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INTERPOLATION PROBLEM FOR MULTIDIMENSIONAL HARMONIZABLE STABLE SEQUENCES

The problem of estimation of unobserved values of stochastic processes is of constant interest in the theory and applications of stochastic processes. The problem of forecasting future values of economic and physical processes, the problem of restoring lost information, cleaning signals, or other data from observations with noise is magnified in an information-laden world. Therefore, the development of estimation methods is one of the main tasks of the modern theory of stochastic processes. In this paper, we consider the problem of optimal linear interpolation of a functional that depends on the unknown values of a vector-valued harmonizable symmetric alpha-stable random sequence from observations of the sequence with noise. We use the classical approach to derive formulas for computing values of the mean-square error and the spectral characteristic of the optimal linear estimate of the functional. The crucial assumption of this approach is that the spectral densities of the involved stochastic sequences are exactly known. However, in practice, complete information on the spectral densities is impossible in most cases. In this situation, one finds a parametric or non-parametric estimate of the unknown spectral density and then applies one of the traditional estimation methods provided that the selected density is the true one. This procedure can result in a significant increase in the value of the estimation error. To avoid this effect, one can search for the estimates that are optimal for all densities from a certain class of admissible spectral densities. These estimates are called minimax because they minimize the maximal values of the errors of estimates for all densities from a given class. Therefore, in the case of spectral uncertainty, we use the minimax approach and propose formulas that determine the least favorable spectral densities and the minimax spectral characteristics of the optimal estimates of the functional for some classes of admissible spectral densities.

Keywords: *harmonizable stable random sequence, periodically harmonizable stable random sequence, optimal linear estimate, minimax-robust estimate, minimax spectral characteristic, least favorable spectral density.*

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Introduction

The problem of estimation of the unknown values of harmonizable random sequences and processes were investigated in papers (Cambanis, 1983; Cambanis, & Soltani, 1984; Hosoya, 1982; Masani, & Wiener, 1957). The interpolation problem for harmonizable symmetric α-stable random sequences were investigated in papers (Weron, 1985; Pourahmadi, 1984).

Basic results concerning estimation of the unknown (missed) values of stochastic processes are based on the assumption that spectral densities of processes are exactly known. In practice, however, complete information on the spectral densities is impossible in most cases. In such situations one finds parametric or nonparametric estimates of the unknown spectral densities. Then the classical estimation method is applied under the assumption that the estimated densities are true. This procedure can result in significant increasing of the value of error as Vastola and Poor have demonstrated with the help of some examples (Vastola, & Poor, 1983). This is a reason to search estimates which are optimal for all densities from a class of admissible spectral densities. These estimates are called minimax since they minimize the maximal value of the error. A survey of results (till 1985) in minimax (robust) methods of data processing can be found in the paper (Kassam, & Poor, 1985). The paper (Grenander, 1957) should be marked as the first one where the minimax extrapolation problem for stationary processes was formulated and solved. Later Franke and Poor (Franke, & Poor, 1984; Franke, 1984, 1985) applied the convex optimization methods for investigation the minimax-robust extrapolation and interpolation problems. The book (Moklyachuk, & Masyutka, 2012) is dedicated to minimax-robust extrapolation, interpolation and filtering problems for vector-valued stationary processes and sequences. In the book (Moklyachuk, & Golichenko, 2016) the minimax-robust extrapolation, interpolation and filtering problems for periodically correlated processes are investigated. In the paper (Moklyachuk, & Ostapenko, 2016) minimax-robust interpolation problems are studied for harmonizable stable sequences.

In this paper the problem of optimal estimation is investigated for the linear functional

$$A_N\vec{\xi}=\sum_{j=0}^{N}(\vec{a}(j))^{\top}\vec{\xi}(j)$$

that depends on the unknown values $\vec{\xi}(j), j=0,1,\dots,N,$ of a vector-valued harmonizable symmetric α-stable random sequence $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}$, from observations of the sequence $\vec{\xi}(j)+\vec{\eta}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$ where $\vec{\xi}(j)$ and $\vec{\eta}(j)$ are mutually independent harmonizable symmetric α-stable random sequences which have the spectral densities $f(\theta)$ and $g(\theta)$ satisfying the minimality condition.

The problem is investigated under the condition of spectral certainty as well as under the condition of spectral uncertainty. Formulas for calculation the value of the error and spectral characteristic of the optimal linear estimate of the functional are derived under the condition of spectral certainty where spectral density of the sequence is exactly known. In the case where spectral density of the sequence is not exactly known sets of admissible spectral densities is available, relations which determine least favorable densities and the minimax-robust spectral characteristics for different classes of spectral densities are found.

1. Harmonizable symmetric α-stable random sequence

Definition 1. (symmetric α-stable random variable) A real-valued random variable ξ is said to be symmetric α-stable, $S\alpha S$, if its characteristic function has the form $Eexp(it\xi)=exp(-c|t|^{\alpha})$ for some $c\geqslant 0$ and $0<\alpha\leqslant 2$. The real random variables $\xi_1,\xi_2,\dots,\xi_n$ are jointly $S\alpha S$ if all linear combinations $\sum_{k=1}^{n}a_k\xi_k$ are $S\alpha S$, or, equivalently, if the characteristic function of the random vector $\vec{\xi}=(\xi_1,\dots,\xi_n)$ is of the form

$$\phi_{\vec{\xi}}(\vec{t})=Eexp\left(i\sum_{k=1}^{n}t_k\xi_k\right)=exp\left\{-\int_{S_n}\left|\sum_{k=1}^{n}t_kx_k\right|^{\alpha}d\Gamma_{\vec{\xi}}(\vec{x})\right\},$$

where $\vec{t}=(t_1,\dots,t_n),t_1,\dots,t_n$ are real numbers and $\Gamma_{\vec{\xi}}(\vec{x})$ is a symmetric measure defined on the unit sphere $S_n\in R^n$, called the spectral measure of the random vector $\vec{\xi}=(\xi_1,\dots,\xi_n)$. There is a one-to-one correspondence between the distribution of $\vec{\xi}$ and its spectral measure $\Gamma_{\vec{\xi}}(\vec{x})$ (Cambanis, 1983).

For real-valued jointly $S\alpha S$ random variables ξ,η with $1<\alpha\leqslant 2$ the covariation of ξ,η is defined by

$$[\xi,\eta]_{\alpha}=\int_{S_2}(x)(y)^{<\alpha-1>}d\Gamma_{\xi,\eta}(x,y),$$

where $(y)^{<\beta>}=|y|^{\beta-1}y$.

For jointly $S\alpha S$ random variables $\xi=\xi_1+i\xi_2$ and $\eta=\eta_1+i\eta_2$ the covariation of ξ with η is defined as (Cambanis, 1983)

$$[\xi,\eta]_{\alpha}=\int_{S_4}(x_1+ix_2)(y_1+iy_2)^{<\alpha-1>}d\Gamma_{\xi_1,\xi_2,\eta_1,\eta_2}(x_1,x_2,y_1,y_2),$$

where $z^{<\beta>}=|z|^{\beta-1}\bar{z}$ for a complex number z and $\beta>0$.

