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ON SCHUR-TYPE THEOREM FOR LEIBNIZ 3-ALGEBRAS

One of the classic results of group theory is the so-called Schur theorem. It states that if the central factor-group $\frac{G}{\zeta(G)}$ of a group G is finite, then its derived subgroup $[G,G]$ is also finite. This theorem was proved by B. Neumann in 1951. This result has numerous generalizations and modifications in group theory. At the same time, similar investigations were conducted in other algebraic structures, namely in modules, linear groups, topological groups, n-groups, associative algebras, Lie algebras, Lie n-algebras. In 2016, L. Kurdachenko, J. Otal and O. Pypka proved an analogue of Schur theorem for Leibniz algebras: if central factor-algebra $\frac{L}{\zeta(L)}$ of Leibniz algebra L has finite dimension, then its derived ideal $[L,L]$ is also finite-dimensional. Moreover, they also proved a slightly modified analogue of Schur theorem: if the codimensions of the left $\zeta^l(L)$ and right $\zeta^r(L)$ centers of Leibniz algebra L are finite, then its derived ideal $[L,L]$ is also finite-dimensional. One of the generalizations of Leibniz algebras is the so-called Leibniz n-algebras. It is worth noting that Leibniz n-algebra theory is currently much less developed than Leibniz algebra theory. One of the directions of development of the general theory of Leibniz n-algebras is the search for analogies with other types of algebras. Therefore, the question of proving analogs of the above results for this type of algebras naturally arises. In this article, we prove the analogues of the two mentioned theorems for Leibniz n-algebras for the case $n = 3$. The obtained results indicate the prospects of further research in this direction.

K e y w o r d s : ***Leibniz algebra, Leibniz 3-algebra, Lie algebra, center, derived ideal, Schur theorem.***

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Introduction

Let L be an algebra over a field F with the binary operations $+$ and $[-,-]$. Then L is called a *Leibniz algebra* (more precisely, a *left Leibniz algebra*) if it satisfies the left Leibniz identity:

$$\big[a,[b,c]\big] = \big[[a,b],c\big] + \big[b,[a,c]\big].$$

for all $a,b,c \in L$. Leibniz algebras first appeared in the paper (Blokh, 1965), but the term "Leibniz algebra" appears in the book (Loday, 1992) and article (Loday, 1993). In (Loday, & Pirashvili, 1993), the authors conducted an in-depth study on Leibniz algebras properties. The theory of Leibniz algebras has developed very intensely in many different directions.

Note that Lie algebras present a partial case of Leibniz algebras. Conversely, if L is a Leibniz algebra in which $[a,a] = 0$ for every element $a \in L$, then it is a Lie algebra. Thus, Lie algebras can be characterized as anticommutative Leibniz algebras.

One of the key tendencies in the development of Leibniz algebra theory is the search for analogues of the basic results of Lie algebra theory. At the same time, there are very significant differences between these two types of algebras (Chupordia et al., 2019).

We recall some necessary definitions. Let L be a Leibniz algebra over a field F. If A, B are subspaces of L, then $[A,B]$ will denote a subspace generated by all elements $[a,b]$ where $a \in A, b \in B$.

A subspace A of L is called a *subalgebra* of L, if $[a,b] \in A$ for every $a,b \in A$. It follows that $[A,A] \leq A$. A subalgebra A of L is called a *left* (respectively *right*) *ideal* of L, if $[b,a] \in A$ (respectively $[a,b] \in A$) for every $a \in A, b \in L$. In other words, if A is a left (respectively right) ideal of L, then $[L,A] \leq A$ (respectively $[A,L] \leq A$). A subalgebra A of L is called an *ideal* of L (more precisely, *two-sided ideal*) if it is both a left ideal and a right ideal. If A is an ideal of L, we can consider a factor-algebra $\frac{L}{A}$. It is not hard to see that this factor-algebra also is a Leibniz algebra.

