interconnected network of modern society significantly deforms the fields of natural, technological, and economic risks. The nodes of this network serve as unique acupuncture points of vulnerability for CI, where small influences can lead to unforeseen consequences on a large scale with irreversible damage (a kind of domino effect). This occurs due to the density of connections between nodes, unprecedented lengthening of causal relationships, and the speed at which material values and information circulate through the links of this network. The behavior of such systems often becomes unstable (Laszlo, 1991).

Therefore, the network society can be seen as a system near an unstable stationary state within a bifurcation zone, where the likelihood of sudden and unforeseen events increases. Very small fluctuations can lead to abrupt changes in the socio-economic and natural-technological characteristics of society, with the amplitude of these jumps potentially being quite significant (Zgurovsky, & Pankratova, 2007; Ermoliev, & von Vinterfeldt, 2012).

Figure 4 illustrates the results of modeling, where a very slight change in the parameter l_i leads to a significant shift in the operating mode of system (4). When $l_i = 1$ the level of structural disruptions reaches maximum $\sum \int Z_i(t)dt = 8077.6$. With a small increase or decrease in l_i by a value of 10^{-9}, the level of structural disruptions significantly decreases (as shown in Figures 4b and 4d). Moreover, in this case the phase portraits of the system differ significantly.

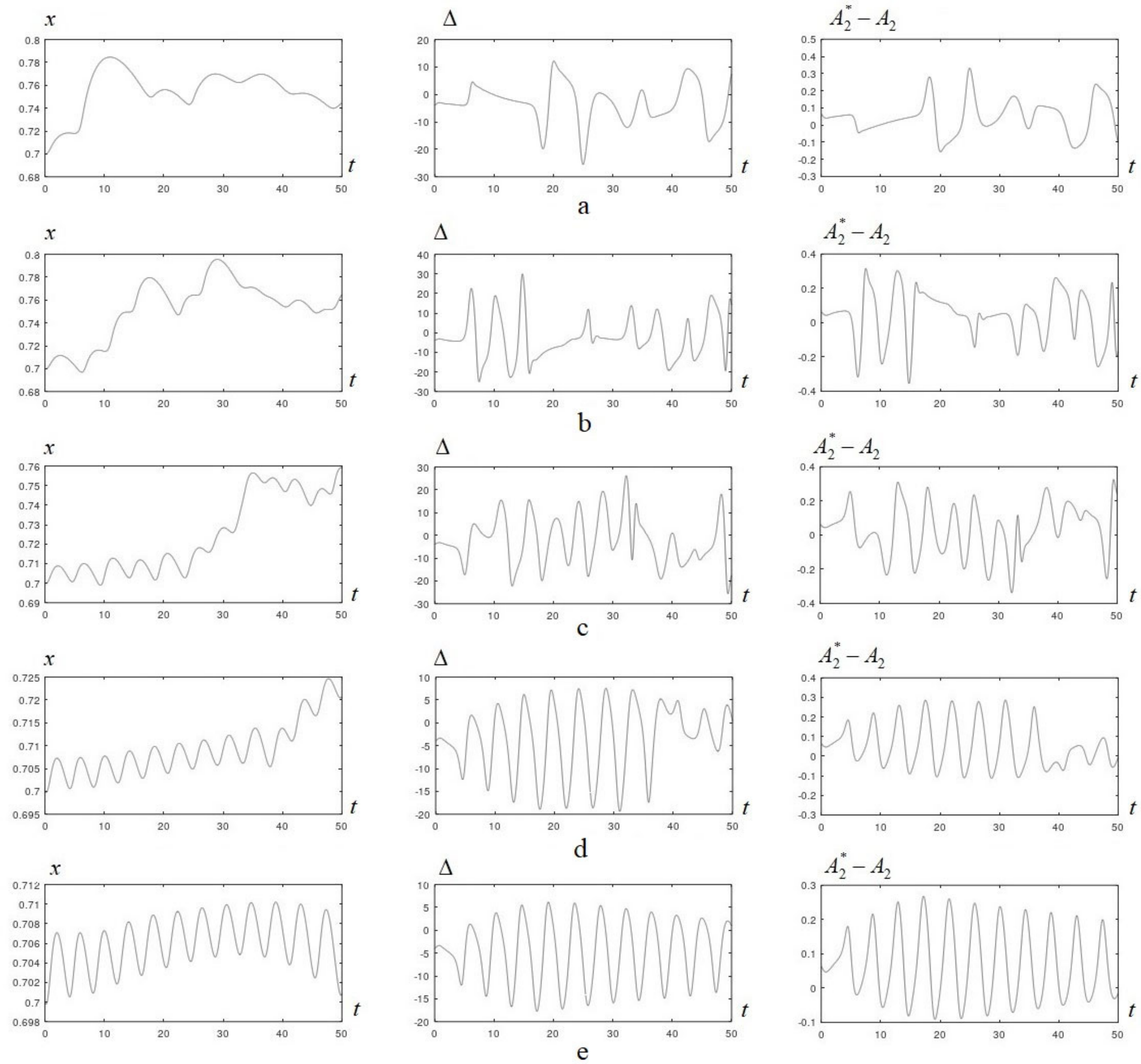

Fig. 1. The impact of changes in the rate of disruption elimination on the dynamics of changes in the vulnerability levels of CI: $b_i = 5/3$ (a); $b_i = 8/3$ (b); $b_i = 12/3$ (c); $b_i = 15.4825/3$ (d); $b_i = 16/3$ (e)

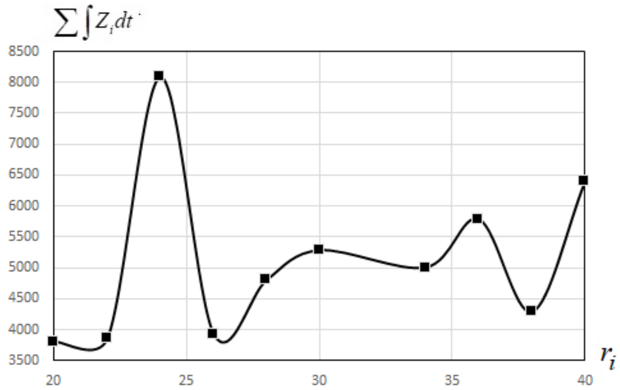

Fig. 2. Dependence of total structural disruptions of CI on parameter r_i.

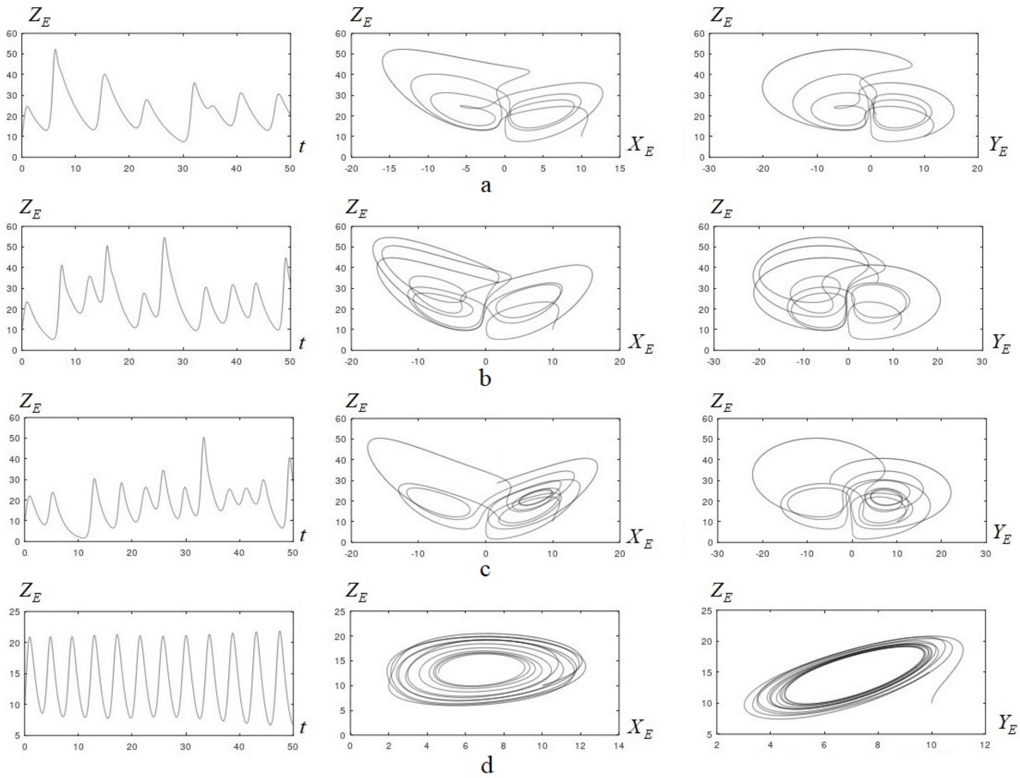

Fig. 3. The impact of changes in the rate of disruption elimination on the dynamics of changes in disturbances in energetic sector: $b_i = 5/3$ (a); $b_i = 8/3$ (b); $b_i = 12/3$ (c); $b_i = 16/3$ (d)