The covariation in general is not symmetric and linear on second argument and for ξ,ξ_1,ξ_2,η jointly $S\alpha S$ has the following properties (Cambanis, 1983; Weron, 1985)

$$[\xi_1+\xi_2,\eta]_{\alpha}=[\xi_1,\eta]_{\alpha}+[\xi_2,\eta]_{\alpha}, \tag{1}$$

$$[a\,\xi,b\,\eta]_{\alpha}=a(b)^{\alpha-1}\,[\xi,\eta]_{\alpha}, \tag{2}$$

$$[\xi,\eta]_{\alpha}=0\quad\text{if } \xi \text{ and } \eta \text{ are independent}, \tag{3}$$

$$[\xi,\eta_1+\eta_2]_{\alpha}=[\xi,\eta_1]_{\alpha}+[\xi,\eta_2]_{\alpha}\quad\text{if } \eta_1 \text{ and } \eta_2 \text{ are independent}, \tag{4}$$

$$|[\xi,\eta]_{\alpha}|\leqslant||\xi||_{\alpha}||\eta||_{\alpha}^{\alpha-1}, \tag{5}$$

the functional

$$||\xi||_{\alpha}=[\xi,\xi]_{\alpha}^{1/\alpha} \tag{6}$$

is a norm in a linear space of $S\alpha S$ random variables which is equivalent to convergence in probability

$$||\sum_{k=1}^{n}\xi_k||_{\alpha}^{\alpha}=\sum_{k=1}^{n}||\xi_k||_{\alpha}^{\alpha} \tag{7}$$

when $\xi_1,\dots,\xi_n$ are independent, the mapping

$$\xi\rightarrow[\xi,\eta]_{\alpha} \tag{8}$$

is a bounded linear functional with the norm $||\eta||_{\alpha}^{\alpha-1}$ on the linear space of $S\alpha S$ random variables, and every bounded linear functional on such a space is of this form for some η.

It should be noted that $||\cdot||_{\alpha}$ is not necessarily the usual L^{α} norm.

Definition 2 (symmetric α-stable stochastic sequence). A stochastic sequence $\{\xi(n),n\in\mathbb{Z}\}$ is called symmetric α-stable, $S\alpha S$, if all linear combinations $\sum_{m=1}^{l}a_m\xi(n_m)$ are $S\alpha S$ random variables.

A vector-valued stochastic sequence $\vec{\xi}(n)=\{\xi_k(n)\}_{k=1}^{T}$, $n\in\mathbb{Z}$, is called symmetric α-stable, $S\alpha S$, stochastic sequence, if all linear combinations $\sum_{k=1}^{T}\sum_{m=1}^{l}a_{m_k}\xi_k(n_m)$ are $S\alpha S$ random variables.

Let $Z=\{Z(t):-\infty<t<\infty\}$ be a complex $S\alpha S$ process with independent increments. The spectral measure of the process Z is defined as $\mu\{(s,t]\}=\|Z(t)-Z(s)\|_{\alpha}^{\alpha}$.

The integrals $\int a(t)dZ(t)$ can be defined for all $a(t)\in L^{\alpha}(\mu)$ with properties for all $a\in L^{\alpha}(\mu), b\in L^{\alpha}(\mu)$ (Cambanis, 1983; Cambanis, & Soltani, 1984; Hosoya, 1982):

$$\left\|\int a(t)dZ(t)\right\|_{\alpha}^{\alpha}=\int |a(t)|^{\alpha}d\mu(t), \tag{9}$$

$$\left[\int a(t)dZ(t),\int b(t)dZ(t)\right]_{\alpha}=\int a(t)(b(t))^{<\alpha-1>}d\mu(t), \tag{10}$$

and for vector-valued functions $\vec{a}(t)=\{a_k(t)\}_{k=1}^{T}, a_k(t)\in L^{\alpha}(\mu)$, $\vec{b}(t)=\{b_k(t)\}_{k=1}^{T}, b_k(t)\in L^{\alpha}(\mu)$, and $\vec{Z}(t)=\{Z_k(t)\}_{k=1}^{T}$

$$\left[\int (\vec{a}(t))^{\top}d\vec{Z}(t),\int (\vec{b}(t))^{\top}d\vec{Z}(t)\right]_{\alpha}=\int (\vec{a}(t))^{\top}d\mu(t)(\vec{b}(t))^{<\alpha-1>}, \tag{11}$$

where μ is the matrix-valued spectral measure corresponding to the process $\vec{Z}(t)$.

Definition 3 (harmonizable symmetric α-stable sequence). A symmetric α-stable, $S\alpha S$, stochastic sequence $\{\xi(n), n\in\mathbb{Z}\}$ is said to be harmonizable, $HS\alpha S$, if there exists a $S\alpha S$ process $Z=\{Z(\theta):\theta\in[-\pi,\pi]\}$ with independent increments and a finite spectral measure μ such that sequence $\xi(n)$ has the spectral representation

$$\xi(n)=\int_{-\pi}^{\pi}e^{in\theta}dZ(\theta),\quad n\in\mathbb{Z},$$

and the covariation has the representation

$$[\xi(n),\xi(m)]_{\alpha}=\int_{-\pi}^{\pi}e^{i(n-m)\theta}d\mu(\theta),\quad m,n\in\mathbb{Z}.$$

A vector-valued symmetric α-stable, $S\alpha S$, stochastic sequence $\vec{\xi}(n)=\{\xi_k(n)\}_{k=0}^{T-1}$, $n\in\mathbb{Z}$, is said to be harmonizable, $HS\alpha S$, if there exists a vector-valued $S\alpha S$ process $\vec{Z}(\theta)=\{Z_k(\theta)\}_{k=0}^{T-1}, \theta\in[-\pi,\pi)$ with independent increments and a finite matrix-valued spectral measure μ such that sequence $\vec{\xi}(n)$ has the spectral representation

$$\vec{\xi}(n)=\int_{-\pi}^{\pi}e^{in\theta}d\vec{Z}(\theta), n\in\mathbb{Z},$$

and the covariation has the representation

$$[\vec{\xi}(n),\vec{\xi}(m)]_{\alpha}=\int_{-\pi}^{\pi}e^{i(n-m)\theta}d\mu(\theta), m,n\in\mathbb{Z}.$$

Definition 4 (periodically harmonizable $S\alpha S$ sequence). A symmetric α-stable, $S\alpha S$, stochastic sequence $\{\xi(n), n\in\mathbb{Z}\}$ is said to be periodically harmonizable, $PHS\alpha S$, if the vector-valued stochastic sequence $\vec{\xi}(n)=\{\xi(nT+k)\}_{k=0}^{T-1}$, $n\in\mathbb{Z}$, is harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequence.

Note that a $HS\alpha S$ stochastic sequence is not necessarily stationary even second order stationary, but for $\alpha=2$ the $HS\alpha S$ sequences are stationary with Gaussian distribution.

In this article we consider the case where $1<\alpha\leqslant 2$.

Denote by $H(\xi)$ the time domain of the $HS\alpha S$ sequence $\{\xi(n), n\in\mathbb{Z}\}$, which is a closed in the norm $\|\cdot\|_{\alpha}$ linear manifold generated by all values of the $HS\alpha S$ sequence $\{\xi(n), n\in\mathbb{Z}\}$. It follows from the spectral representation of the $HS\alpha S$ sequence $\{\xi(n), n\in\mathbb{Z}\}$ that the mapping $\xi(n)\leftrightarrow e^{in\theta}, n\in\mathbb{Z}$, extents to an isomorphism between the spaces $H(\xi)$ and $L^{\alpha}(\mu)$. Under this isomorphism to each $\eta\in H(\xi)$ corresponds a unique $f\in L^{\alpha}(\mu)$ such that $\eta=\int_{-\pi}^{\pi}f(\theta)dZ(\theta)$.