The *left* $\zeta^l(L)$ and *right* $\zeta^r(L)$ centers of a Leibniz algebra L are defined by the rules:

$$\zeta^l(L) = \{a \in L |\ [a,b] = 0 \text{ for all } b \in L\},$$
$$\zeta^r(L) = \{a \in L |\ [b,a] = 0 \text{ for all } b \in L\}.$$

It is not hard to prove that the left center of L is an ideal, but it is not true for the right center. The right center is a subalgebra of L, and, in general, the left and right centers are different. They even may have different dimensions (Kirichenko et al., 2017).

The center $\zeta(L)$ of L is the intersection of the left and right centers, that is

$$\zeta(L) = \{a \in L |\ [a,b] = 0 = [b,a] \text{ for all } b \in L\}.$$

Clearly, the center $\zeta(L)$ is an ideal of L. In particular, we can consider the factor-algebra $\frac{L}{\zeta(L)}$.

There is a very close connection between the center $\zeta(L)$ of the Leibniz algebra L and its derived ideal $[L,L]$. In (Kurdachenko, Otal, Pypka, 2016), the authors prove that if the central factor-algebra $\frac{L}{\zeta(L)}$ has finite dimension d, then the derived ideal $[L,L]$ is also finite-dimensional and $dim_F([L,L]) \leq d^2$. Moreover, they proved some modification of this result: if $codim_F\left(\zeta^l(L)\right) = d$ and $codim_F\left(\zeta^r(L)\right) = r$ are finite, then $dim_F([L,L]) \leq d(d+r)$.

The first of the mentioned results is a direct analogue of the famous group-theoretical Schur theorem. More precisely, Schur theorem states that if the central factor-group $\frac{G}{\zeta(G)}$ of a group G is finite, then the derived subgroup $[G,G]$ is also finite. In this formulation, for the first time it appears in the paper (Neumann, 1951). This theorem was obtained also in (Baer, 1952).

In (Casas, Loday, Pirashvili, 2002), the authors introduced the concept of Leibniz n-algebras. Let L be an n-algebra over a field F with the binary operations $+$ and an n-linear bracket $[-,\dots,-]$. Then L is called a *left Leibniz n-algebra* if it satisfies the following *left Leibniz n-identity*:

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$$[b_1, \dots, b_{n-1}, [a_1, \dots, a_n]] = \sum_{i=1}^{n} [a_1, \dots, a_{i-1}, [b_1, \dots, b_{n-1}, a_i], a_{i+1}, \dots, a_n]$$

for any $a_1, \dots, a_n, b_1, \dots, b_{n-1} \in L$. The theory of Leibniz n-algebras is much less developed than the theory of Leibniz algebras. Our goal is to establish connections between these types of algebras. In particular, in this paper we will prove analogs of the two above-mentioned results from the theory of Leibniz algebras for Leibniz n-algebras for $n = 3$.

1. Main results

Let L be a 3-algebra over a field F with the binary operation $+$ and ternary operation $[-,-,-]$. Then L is called a *Leibniz* 3-*algebra* (more precisely, a *left Leibniz* 3-*algebra*) if for all elements $a, b, c, x, y \in L$ it satisfies the *left Leibniz* 3-*identity*:

$$[x, y, [a, b, c]] = [[x, y, a], b, c] + [a, [x, y, b], c] + [a, b, [x, y, c]].$$

If A, B, C are subspaces of L, then $[A, B, C]$ will denote a subspace generated by all elements $[a, b, c]$ where $a \in A, b \in B, c \in C$. As usual, a subspace A of L is called a *subalgebra* of L if $[a, b, c] \in A$ for all elements $a, b, c \in A$. In other words, $[A, A, A] \leq A$. A subalgebra A is called a *left* (respectively *middle*, *right*) *ideal* of L if $[x, y, a] \in A$ (respectively $[x, a, y] \in A$, $[a, x, y] \in A$) for every $x, y \in L$, $a \in A$. In other words, if A is a left (respectively middle, right) ideal of L, then $[L, L, A] \leq A$ (respectively $[L, A, L] \leq A$, $[A, L, L] \leq A$). A subalgebra A of L is called an *ideal* of L (or *three-sided ideal*) if it is a left, middle and a right ideal of L. If A is an ideal of L, we can say about the factor-algebra $\frac{L}{A}$. It is not hard to see that $\frac{L}{A}$ is also a Leibniz 3-algebra.