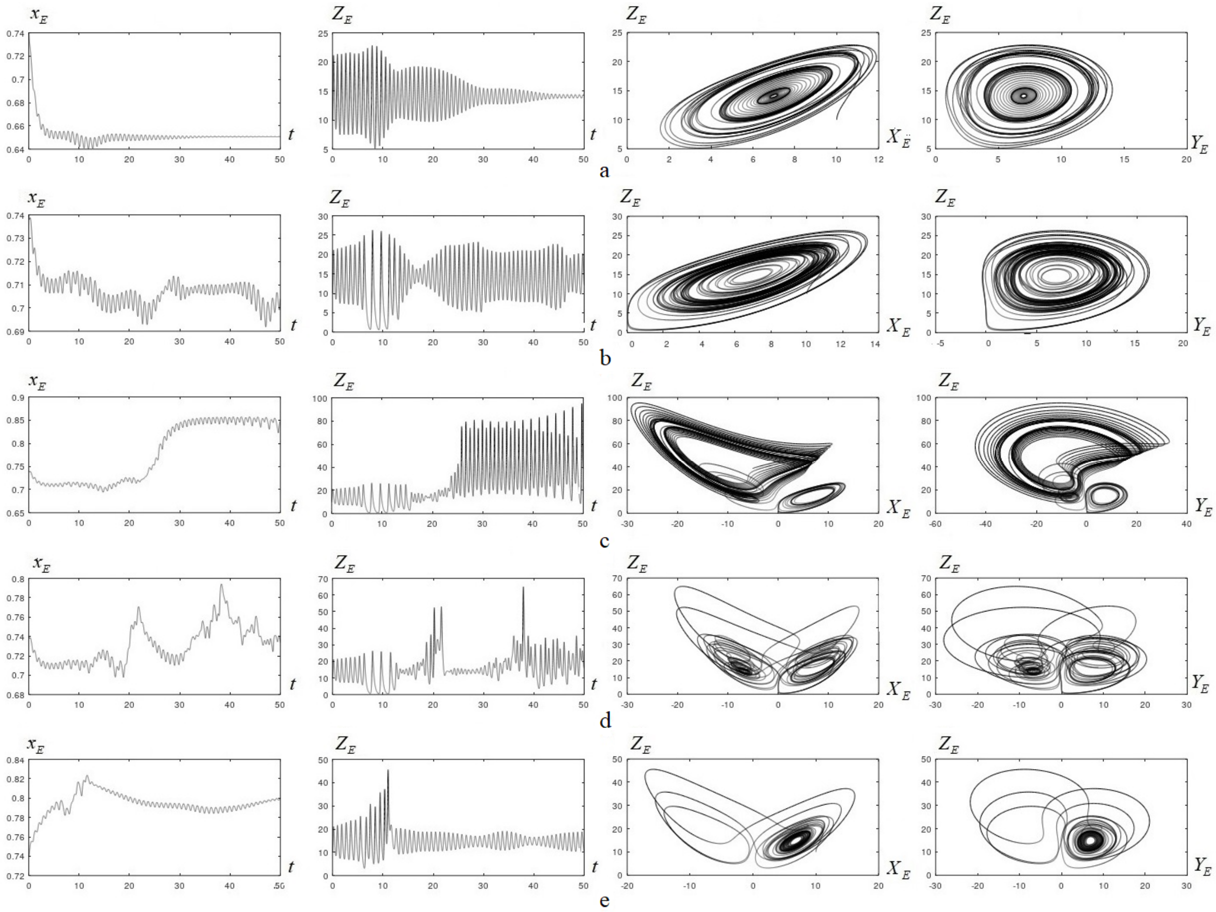

Fig. 4. Changes in the vulnerability of CI and in the level of structural disruptions in the energy sector with an increase in parameter l_i : $l_i = 0.954$, $\sum\int Z_i(t)dt = 3853.6$ (a); $l_i = 0.999999999$, $\sum\int Z_i(t)dt = 3870.9$ (b); $l_i = 1.0$, $\sum Z_i(t)dt = 8077.6$ (c); $l_i = 1.000000001$, $\sum\int Z_i(t)dt = 5192.8$ (d); $l_i = 1.046$, $\sum\int Z_i(t)dt = 4159.6$ (e)

Thus, a slight change in the balance of supply and demand can significantly alter the development trajectory of the system, change the vulnerability of CI, and affect the scale of negative consequences in case of its disruption. At the same time, due to this high sensitivity to small parameter changes under chaotic conditions, it is possible to shift the system from one unstable limit cycle to another, thereby qualitatively altering the system's behavior with minimal costs for changes in the interconnected sectors of the economy.