For a closed linear subspace $M\subseteq L^{\alpha}(\mu)$ and $f\in L^{\alpha}(\mu)$, there exists a unique element from M which minimizes the distance to f. This element is called projection of f onto M or the best approximation of f in M. This projection is denoted by $P_M f$ and is uniquely determined by the condition (Singer, 1970)

$$\int_{-\pi}^{\pi}g\left(f-P_M f\right)^{<\alpha-1>}d\mu=0,\quad g\in M. \tag{12}$$

Similarly, for $HS\alpha S$ stochastic sequence $\{\xi(n), n\in\mathbb{Z}\}$ and a closed linear subspace $H^{-}(\xi)$ of the space $H(\xi)$ there is a uniquely determined element $\hat{\xi}(n)\in H^{-}(\xi)$ which minimizes the distance to $\xi(n)$ and is uniquely determined from the condition

$$\left[\eta,\xi(n)-\hat{\xi}(n)\right]_{\alpha}=0,\quad \eta\in H^{-}(\xi). \tag{13}$$

From linearity of the covariation with respect to the first argument from this relation we have that

$$||\xi(n)-\hat{\xi}(n)||_{\alpha}^{\alpha}=\left[\xi(n),\xi(n)-\hat{\xi}(n)\right]_{\alpha}-\left[\hat{\xi}(n),\xi(n)-\hat{\xi}(n)\right]_{\alpha}=\left[\xi(n),\xi(n)-\hat{\xi}(n)\right]_{\alpha}. \tag{14}$$

This relation plays a fundamental role in the characterization of minimal $HS\alpha S$ stochastic sequences $\{\xi(n), n\in\mathbb{Z}\}$.

2. Interpolation problem. Observations with noise. Projection approach

Consider the problem of the optimal estimation of the linear functional

$$A_N\vec{\xi}=\sum_{j=0}^{N}(\vec{a}(j))^{\top}\vec{\xi}(j)=\int_{-\pi}^{\pi}(A_N(e^{i\theta}))^{\top}d\vec{Z}^{\xi}(\theta),$$

where

$$A_N(e^{i\theta})=\sum_{j=0}^{N}\vec{a}(j)e^{ij\theta},$$

that depends on the unknown values $\vec{\xi}(j), j=0,1,\dots,N$, of a vector-valued harmonizable symmetric α-stable random sequence $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}$, from observations of the sequence $\vec{\xi}(j)+\vec{\eta}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$.

We consider the problem for mutually independent vector-valued harmonizable symmetric α-stable random sequences $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}$ and $\vec{\eta}(j)=\{\eta_k(j)\}_{k=1}^{T}$ which have absolutely continuous spectral measures and the spectral densities $f(\theta)$ and $g(\theta)$, satisfying the minimality condition (Pourahmadi, 1984; Weron, 1985)

$$\int_{-\pi}^{\pi}\mathrm{Tr}\left[(f(\theta)+g(\theta))^{-1/(\alpha-1)}\right]d\theta<\infty. \tag{15}$$

Denote by $H^N(\xi+\eta)$ the closed in the $||\cdot||_\alpha$ norm linear manifold generated by values of the harmonizable symmetric α-stable stochastic sequence $\vec{\xi}(j)+\vec{\eta}(j)$, $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$ in the space $H(\xi+\eta)$ generated by all values of the harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequence $\vec{\xi}(j)+\vec{\eta}(j)$, $j\in\mathbb{Z}$.

The optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ is a projection of $A_N\vec{\xi}$ on the subspace $H^N(\xi+\eta)$ which is determined by the relations

$$[\eta,A_N\vec{\xi}-\hat{A}_N\vec{\xi}]_\alpha=0,\quad\forall\eta\in H^N(\xi+\eta),$$

or, equivalently, by relations

$$[\xi_k(j)+\eta_k(j),A_N\vec{\xi}-\hat{A}_N\vec{\xi}]_\alpha=0,\ j\in\mathbb{Z}\setminus\{0,1,\dots,N\},\ k=\overline{1,T}. \tag{16}$$

It follows from the isomorphism between the spaces $H(\xi+\eta)$ and $L^\alpha(F+G)$ that the optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ is of the form

$$\hat{A}_N\vec{\xi}=\int_{-\pi}^{\pi}(h(\theta))^{\top}\left(d\vec{Z}^{\xi}(\theta)+d\vec{Z}^{\eta}(\theta)\right). \tag{17}$$

It is determined by the spectral characteristic $h(\theta)$ of the estimate which is from the subspace $L^\alpha_N(F+G)$ of the $L^\alpha(F+G)$ space generated by functions

$$e^{ij\theta}\delta_k,\ \delta_k=\{\delta_{kl}\}_{l=1}^{T},\ k=1,\dots,T,\ j\in\mathbb{Z}\setminus\{0,1,\dots,N\}.$$

The spectral characteristic $h(\theta)=\{h_k(\theta)\}_{k=1}^{T}$ of the optimal estimate satisfies the following equations

$$\int_{-\pi}^{\pi}e^{ij\theta}\left[f(\theta)\left(A(e^{i\theta})-h(\theta)\right)^{<\alpha-1>}-g(\theta)\,(h(\theta))^{<\alpha-1>}\right]d\theta=0,\ j\in\mathbb{Z}\setminus\{0,1,\dots,N\}. \tag{18}$$

It follows from these equations that the spectral characteristic $h(\theta)$ of the estimate is determined by the equation

$$\left[f(\theta)\left(A(e^{i\theta})-h(\theta)\right)^{<\alpha-1>}-g(\theta)\,(h(\theta))^{<\alpha-1>}\right]=C_N(e^{i\theta}), \tag{19}$$

$$C_N(e^{i\theta})=\sum_{j=0}^{N}\vec{c}(j)e^{-ij\theta},$$

where $\vec{c}(j)=\{c_k(j)\}_{k=1}^{T}, j=0,1,\dots,N$ are unknown coefficients.

The unknown coefficients $\vec{c}(j), j=0,1,\dots,N$, are determined from the condition $h(\theta)\in L^\alpha_N(f+g)$, which gives us the system of equations

$$\int_{-\pi}^{\pi}e^{-i\theta k}\ h(\theta)d\theta=0,\quad k=0,1,\dots,N. \tag{20}$$

The variance of the optimal estimate of the functional is calculated by the formula

$$\left\|A_N\xi-\hat{A}_N\xi\right\|_\alpha^\alpha=\int_{-\pi}^{\pi}\left(A_N(e^{i\theta})-h(\theta)\right)^{\top}f(\theta)\left(A_N(e^{i\theta})-h(\theta)\right)^{<\alpha-1>}d\theta+$$

$$+\int_{-\pi}^{\pi}(h(\theta))^{\top}g(\theta)\,(h(\theta))^{<\alpha-1>}d\theta. \tag{21}$$

In the case where the dimension of the sequence $T = 1$, this formula can be written in the form

$$\left\|A_N\xi - \hat{A}_N\xi\right\|_\alpha^\alpha = \int_{-\pi}^{\pi} \left|A_N(e^{i\theta}) - h(\theta)\right|^\alpha f(\theta)d\theta + \int_{-\pi}^{\pi} |h(\theta)|^\alpha g(\theta)d\theta. \tag{22}$$

We can conclude that the following theorems hold true.

Theorem 1. *Let $\vec{\xi}(j) = \{\xi_k(j)\}_{k=1}^T$, $j \in \mathbb{Z}$ and $\vec{\eta}(j) = \{\eta_k(j)\}_{k=1}^T$, $j \in \mathbb{Z}$ be mutually independent harmonizable symmetric α-stable $HS\alpha S$, stochastic sequences which have absolutely continuous spectral measures and the spectral densities $f(\theta)$ and $g(\theta)$ satisfying the minimality condition (15). The optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi} = \sum_{j=0}^N (\vec{a}(j))^\top \vec{\xi}(j)$, that depends on the unknown values $\vec{\xi}(j), j = 0, 1, \ldots, N$, of the sequence $\vec{\xi}(j)$, from observations of the sequence $\vec{\xi}(j) + \vec{\eta}(j)$ at points $j \in \mathbb{Z}\backslash\{0, 1, \ldots, N\}$ is calculated by formula (17). The spectral characteristic $h(\theta)$ of the estimate is determined by equation (19), where the unknown coefficients $\vec{c}(j), j = 0, 1, \ldots, N$, are determined from the system of equations (20). The variance of the optimal estimate of the functional is calculated by formula (21) (by formula (22) in the case where dimension $T = 1$).*