Let L be a Leibniz 3-algebra over a field F, M be a non-empty subset of L, and H be a subalgebra of L. Put

$$Ann_H^l(M) = \{a \in H |\ [a, M, M] = < 0 >\},$$
$$Ann_H^m(M) = \{a \in H |\ [M, a, M] = < 0 >\},$$
$$Ann_H^r(M) = \{a \in H |\ [M, M, a] = < 0 >\}.$$

The subset $Ann_H^l(M)$ (respectively $Ann_H^m(M)$, $Ann_H^r(M)$) is called the *left* (respectively *middle*, *right*) *annihilator* of M in H. The intersection

$$Ann_H(M) = Ann_H^l(M) \cap Ann_H^m(M) \cap Ann_H^r(M)$$

is called *annihilator* of M in H. We note the following basic properties of annihilators.

Lemma 1. *Let L be a Leibniz 3-algebra over a field F, M be an ideal of L, and H be a subalgebra of L. Then $Ann_H^l(M)$, $Ann_H^m(M)$, $Ann_H^r(M)$, $Ann_H(M)$ are subalgebras of L.*

Let L be a Leibniz 3-algebra over a field F. Put

$$\zeta(L) = \{a \in L |\ [a, x, y] = [x, a, y] = [x, y, a] = 0 \text{ for all } x, y \in L\}.$$

The subset $\zeta(L)$ is called the *center* of L. We note that the center of L is the annihilator of L in L. The left, middle and right annihilators lead us to the following subsets. Put

$$\zeta^l(L) = \{a \in L |\ [a, x, y] = 0 \text{ for all } x, y \in L\},$$
$$\zeta^m(L) = \{a \in L |\ [x, a, y] = 0 \text{ for all } x, y \in L\},$$
$$\zeta^r(L) = \{a \in L |\ [x, y, a] = 0 \text{ for all } x, y \in L\}.$$

The subset $\zeta^l(L)$ (respectively $\zeta^m(L)$, $\zeta^r(L)$) is called the *left* (respectively *middle*, *right*) *center* of L. Obviously,

$$\zeta(L) = \zeta^l(L) \cap \zeta^m(L) \cap \zeta^r(L).$$

In particular, we can define the *lm-center* of L by the rule

$$\zeta^{lm}(L) = \zeta^l(L) \cap \zeta^m(L).$$

We note that, in general, $\zeta^l(L), \zeta^m(L), \zeta^r(L)$ are not ideals of L. However, we have the following result.

Lemma 2. *Let L be a Leibniz 3-algebra over a field F. Then the following assertions hold.*

(i) $\zeta^l(L), \zeta^m(L), \zeta^r(L), \zeta(L)$ *are subalgebras of* L.

(ii) $\zeta^{lm}(L), \zeta(L)$ *are ideals of* L.

Let L be a Leibniz 3-algebra over a field F. A linear transformation f of L is called a *derivation* of L if

$$f([a, b, c]) = [f(a), b, c] + [a, f(b), c] + [a, b, f(c)]$$

for all $a, b, c \in L$.

Consider the mapping $l_{a,b}: L \to L$ defined by the rule $l_{a,b}(x) = [a, b, x]$, $a, b \in L$. We note some basic properties of $l_{a,b}$. Let $x, y, z \in L$, $\lambda \in F$. Then

$$l_{a,b}(x + y) = [a, b, x + y] = [a, b, x] + [a, b, y] = l_{a,b}(x) + l_{a,b}(y),$$
$$l_{a,b}(\lambda x) = [a, b, \lambda x] = \lambda [a, b, x] = \lambda l_{a,b}(x),$$
$$l_{a,b}([x, y, z]) = [a, b, [x, y, z]] = [[a, b, x], y, z] + [x, [a, b, y], z] + [x, y, [a, b, z]] =$$
$$= [l_{a,b}(x), y, z] + [x, l_{a,b}(y), z] + [x, y, l_{a,b}(z)].$$

These equalities show that $l_{a,b}$ is a derivation of L.