The further development of this work will be aimed at determining the dependence of the control parameters A_i and A_2 of model (1) on the technological, environmental, economic, and social factors that characterize the functioning of CI. These parameters may be determined using approaches, elaborated in (Atoyev et al., 2017; Ermoliev et al., 2020; Atoyev, & Knopov, 2023; Pepelyaev, Golodnikov, & Golodnikova, 2023; Shulzhenko, Nechaieva, & Leshchenko, 2024).

Discussion and conclusions

A mathematical approach has been proposed to assess the security level of critical infrastructure (CI) objects and the consequences of their destruction for interrelated sectors of the economy. This approach allows for the evaluation of threat levels, defined using the formalism of smooth function theory, based on the proximity of CI parameters to their bifurcation values, the achievement of which sharply increases the likelihood of their destruction. This way, the "safety margin" of objects is calculated. It is assumed that the probability of an attack on a CI object is higher the greater its vulnerability level. The assessment of the impact of CI destruction consequences on the economy is carried out using a multi-sectoral Lorenz model, which determines the dynamics of productivity levels, employment, and structural disruptions in sectors directly related to CI activities: energy, transportation, hydraulic structures and water supply networks, chemical industry, production, storage and transportation of hazardous materials, food and medical supply facilities, and information technology

The study investigated the impact of changes in the speed of eliminating structural disruptions on the dynamics of vulnerability levels in CI. Conditions for the transition of interrelated sectors of the economy into a state of deterministic chaos were identified as a result of changes in the specific demands and supplies of products in production systems and the number of occupied jobs, as well as changes in the speed of eliminating structural disruptions. It was demonstrated that there are regions of parameter values in the model where minor changes can lead to significant transformations in the CI system and its security space.

Authors' contribution: Konstantin Atoyev – conceptualization, methodology, empirical data collection and validation, empirical research, analysis of sources, preparation of literature review or theoretical foundations of research; Pavel Knopov – methodology, analysis of sources, preparation of literature review or theoretical foundations of research.

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ШЕСТИСЕКТОРНА МОДЕЛЬ ЛОРЕНЦА ДЛЯ УПРАВЛІННЯ РИЗИКАМИ КРИТИЧНОЇ ІНФРАСТРУКТУРИ

Присвячено проблемі розроблення математичних методів оцінки вразливості критичної інфраструктури (КІ). Основною метою пропонованого дослідження є створення математичної моделі для прогнозування кризових явищ, ранжування рівнів загроз і виявлення слабких ланок КІ, що суттєво впливають на деформацію простору безпеки. Модель охоплює економічний блок і блок оцінювання ризиків. Економічний блок описано шестисекторною моделлю Лоренца зі змінними коефіцієнтами, що об'єднує схожі досліджені сектори економіки в єдину структуру. Кожен сектор розглянуто з позиції продуктивності, зайнятості та структурних порушень у сферах, безпосередньо пов'язаних з діяльністю КІ: енергетика, транспортна інфраструктура, мережі водопостачання та гідротехнічні споруди, хімічна промисловість, виробництво, зберігання та транспортування небезпечних матеріалів, а також харчові та медичні товари й інформаційні технології. Блок оцінювання ризиків використовує методи теорії катастроф для визначення вразливості КІ. Оцінювання ризику здійснюється за наближеністю параметрів моделі до їхніх біфуркаційних значень, у разі досягнення яких система переходить від стаціонарного стану, що характеризується прийнятною вразливістю, до стану підвищеної вразливості. Проведено дослідження того, як зміна швидкості усунення структурних порушень впливає на динаміку змін рівнів уразливості КІ. Основні результати роботи пов'язані з чисельним дослідженням розробленої моделі. Ідентифіковано умови для переходу взаємопов'язаних секторів економіки до режиму детермінованого хаосу внаслідок зміни питомого попиту та пропозиції на продукцію виробничої системи та кількості робочих місць у них, а також зміни темпів ліквідації структурних порушень. Показано, що в моделі існують діапазони значень параметрів, де невеликі зміни призводять до значних трансформацій у системі КІ та її безпеці. Подальші дослідження будуть зосереджені на визначенні залежності керуючих параметрів розробленої моделі від технологічних, екологічних, економічних і соціальних факторів, що характеризують функціонування КІ.

Ключові слова: ***модель Лоренца, математичне моделювання, критична інфраструктура, детермінований хаос, оцінювання ризику.***

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