Theorem 2. *Let the vector-valued harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequences*

$$\vec{\xi}(n) = \{\xi(nT + k)\}_{k=0}^{T-1}, \ \vec{\eta}(n) = \{\eta(nT + k)\}_{k=0}^{T-1}, \ n \in \mathbb{Z},$$

that correspond to periodically harmonizable symmetric α-stable, $PHS\alpha S$, stochastic sequences $\xi(n)$ and $\eta(n)$, $n \in \mathbb{Z}$, have absolutely continuous spectral measures and the spectral densities $f(\theta), g(\theta)$, satisfying the minimality condition (15). The optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional

$$A_N\vec{\xi} = \sum_{j=0}^{N} (\vec{a}(j))^\top \vec{\xi}(j) = \sum_{j=0}^{N}\sum_{k=0}^{T-1} a_k(j)\xi(jT + k),$$

where $\vec{a}(j) = \{a_k(j)\}_{k=0}^{T-1}$, $\vec{\xi}(j) = \{\xi(jT + k)\}_{k=0}^{T-1}$, that depends on the unknown values $\xi(j), j = 0, 1, \ldots, T(N + 1) - 1$, of the sequence $\xi(j)$ from observations of the sequence $\xi(j) + \eta(j)$ at points $j \in \mathbb{Z} \setminus \{0, 1, \ldots, T(N + 1) - 1\}$ is calculated by the formula (17). The spectral characteristic $h(\theta)$ of the estimate is determined by equation (19), where the unknown coefficients $\vec{c}(j) = \{c_k(j)\}_{k=0}^{T-1}, j = 0, 1, \ldots, T(N+1)-1$, are determined from the system of equations (20). The variance of the optimal estimate of the functional is calculated by the formula (21) (by formula (22) in the case where period $T = 1$).

3. Interpolation problem. Observations without noise. Projection approach

Consider the problem of the optimal estimation of the linear functional

$$A_N\vec{\xi} = \sum_{j=0}^{N} (\vec{a}(j))^\top \vec{\xi}(j) = \int_{-\pi}^{\pi} (A_N(e^{i\theta}))^\top d\vec{Z}^\xi(\theta),$$

where

$$A_N(e^{i\theta}) = \sum_{j=0}^{N} \vec{a}(j)e^{ij\theta},$$

that depends on the unknown values $\vec{\xi}(j), j = 0, 1, \ldots, N$, of a vector-valued harmonizable symmetric α-stable random sequence $\vec{\xi}(j) = \{\xi_k(j)\}_{k=1}^T$, from observations of the sequence $\vec{\xi}(j)$ at points $j \in \mathbb{Z} \setminus \{0, 1, \ldots, N\}$.

We consider the problem for vector-valued harmonizable symmetric α-stable random sequence $\vec{\xi}(j) = \{\xi_k(j)\}_{k=1}^T$ which have absolutely continuous spectral measure μ and the spectral density $f(\theta)$, satisfying the minimality condition (Pourahmadi, 1984; Weron, 1985)

$$\int_{-\pi}^{\pi} \mathrm{Tr}\left[(f(\theta))^{-1/(\alpha-1)}\right] d\theta < \infty. \tag{23}$$

Denote by $H^N(\xi)$ the closed in the $||\cdot||_\alpha$ norm linear manifold generated by values of the harmonizable symmetric α-stable random sequence $\vec{\xi}(j)$, $j \in \mathbb{Z} \setminus \{0, 1, \ldots, N\}$ in the space $H(\xi)$ generated by all values of the $HS\alpha S$ sequence $\vec{\xi}(j) = \{\xi_k(j)\}_{k=1}^T$, $j \in \mathbb{Z}$.

The optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ is a projection of $A_N\vec{\xi}$ on the subspace $H^N(\xi)$ which is determined by the relations

$$[\eta, A_N\vec{\xi} - \hat{A}_N\vec{\xi}]_\alpha = 0, \quad \forall\eta \in H^N(\xi),$$

or, equivalently, by relations

$$[\xi_k(j), A_N\vec{\xi} - \hat{A}_N\vec{\xi}]_\alpha = 0, \ j \in \mathbb{Z} \setminus \{0, 1, \ldots, N\}, \ k = \overline{1, T}. \tag{24}$$

It follows from the isomorphism between the spaces $H(\xi)$ and $L^\alpha(\mu)$ that the optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ is of the form

$$\hat{A}_N\vec{\xi} = \int_{-\pi}^{\pi} (h(\theta))^\top d\vec{Z}^\xi(\theta). \tag{25}$$

It is determined by the spectral characteristic $h(\theta)$ of the estimate which is from the subspace $L_N^{\alpha}(\mu)$ of the $L^{\alpha}(\mu)$ space generated by functions

$$e^{ij\theta}\delta_k,\ \delta_k=\{\delta_{kl}\}_{l=1}^{T},\ k=1,\dots,T, j\in\mathbb{Z}\setminus\{0,1,\dots,N\}.$$

The spectral characteristic $h(\theta)=\{h_k(\theta)\}_{k=1}^{T}$ of the optimal estimate satisfies the following equations

$$\int_{-\pi}^{\pi} e^{ij\theta}f(\theta)\left(A_N(e^{i\theta})-h(\theta)\right)^{<\alpha-1>}d\theta=0,\quad j\in\mathbb{Z}\setminus\{0,1,\dots,N\}. \tag{26}$$

It follows from these equations that the spectral characteristic $h(\theta)$ of the estimate is determined by the equation

$$f(\theta)\left(A_N(e^{i\theta})-h(\theta)\right)^{<\alpha-1>}=C_N(e^{i\theta}),\quad C_N(e^{i\theta})=\sum_{j=0}^{N}\vec{c}(j)e^{-ij\theta},$$

where $\vec{c}(j)=\{c_k(j)\}_{k=1}^{T}, j=0,1,\dots,N$ are unknown coefficients. It follows from the last relation that the spectral characteristic $h(\theta)$ of the estimate is of the form

$$h(\theta)=A_N(e^{i\theta})-\left(f^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}. \tag{27}$$

The unknown coefficients $\vec{c}(j), j=0,1,\dots,N$, are determined from the condition $h(\theta)\in L_N^{\alpha}(\mu)$, which gives us the system of equations

$$\int_{-\pi}^{\pi} e^{-i\theta k}\ h(\theta)d\theta=0,\quad k=0,1,\dots,N. \tag{28}$$

These equations are of the form

$$\int_{-\pi}^{\pi} e^{-i\theta k}\left[\left(\sum_{j=0}^{N}\vec{a}(j)e^{ij\theta}\right)-\left((f(\theta))^{-1}\left(\sum_{j=0}^{N}\vec{c}(j)e^{-ij\theta}\right)\right)^{<\frac{1}{\alpha-1}>}\right]d\theta=0,\ k=0,1,\dots,N. \tag{29}$$

The variance of the optimal estimate of the functional is calculated by the formula

$$\left\|A_N\vec{\xi}-\hat{A}_N\vec{\xi}\right\|_{\alpha}^{\alpha}=\int_{-\pi}^{\pi}\left[\left(f^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\right]^{\top}f(\theta)\left(\overline{f^{-1}(\theta)C_N(e^{i\theta})}\right)^{<\frac{\alpha-1}{\alpha-1}>}d\theta. \tag{30}$$

In the case where the dimension of the sequence $T=1$, this formula can be written in the form

$$\left\|A_N\xi-\hat{A}_N\xi\right\|_{\alpha}^{\alpha}=\int_{-\pi}^{\pi}\left|\left(f^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\right|^{\alpha}f(\theta)d\theta. \tag{31}$$

We can conclude that the following theorems hold true.