We now present the main result of this paper.

Theorem 3. *Let L be a Leibniz 3-algebra over a field F. If $codim_F\left(\zeta^{lm}(L)\right) = d$ and $codim_F\left(\zeta^r(L)\right) = r$ are finite, then $dim_F([L, L, L]) \leq d^2(d + r)$.*

2. Proofs

We begin with the proofs of the two lemmas given above.

Proof of Lemma 1. Let $A = Ann_H^l(M)$, $a, b, c \in A$, $x, y \in M$. We have:

$$[[a, b, c], x, y] = [a, b, [c, x, y]] - [c, [a, b, x], y] - [c, x, [a, b, y]].$$

Since $c \in A$, $[c, x, y] = 0$. The fact that M is an ideal of L implies that $[a, b, x] \in M$ and $[a, b, y] \in M$, so that $[c, [a, b, x], y] = [c, x, [a, b, y]] = 0$. Thus, $[[a, b, c], x, y] = 0$.

Now, let $B = Ann_H^m(M)$, $a, b, c \in B$, $x, y \in M$. We have:

$$[x, [a, b, c], y] = [a, b, [x, c, y]] - [[a, b, x], c, y] - [x, c, [a, b, y]].$$

Since $c \in B$, $[x,c,y]=0$. Furthermore, M is an ideal of L, so that $[a,b,x] \in M$ and $[a,b,y] \in M$. Therefore, $\big[[a,b,x],c,y\big] = \big[x,c,[a,b,y]\big]=0$. Thus, $[x,[a,b,c],y]=0$.

Finally, let $C=Ann_H^r(M)$, $a,b,c \in C$, $x,y \in M$. We have:

$$\big[x,y,[a,b,c]\big]=\big[[x,y,a],b,c\big]+[a,[x,y,b],c]+\big[a,b,[x,y,c]\big].$$

Since $a,b,c \in C$, $[x,y,a]=[x,y,b]=[x,y,c]=0$. Thus, $\big[x,y,[a,b,c]\big]=0$.

Since $Ann_H(M)=A\cap B\cap C$, $Ann_H(M)$ is also a subalgebra of L. The lemma is proved.

Proof of Lemma 2. (i) Since

$$Ann_L^l(L)=\zeta^l(L), Ann_L^m(L)=\zeta^m(L), Ann_L^r(L)=\zeta^r(L), Ann_L(L)=\zeta(L),$$

Lemma 1 shows that $\zeta^l(L),\zeta^m(L),\zeta^r(L),\zeta(L)$ are subalgebras of L.

(ii) Let $D=\zeta^{lm}(L)$, $v \in D$, $a,b,x,y \in L$. We have:

$$\big[[x,y,v],a,b\big]=\big[x,y,[v,a,b]\big]-[v,[x,y,a],b]-\big[v,a,[x,y,b]\big],$$
$$[a,[x,y,v],b]=\big[x,y,[a,v,b]\big]-\big[[x,y,a],v,b\big]-\big[a,v,[x,y,b]\big].$$

Since $v \in D$,

$$\big[[x,y,v],a,b\big]=[x,y,0]-0-0=0,$$
$$[a,[x,y,v],b]=[x,y,0]-0-0=0.$$

This means that D is a left ideal of L. Furthermore,

$$\big[[x,v,y],a,b\big]=\big[x,v,[y,a,b]\big]-[y,[x,v,a],b]-\big[y,a,[x,v,b]\big]=0-[y,0,b]-[y,a,0]=0,$$
$$[a,[x,v,y],b]=\big[x,v,[a,y,b]\big]-\big[[x,v,a],y,b\big]-\big[a,y,[x,v,b]\big]=0-[0,y,b]-[a,y,0]=0,$$

which shows that D is a middle ideal of L. Finally, we have:

$$\big[[v,x,y],a,b\big]=\big[v,x,[y,a,b]\big]-[y,[v,x,a],b]-\big[y,a,[v,x,b]\big]=0-[y,0,b]-[y,a,0]=0,$$
$$[a,[v,x,y],b]=\big[v,x,[a,y,b]\big]-\big[[v,x,a],y,b\big]-\big[a,y,[v,x,b]\big]=0-[0,y,b]-[a,y,0]=0.$$

In other words, D is a right ideal of L. Thus, D is an ideal of L.

In the same way we can show that the center $\zeta(L)$ of L is an ideal of L. The lemma is proved.

Finally, we present the proof of the main result of this article.

Proof of Theorem 3. We have: $L=\zeta^{lm}(L)\oplus E$ for some subspace E. Choose in E a basis $\{e_1,\dots,e_d\}$. If x,y,z are arbitrary elements of L, then

$$x=\alpha_1e_1+\dots+\alpha_de_d+s_1,\ y=\beta_1e_1+\dots+\beta_de_d+s_2,\ z=\gamma_1e_1+\dots+\gamma_de_d+s_3$$

for $\alpha_i,\beta_i,\gamma_i \in F$, $i \in \{1,\dots,d\}$, and $s_j \in \zeta^{lm}(L)$, $j \in \{1,2,3\}$. Then we have

$$[x,y,z]=\left[\sum_{1\le i\le d}\alpha_ie_i+s_1,\ \sum_{1\le j\le d}\beta_je_j+s_2,\ \sum_{1\le k\le d}\gamma_ke_k+s_3\right]=$$
$$\sum_{1\le i,j,k\le d}\alpha_i\beta_j\gamma_k\big[e_i,e_j,e_k\big]+\sum_{1\le i,j\le d}\alpha_i\beta_j\big[e_i,e_j,s_3\big].$$

It follows that the subspace S generated by the elements $\big[e_i,e_j,e_k\big]$, $1\le i,j,k\le d$, and the subspaces $\big[e_i,e_j,\zeta^{lm}(L)\big]$, $1\le i,j\le d$, include $[L,L,L]$.

Put $Z=\zeta^{lm}(L)$ and let a,b be arbitrary elements of L. We define a mapping $l_{a,b}(x)\colon Z\to Z$ by the rule $l_{a,b}(z)=[a,b,z]$, $z \in Z$. As we noted above, this mapping is linear, $Im\big(l_{a,b}\big)=[a,b,Z]$ and $Ker\big(l_{a,b}\big)=Ann_Z^r\big((a,b)\big)$ where (a,b) is an ordered pair of elements a and b. Hence

$$[a,b,Z]=\frac{Im\big(l_{a,b}\big)\cong_F Z}{Ker\big(l_{a,b}\big)}=\frac{Z}{Ann_Z^r\big((a,b)\big)}.$$

Since $\zeta^r(L)\le Ann_L^r\big((a,b)\big)$, we have $codim_F\left(Ann_L^r\big((a,b)\big)\right)\le r$ and so

$$dim_F([a,b,Z])=dim_F\left(\frac{Z}{Ann_Z^r\big((a,b)\big)}\right)\le r.$$

In particular, $dim_F\big(\big[e_i,e_j,Z\big]\big)\le r$ for every $1\le i,j\le d$. It follows that the subspace S has dimension at most $d^3+d^2r=d^2(d+r)$, as required.

3. Examples

Let's consider two partial cases. Firstly, Theorem 3 directly implies the following.

Corollary 4. *Let L be a Leibniz 3-algebra over a field F. If $codim_F\big(\zeta(L)\big)=d$ is finite, then $dim_F([L,L,L])\le d^3$.*

Indeed, we have $\zeta(L)=\zeta^{lm}(L)\cap\zeta^r(L)$. Since $codim_F\left(\zeta^{lm}(L)\right)$ *and* $codim_F\big(\zeta^r(L)\big)$ *are finite*, $codim_F\big(\zeta(L)\big)$ is finite. Then it suffices to take the proof of Theorem 3 into account to obtain Corollary 4.