Theorem 3. *Let $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}, j\in\mathbb{Z}$ be a vector-valued harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequence which has absolutely continuous spectral measure $\mu(\theta)$ and the spectral density $f(\theta)$, satisfying the minimality condition (23). The optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}=\sum_{j=0}^{N}(\vec{a}(j))^{\top}\vec{\xi}(j)$, that depends on the unknown values $\vec{\xi}(j), j=0,1,\dots,N$, of the sequence $\vec{\xi}(j)$, from observations of the sequence $\vec{\xi}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$ is calculated by the formula (25). The spectral characteristic $h(\theta)$ of the optimal estimate is determined by the equation (27), where the unknown coefficients $\vec{c}(j), j=0,\dots,N$, are determined from the system of equations (29). The variance of the optimal estimate of the functional is calculated by the formula (30) (by formula (31) in the case where dimension of the sequence $T=1$).*

Theorem 4. *Let the vector-valued harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequence $\vec{\xi}(n)=\{\xi(nT+k)\}_{k=0}^{T-1}$, $n\in\mathbb{Z}$, that corresponds to periodically harmonizable symmetric α-stable, $PHS\alpha S$, stochastic sequence $\{\xi(n), n\in\mathbb{Z}\}$, has absolutely continuous spectral measure $\mu(\theta)$ and the spectral density $f(\theta)$, satisfying the minimality condition (23). The optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional*

$$A_N\vec{\xi}=\sum_{j=0}^{N}(\vec{a}(j))^{\top}\vec{\xi}(j)=\sum_{j=0}^{N}\sum_{k=0}^{T-1}a_k(j)\xi(jT+k),$$

$\vec{a}(j)=\{a_k(j)\}_{k=0}^{T-1}$, $\vec{\xi}(j)=\{\xi(jT+k)\}_{k=0}^{T-1}$, that depends on the unknown values $\xi(j), j=0,1,\dots,T(N+1)-1$, of the sequence $\xi(j)$ from observations of the sequence $\xi(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,T(N+1)-1\}$ is calculated by the formula (25). The spectral characteristic $h(\theta)$ of the optimal estimate is of the form (27), where the unknown coefficients $\vec{c}(j)=\{c_k(j)\}_{k=0}^{T-1}, j=0,1,\dots,T(N+1)-1$, are determined from the system of equations (29). The variance of the optimal estimate of the functional is calculated by the formula (30) (by formula (31) in the case where period $T=1$).

Example. Consider the problem of the optimal linear estimation of the functional $A_0\xi=a\xi(0)$, that depends on the unknown value $\xi(0)$ of a harmonizable symmetric α-stable, $HS\alpha S$, stochastic sequence $\xi(n)$ that has absolutely continuous spectral measure $\mu(\theta)$ and the spectral density $f(\theta)$, satisfying the minimality condition (23), from observations of the sequence $\xi(n)$ at points $n \in \mathbb{Z} \setminus \{0\}$ (Moklyachuk, & Ostapenko, 2016).

In this case the spectral characteristic $h(\theta)$ of the optimal estimate of the functional is of the form

$$h(\theta) = a - c^{<\frac{1}{\alpha-1}>} (f(\theta))^{\frac{-1}{\alpha-1}}\,.$$

The variance of the optimal estimate of the functional is calculated by the formula

$$\left\|\hat{A}_0\xi - A_0\xi\right\|_{\alpha}^{\alpha} = \int_{-\pi}^{\pi} \left|c^{<\frac{1}{\alpha-1}>} (f(\theta))^{-\frac{1}{\alpha-1}}\right|^{\alpha} f(\theta)d\theta,$$

where the constant c is a solution of the equation

$$\int_{-\pi}^{\pi} \left(a - c^{<\frac{1}{\alpha-1}>} (f(\theta))^{-\frac{1}{\alpha-1}}\right) d\theta = 0,$$

$$c = \frac{(2\pi a)^{<\alpha-1>}}{\left(\int_{-\pi}^{\pi} (f(\theta))^{-\frac{1}{\alpha-1}} d\theta\right)^{<\alpha-1>}}.$$

In the case of $\alpha = 2$ we have the Kolmogorov (Kolmogorov, 1992) results

$$h(\theta) = a - c\,(f(\theta))^{-1}\,,$$

$$c = a\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} ((f(\theta))^{-1})d\theta\right)^{-1},$$

$$\left\|\hat{A}\xi - A\xi\right\|_{2}^{2} = 2\pi|a|^2\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} ((f(\theta))^{-1})d\theta\right)^{-1}.$$

4. Interpolation problem. Minimax approach

The value of the error

$$\Delta\left(h(f_0, g_0); f, g\right) = \left\|A_N\vec{\xi} - \hat{A}_N\vec{\xi}\,\right\|_{\alpha}^{\alpha}$$

and the spectral characteristic $h(f,g) = h(\theta)$ of the optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi} = \sum_{j=0}^{N}(\vec{a}(j))^{\top}\vec{\xi}(j)$ that depends on the unknown values $\vec{\xi}(j), j = 0, 1, \ldots, N$, of the sequence $\vec{\xi}(j)$, from observations of the sequence $\vec{\xi}(j) + \vec{\eta}(j)$ at points $j \in \mathbb{Z} \setminus \{0, 1, \ldots, N\}$ can be calculated by the proposed formulas only in the case where we know the spectral densities $f(\theta)$ and $g(\theta)$ of the mutually independent harmonizable symmetric α-stable $HS\alpha S$, stochastic sequences $\vec{\xi}(j) = \{\xi_k(j)\}_{k=1}^{T}$, $j \in \mathbb{Z}$ and $\vec{\eta}(j) = \{\eta_k(j)\}_{k=1}^{T}$, $j \in \mathbb{Z}$.

However, in practice we can't exactly evaluate the spectral densities of stochastic sequences, but, instead, we often can have a set $D = D_f \times D_g$ of admissible spectral densities. In this case we can apply the minimax-robust method of estimation to the interpolation problem. This method let us find an estimate that minimizes the maximum of the errors for all spectral densities from the given set $D = D_f \times D_g$ of admissible spectral densities simultaneously (Moklyachuk, & Masyutka, 2012; Moklyachuk, & Golichenko, 2016).

Definition 5. For a given class of spectral densities $D = D_f \times D_g$ the spectral densities $f_0(\theta) \in D_f$, $g_0(\theta) \in D_g$ are called the least favorable in $D = D_f \times D_g$ for the optimal linear estimation $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$, if the following relation hold true

$$\Delta\left(f_0, g_0\right) = \Delta\left(h\left(f_0, g_0\right); f_0, g_0\right) = \max_{(f,g)\in D_f\times D_g} \Delta\left(h\left(f, g\right); f, g\right).$$

Definition 6. For a given class of spectral densities $D = D_f \times D_g$ the spectral characteristic $h^0 = h(f_0)$ of the optimal estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ is called minimax (robust) for the optimal linear estimation $A_N\vec{\xi}$, if the following relations hold true

$$h^0(\theta) \in H_D = \bigcap_{(f,g)\in D_f\times D_g} L^{\alpha}(f+g),$$

$$\min_{h\in H_D}\max_{(f,g)\in D} \Delta\left(h; f, g\right) = \max_{(f,g)\in D} \Delta\left(h^0; f, g\right).$$