Secondly, recall (Filippov, 1985) that a 3-algebra L over a field F $(char(F)\ne 2)$ with the binary operations $+$ and a 3-linear bracket $[-,-,-]$ is called a *Lie 3-algebra* if it satisfies the following conditions: (i) Lie 3-bracket is antisymmetric, that is $[a_1,a_2,a_3]=sign(\sigma)\big[a_{\sigma(1)},a_{\sigma(2)},a_{\sigma(3)}\big]$; (ii) Lie 3-bracket satisfies the generalized Jacobi identity (or Jacobi 3-identity), that is

$$\big[x,y,[a,b,c]\big]=\big[[x,y,a],b,c\big]+[a,[x,y,b],c]+\big[a,b,[x,y,c]\big]$$

for any $a,b,c,x,y \in L$ and any permutation $\sigma \in S_3$. Note that since $char(F)\ne 2$ the first condition equivalent to $[a_1,a_2,a_3]=0$ whenever $a_i=a_j$ for some $i\ne j$, $1\le i,j\le 3$. Obviously, every Lie 3-algebra is a Leibniz 3-algebra. Therefore, we can apply the previous results to this partial case. We have the following.

Corollary 5. *Let L be a Lie 3-algebra over a field F. If $codim_F\big(\zeta(L)\big)=d$ is finite, then* $([L,L,L])\le\frac{d(d-1)(d-2)}{6}$.

Discussion and conclusions

The obtained results illustrate that the transition from Leibniz algebras to Leibniz 3-algebras is possible, but sometimes requires additional restrictions. The experience gained during the study of Leibniz algebras can be used to study similar

questions of the general theory of Leibniz 3-algebras, and in the future, Leibniz n-algebras. The results proved in the article add to the large list of algebraic structures for which an analogue of Schur theorem and its modification holds true. All this points to the prospects of further research in this direction.

Authors' contribution: Pavlo Minaiev – conceptualization, methodology, analysis of sources, preparation of literature review and theoretical foundations of research; Oleksandr Pypka – conceptualization, methodology, analysis of sources, preparation of literature review and theoretical foundations of research.

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ПРО АНАЛОГ ТЕОРЕМИ ШУРА ДЛЯ 3-АЛГЕБР ЛЕЙБНІЦА

Одним із класичних результатів теорії груп є так звана теорема Шура, яка стверджує: якщо центральна фактор-група $\frac{G}{\zeta(G)}$ групи G скінченна, то її комутант $[G,G]$ також скінченний. Цю теорему довів Б. Нейман у 1951 р. Такий результат має численні узагальнення та модифікації в теорії груп. Водночас аналогічні дослідження проводили і в інших алгебричних структурах, а саме в модулях, лінійних групах, топологічних групах, n-групах, асоціативних алгебрах, алгебрах Лі, n-алгебрах Лі. У 2016 р. Л. Курдаченко, Х. Отал й О. Пипка довели аналог теореми Шура для алгебр Лейбніца: якщо центральна фактор-алгебра $\frac{L}{\zeta(L)}$ алгебри Лейбніца L має скінченну розмірність, то її похідний ідеал $[L,L]$ також скінченновимірний. Ба більше, вони довели й дещо модифікований аналог теореми Шура: якщо ковимірності лівого $\zeta^l(L)$ та правого $\zeta^r(L)$ центрів алгебри Лейбніца L скінченні, то її похідний ідеал $[L,L]$ також скінченновимірний. Одним з узагальнень алгебр Лейбніца є так звані n-алгебри Лейбніца. Варто зауважити, що теорія n-алгебр Лейбніца натепер є значно менш розвиненою за теорію алгебр Лейбніца. Одним із напрямків розвитку загальної теорії n-алгебр Лейбніца є пошук аналогій з іншими типами алгебр. Тож цілком природно постає питання про доведення аналогів наведених вище результатів для цього типу алгебр. У пропонованій статті ми доводимо аналоги двох згаданих теорем для n-алгебр Лейбніца для випадку $n = 3$. Отримані результати вказують на перспективність подальших досліджень у цьому напрямі.

Ключові слова: ***алгебра Лейбніца, 3-алгебра Лейбніца, алгебра Лі, центр, похідний ідеал, теорема Шура.***

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