The least favorable spectral densities $f_0(\theta)$, $g_0(\theta)$ and the minimax spectral characteristic $h^0 = h(f_0, g_0)$ form a saddle point of the function $\Delta\left(h; f, g\right)$ on the set $H_D \times D$. The saddle point inequalities

$$\Delta\left(h; f_0, g_0\right) \geqslant \Delta\left(h^0; f_0, g_0\right) \geqslant \Delta\left(h^0; f, g\right),\ \forall h \in H_D,\ \forall f \in D_f,\ \forall g \in D_g$$

hold true if $h^0 = h(f_0, g_0)$ and $h(f_0, g_0) \in H_D$, where (f_0, g_0) is a solution to the conditional extremum problem

$$\max_{(f,g)\in D_f\times D_g} \Delta\left(h(f_0, g_0); f, g\right) = \Delta\left(h(f_0, g_0); f_0, g_0\right), \qquad (32)$$

$$\Delta\left(h(f_0,g_0);f,g\right)=\left\|A_N\vec{\xi}-\hat{A}_N\vec{\xi}\right\|_{\alpha}^{\alpha}=\int_{-\pi}^{\pi}\left(A_N(e^{i\theta})-h^0(\theta)\right)^{\top}f(\theta)\left(A_N(e^{i\theta})-h^0(\theta)\right)^{<\alpha-1>}d\theta+$$

$$+\int_{-\pi}^{\pi}\left(h^0(\theta)\right)^{\top}g(\theta)\left(h^0(\theta)\right)^{<\alpha-1>}d\theta. \tag{33}$$

The constrained optimization problem (32) is equivalent to the unconditional extremum problem

$$\Delta_D(f,g)=-\Delta(h(f_0,g_0);f,g)+\delta(f,g\,|D_f\times D_g\,)\to \text{inf}, \tag{34}$$

where $\delta(f,g\,|D_f\times D_g\,)$ is the indicator function of the set $D=D_f\times D_g$. Solution (f_0,g_0) to the problem (34) is characterized by the condition $0\in\partial\Delta_D(f_0,g_0)$, where $\partial\Delta_D(f_0)$ is the subdifferential of the convex functional $\Delta_D(f,g)$ at point (f_0,g_0). This condition makes it possible to find the least favorable spectral densities in some special classes of spectral densities $D=D_f\times D_g$ (Ioffe, & Tihomirov, 1979; Pshenichnyj, 1971; Rockafellar, 1970).

Note, that the form of the functional $\Delta(h(f_0,g_0);f,g)$ is convenient for application the Lagrange method of indefinite multipliers for finding solution to the problem (32). Making use the method of Lagrange multipliers and the form of subdifferentials of the indicator functions we describe relations that determine least favourable spectral densities in some special classes of spectral densities.

Summing up the derived formulas and the introduced definitions we come to conclusion that the following lemmas hold true

Lemma 1. *Let $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}$, $j\in\mathbb{Z}$ and $\vec{\eta}(j)=\{\eta_k(j)\}_{k=1}^{T}$, $j\in\mathbb{Z}$ be mutually independent harmonizable symmetric α-stable random sequences which have absolutely continuous spectral measures and the spectral densities $f_0(\theta)$ and $g_0(\theta)$ satisfying the minimality condition (15). Let the spectral densities $(f_0,g_0)\in D_f\times D_g$ give a solution to the constrained optimization problem (32). The spectral densities (f_0,g_0) are the least favorable ones in $D_f\times D_g$ and $h^0=h(f_0,g_0)$ is the minimax spectral characteristic of the optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ that depends on the unknown values $\vec{\xi}(j), j=0,1,\dots,N$, of the sequence $\vec{\xi}(j)$, from observations of the sequence $\vec{\xi}(j)+\vec{\eta}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$, if $h^0=h(f_0,g_0)\in H_D$.*

Lemma 2. *Let $\vec{\xi}(j)=\{\xi_k(j)\}_{k=1}^{T}$, $j\in\mathbb{Z}$ be a harmonizable symmetric α-stable random sequence which has absolutely continuous spectral measure and the spectral density $f_0(\theta)$ satisfying the minimality condition (23). Let the spectral density $f_0\in D_f$ gives a solution to the constrained optimization problem*

$$\max_{f\in D_f}\Delta\left(h(f_0);f\right)=\Delta\left(h(f_0);f_0\right), \tag{35}$$

where

$$\Delta\left(h(f_0);f\right)=\left\|A_N\vec{\xi}-\hat{A}_N\vec{\xi}\,\right\|_{\alpha}^{\alpha}=\int_{-\pi}^{\pi}\left[\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\right]^{\top}f(\theta)\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^{<\frac{\alpha-1}{\alpha-1}>}d\theta. \tag{36}$$

The spectral density f_0 is the least favorable spectral density in D_f and $h^0=h(f_0)$ is the minimax spectral characteristic of the optimal linear estimate $\hat{A}_N\vec{\xi}$ of the functional $A_N\vec{\xi}$ that depends on the unknown values $\vec{\xi}(j), j=0,1,\dots,N$, of the sequence $\vec{\xi}(j)$, from observations of the sequence $\vec{\xi}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$, if $h^0=h(f_0)\in H_D$.

5. Least favorable spectral density in classes $D=D_0\times D_\varepsilon$

Consider the problem of the mean-square optimal interpolation of the functional $A_N\vec{\xi}$ in the case when spectral densities of sequences belong to the class of admissible spectral densities $D=D_0^k\times D_\varepsilon^k, k=1,2,3,4,$ where

$$D_0^1=\left\{f(\theta)\left|\frac{1}{2\pi}\int_{-\pi}^{\pi}\text{Tr}\,f(\theta)d\theta=p\right.\right\},$$

$$D_\varepsilon^1=\left\{g(\theta)\left|\text{Tr}\,g(\theta)=(1-\varepsilon)\text{Tr}\,g_1(\theta)+\varepsilon\text{Tr}\,w(\theta),\frac{1}{2\pi}\int_{-\pi}^{\pi}\text{Tr}\,g(\theta)d\theta=q\right.\right\};$$

$$D_0^2=\left\{f(\theta)\left|\frac{1}{2\pi}\int_{-\pi}^{\pi}f_{kk}(\theta)d\theta=p_k,k=\overline{1,T}\right.\right\},$$

$$D_\varepsilon^2=\left\{g(\theta)\left|g_{kk}(\theta)=(1-\varepsilon)g_{kk}^1(\theta)+\varepsilon w_{kk}(\theta),\frac{1}{2\pi}\int_{-\pi}^{\pi}g_{kk}(\theta)d\theta=q_k,k=\overline{1,T}\right.\right\};$$

$$D_0^3=\left\{f(\theta)\left|\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\langle B_1,f(\theta)\right\rangle d\theta=p\right.\right\},$$

$$D_\varepsilon^3=\left\{g(\theta)\left|\left\langle B_2,g(\theta)\right\rangle=(1-\varepsilon)\left\langle B_2,g_1(\theta)\right\rangle+\varepsilon\left\langle B_2,w(\theta)\right\rangle,\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\langle B_2,g(\theta)\right\rangle d\theta=q\right.\right\};$$

$$D_0^4=\left\{f(\theta)\left|\frac{1}{2\pi}\int_{-\pi}^{\pi}f(\theta)d\theta=P\right.\right\},$$

$$D_\varepsilon^4=\left\{g(\theta)\left|g(\theta)=(1-\varepsilon)g_1(\theta)+\varepsilon w(\theta),\frac{1}{2\pi}\int_{-\pi}^{\pi}g(\theta)d\theta=Q\right.\right\},$$

where $g_1(\theta)$ is a known and fixed spectral density matrix while $w(\theta)$ is an unknown spectral density matrix, p, q, p_k, q_k, $k=\overline{1,T}$ are given numbers, P, Q, B_1, B_2 are given matrices.

From the condition $0 \in \partial\Delta_D(f_0, g_0)$ we find the following equations which determine the least favourable spectral densities for these given sets of admissible spectral densities.

For the first pair $D_0^1 \times D_\varepsilon^1$ we have equations

$$\left(A_N(e^{i\theta}) - h^0(\theta)\right)\left((A_N(e^{i\theta}) - h^0(\theta))^\top\right)^{<\alpha-1>} = \alpha^2 E, \tag{37}$$

$$h^0(\theta)\left((h^0(\theta))^\top\right)^{<\alpha-1>} = \beta^2 + \gamma(\theta), \tag{38}$$

where $\gamma(\theta) \leqslant 0$ and $\gamma(\theta) = 0$ if $\mathrm{Tr}\, g_0(\theta) > (1-\varepsilon)\mathrm{Tr}\, g_1(\theta)$.

For the second pair $D_0^2 \times D_\varepsilon^2$ we have equations

$$\left(A_N(e^{i\theta}) - h^0(\theta)\right)\left((A_N(e^{i\theta}) - h^0(\theta))^\top\right)^{<\alpha-1>} = \left\{\alpha_k^2 \delta_{kl}\right\}_{k,l=1}^T, \tag{39}$$

$$h^0(\theta)\left((h^0(\theta))^\top\right)^{<\alpha-1>} = \left\{(\beta_k^2 + \gamma_k(\theta))\delta_{kl}\right\}_{k,l=1}^T, \tag{40}$$

where $\gamma_k(\theta) \leqslant 0$ and $\gamma_k(\theta) = 0$ if $g_{kk}^0(\theta) > (1-\varepsilon)g_{kk}^1(\theta)$.

For the third pair $D_0^3 \times D_\varepsilon^3$ we have equations

$$\left(A_N(e^{i\theta}) - h^0(\theta)\right)\left((A_N(e^{i\theta}) - h^0(\theta))^\top\right)^{<\alpha-1>} = \alpha^2 B_1, \tag{41}$$

$$h^0(\theta)\left((h^0(\theta))^\top\right)^{<\alpha-1>} = (\beta^2 + \gamma'(\theta))B_2, \tag{42}$$

where $\gamma'(\theta) \leqslant 0$ and $\gamma'(\theta) = 0$ if $\langle B_2, g_0(\theta)\rangle > (1-\varepsilon)\langle B_2, g_1(\theta)\rangle$.

For the fourth pair $D_0^4 \times D_\varepsilon^4$ we have equations

$$\left(A_N(e^{i\theta}) - h^0(\theta)\right)\left((A_N(e^{i\theta}) - h^0(\theta))^\top\right)^{<\alpha-1>} = \vec{\alpha} \cdot \vec{\alpha}^*, \tag{43}$$

$$h^0(\theta)\left((h^0(\theta))^\top\right)^{<\alpha-1>} = \vec{\beta} \cdot \vec{\beta}^* + \Gamma(\theta), \tag{44}$$

where $\Gamma(\theta) \leqslant 0$ and $\Gamma(\theta) = 0$ if $g_0(\theta) > (1-\varepsilon)g_1(\theta)$.

Thus, the following statements hold true.

Theorem 5. *Let the minimality condition (15) hold true. The least favorable spectral densities $f_0(\theta)$, $g_0(\theta)$ in the classes $D_0 \times D_\varepsilon$ for the optimal linear interpolation of the functional $A_N\vec{\xi}$ are determined by relations (37), (38) for the first pair $D_0^1 \times D_\varepsilon^1$ of sets of admissible spectral densities; (39), (40) for the second pair $D_0^2 \times D_\varepsilon^2$ of sets of admissible spectral densities; (41), (42) for the third pair $D_0^3 \times D_\varepsilon^3$ of sets of admissible spectral densities; (43), (44) for the fourth pair $D_0^4 \times D_\varepsilon^4$ of sets of admissible spectral densities; constrained optimization problem (32) and restrictions on densities from the corresponding classes $D_0 \times D_\varepsilon$. The minimax-robust spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (19).*

Corollary 1. *Let the minimality condition (23) hold true. The least favorable spectral densities $f_0(\theta)$ in the classes D_0^k, $k = 1, 2, 3, 4$, for the optimal linear interpolation of the functional $A_N\vec{\xi}$, which depends on the unknown values of the sequence $\vec{\xi}(j)$ based on observations of the sequence $\vec{\xi}(j)$ at points $j \in \mathbb{Z} \setminus \{0, 1, \dots, N\}$, are determined by the following equations, respectively,*

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^\top\right)^{<\frac{\alpha-1}{\alpha-1}>} = \alpha^2 E, \tag{45}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^\top\right)^{<\frac{\alpha-1}{\alpha-1}>} = \left\{\alpha_k^2 \delta_{kl}\right\}_{k,l=1}^T, \tag{46}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^\top\right)^{<\frac{\alpha-1}{\alpha-1}>} = \alpha^2 B_2, \tag{47}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^\top\right)^{<\frac{\alpha-1}{\alpha-1}>} = \vec{\alpha} \cdot \vec{\alpha}^*, \tag{48}$$

constrained optimization problem (35) and restrictions on densities from the corresponding classes D_0^k, $k = 1, 2, 3, 4$. The minimax spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (27).

Corollary 2. *Let the minimality condition (23) hold true. The least favorable spectral densities $f_0(\theta)$ in the classes D_ε^k, $k = 1, 2, 3, 4$, for the optimal linear interpolation of the functional $A_N\vec{\xi}$, which depends on the unknown values of the sequence $\vec{\xi}(j)$ based on observations of the sequence $\vec{\xi}(j)$ at points $j \in \mathbb{Z} \setminus \{0, 1, \dots, N\}$, are determined by the following equations, respectively,*

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^\top\right)^{<\frac{\alpha-1}{\alpha-1}>} = (\beta^2 + \gamma(\theta))E, \tag{49}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\left\{(\beta_k^2+\gamma_k(\theta))\delta_{kl}\right\}_{k,l=1}^{T}, \tag{50}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=(\beta^2+\gamma'(\theta))B_2, \tag{51}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left(\left(\overline{f_0^{-1}(\theta)C_N(e^{i\theta})}\right)^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\vec{\beta}\cdot\vec{\beta}^{*}+\Gamma(\theta), \tag{52}$$

constrained optimization problem (35) and restrictions on densities from the corresponding classes D_{ε}^{k}, $k=1,2,3,4.$ The minimax spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (27).

6. Least favorable spectral density in classes $D=D_V^U\times D_{2\delta}$

Consider the problem of mean square optimal interpolation of the functional $A_N\vec{\xi}$ in the case when spectral densities of the sequences belong to the class of admissible spectral densities $D=D_V^U\times D_{2\delta}$,

$$D_V^{U^1}=\left\{f(\theta)\middle|\operatorname{Tr}v(\theta)\leqslant\operatorname{Tr}f(\theta)\leqslant\operatorname{Tr}u(\theta),\ \frac{1}{2\pi}\int_{-\pi}^{\pi}\operatorname{Tr}f(\theta)d\theta=p\right\},$$

$$D_{2\delta}^{1}=\left\{g(\theta)\middle|\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\operatorname{Tr}(g(\theta)-g_1(\theta))\right|^2d\theta\leqslant\delta\right\};$$

$$D_V^{U^2}=\left\{f(\theta)\middle|v_{kk}(\theta)\leqslant f_{kk}(\theta)\leqslant u_{kk}(\theta),\ \frac{1}{2\pi}\int_{-\pi}^{\pi}f_{kk}(\theta)d\theta=p_k,k=\overline{1,T}\right\},$$

$$D_{2\delta}^{2}=\left\{g(\theta)\middle|\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|g_{kk}(\theta)-g_{kk}^{1}(\theta)\right|^2d\theta\leqslant\delta_k,k=\overline{1,T}\right\};$$

$$D_V^{U^3}=\left\{f(\theta)\middle|\left\langle B_1,v(\theta)\right\rangle\leqslant\left\langle B_1,f(\theta)\right\rangle\leqslant\left\langle B_1,u(\theta)\right\rangle,\ \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\langle B_1,f(\theta)\right\rangle d\theta=p\right\},$$

$$D_{2\delta}^{3}=\left\{g(\theta)\middle|\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left\langle B_2,g(\theta)-g_1(\theta)\right\rangle\right|^2d\theta\leqslant\delta\right\};$$

$$D_V^{U^4}=\left\{f(\theta)\middle|v(\theta)\leqslant f(\theta)\leqslant u(\theta),\ \frac{1}{2\pi}\int_{-\pi}^{\pi}f(\theta)d\theta=P\right\},$$

$$D_{2\delta}^{4}=\left\{g(\theta)\middle|\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|g_{ij}(\theta)-g_{ij}^{1}(\theta)\right|^2d\theta\leqslant\delta_{ij},i,j=\overline{1,T}\right\},$$

where spectral densities $v(\theta),u(\theta),g_1(\theta)$ are known and fixed. The class D_V^U describes the "strip" model of stochastic processes, $D_{2\delta}$ describes "δ-district" in the space L_2 of the given bounded spectral density $g_1(\theta)$.

From the condition $0\in\partial\Delta_D(f_0,g_0)$ we find the following equations which determine the least favourable spectral densities for these given sets of admissible spectral densities.

For the first pair $D_V^{U^1}\times D_{2\delta}^{1}$ we have equations

$$\left(A_N(e^{i\theta})-h^0(\theta)\right)\left((A_N(e^{i\theta})-h^0(\theta))^{\top}\right)^{<\alpha-1>}=(\alpha^2+\gamma_1(\lambda)+\gamma_2(\lambda))E, \tag{53}$$

$$h^0(\theta)\left((h^0(\theta))^{\top}\right)^{<\alpha-1>}=\beta^2\operatorname{Tr}(g_0(\theta)-g_1(\theta))E, \tag{54}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\operatorname{Tr}(g_0(\theta)-g_1(\theta))\right|^2d\theta=\delta, \tag{55}$$

where $\gamma_1\theta)\leqslant 0$ and $\gamma_1(\theta)=0$ if $\operatorname{Tr}f_0(\theta)>\operatorname{Tr}v(\theta)$, $\gamma_2(\theta)\geqslant 0$ and $\gamma_2(\theta)=0$ if $\operatorname{Tr}f_0(\lambda)<\operatorname{Tr}u(\theta)$.

For the second pair $D_V^{U^2}\times D_{2\delta}^{2}$ we have equations

$$\left(A_N(e^{i\theta})-h^0(\theta)\right)\left((A_N(e^{i\theta})-h^0(\theta))^{\top}\right)^{<\alpha-1>}=\left\{(\alpha_k^2+\gamma_{1k}(\theta)+\gamma_{2k}(\theta))\delta_{kl}\right\}_{k,l=1}^{T}, \tag{56}$$

$$h^0(\theta)\left((h^0(\theta))^{\top}\right)^{<\alpha-1>}=\left\{\beta_k^2(g_{kk}^0(\theta)-g_{kk}^1(\theta))\delta_{kl}\right\}_{k,l=1}^{T}, \tag{57}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|g_{kk}^0(\theta)-g_{kk}^1(\theta)\right|^2d\theta=\delta_k,\ k=\overline{1,T}, \tag{58}$$

where $\gamma_{1k}(\theta)\leqslant 0$ and $\gamma_{1k}(\theta)=0$ if $f_{kk}^0(\theta)>v_{kk}(\theta)$, $\gamma_{2k}(\theta)\geqslant 0$ and $\gamma_{2k}(\theta)=0$ if $f_{kk}^0(\theta)<u_{kk}(\theta)$.

For the third pair $D_V^{U^3}\times D_{2\delta}^{3}$ we have equations

$$\left(A_N(e^{i\theta})-h^0(\theta)\right)\left((A_N(e^{i\theta})-h^0(\theta))^{\top}\right)^{<\alpha-1>}=(\alpha^2+\gamma_1'(\theta)+\gamma_2'(\theta))B_1, \tag{59}$$

$$h^0(\theta)\left((h^0(\theta))^{\top}\right)^{<\alpha-1>}=\beta^2\left\langle B_2,g_0(\theta)-g_1(\theta)\right\rangle I, \tag{60}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\langle B_2, g_0(\theta)-g_1(\theta)\rangle\right|^2 d\theta=\delta, \tag{61}$$

where $\gamma_1'(\theta)\leqslant 0$ and $\gamma_1'(\theta)=0$ if $\langle B_1, f_0(\theta)\rangle>\langle B_1, v(\theta)\rangle$, $\gamma_2'(\theta)\geqslant 0$ and $\gamma_2'(\theta)=0$ if $\langle B_1, f_0(\theta)\rangle<\langle B_1, u(\theta)\rangle$, I is a matrix of ones.

For the fourth pair $D_V^{U4}\times D_{2\delta}^4$ we have equations

$$\left(A_N(e^{i\theta})-h^0(\theta)\right)\left(\overline{(A_N(e^{i\theta})-h^0(\theta))}^{\top}\right)^{<\alpha-1>}=\vec{\alpha}\cdot\vec{\alpha}^*+\Gamma_1(\theta)+\Gamma_2(\theta) \tag{62}$$

$$h^0(\theta)\left(\overline{(h^0(\theta))}^{\top}\right)^{<\alpha-1>}=\left\{\beta_{ij}(g_{ij}^0(\theta)-g_{ij}^1(\theta))\right\}_{i,j=1}^{T}, \tag{63}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|g_{ij}^0(\theta)-g_{ij}^1(\theta)\right|^2 d\theta=\delta_{ij},\ i,j=\overline{1,T}, \tag{64}$$

where $\Gamma_1(\theta)\leqslant 0$ and $\Gamma_1(\theta)=0$ if $f_0(\theta)>v(\theta)$, $\Gamma_2(\theta)\geqslant 0$ and $\Gamma_2(\theta)=0$ if $f_0(\theta)<u(\theta)$.

The following theorem and corollaries hold true.

Theorem 6. *Let the minimality condition (15) hold true. The least favorable spectral densities $f_0(\theta)$, $g_0(\theta)$ in the classes $D=D_V^U\times D_{2\delta}$ for the optimal linear interpolation of the functional $A_N\vec{\xi}$ are determined by relations (53) – (55) for the first pair $D_V^{U1}\times D_{2\delta}^1$ of sets of admissible spectral densities; (56) – (58) for the second pair $D_V^{U2}\times D_{2\delta}^2$ of sets of admissible spectral densities; (59) – (61) for the third pair $D_V^{U3}\times D_{2\delta}^3$ of sets of admissible spectral densities; (62) – (64) for the fourth pair $D_V^{U4}\times D_{2\delta}^4$ of sets of admissible spectral densities; constrained optimization problem (32) and restrictions on densities from the corresponding classes $D=D_V^U\times D_{2\delta}$. The minimax-robust spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (19).*

Corollary 3. *Let the minimality condition (23) hold true. The least favorable spectral densities $f_0(\theta)$ in the classes D_V^{Uk}, $k=1,2,3,4$, for the optimal linear interpolation of the functional $A_N\vec{\xi}$, which depends on the unknown values of the sequence $\vec{\xi}(j)$ based on observations of the sequence $\vec{\xi}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$, are determined by the following equations, respectively,*

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=(\alpha^2+\gamma_1(\theta)+\gamma_2(\theta))E, \tag{65}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=$$
$$=\left\{(\alpha_k^2+\gamma_{1k}(\theta)+\gamma_{2k}(\theta))\delta_{kl}\right\}_{k,l=1}^{T}, \tag{66}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=(\alpha^2+\gamma_1'(\theta)+\gamma_2'(\theta))B_1, \tag{67}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\vec{\alpha}\cdot\vec{\alpha}^*+\Gamma_1(\theta)+\Gamma_2(\theta), \tag{68}$$

constrained optimization problem (35) and restrictions on densities from the corresponding classes D_V^{Uk}. The minimax spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (27).

Corollary 4. *Let the minimality condition (23) hold true. The least favorable spectral densities $f_0(\theta)$ in the classes $D_{2\delta}^k$, $k=1,2,3,4$, for the optimal linear interpolation of the functional $A_N\vec{\xi}$, which depends on the unknown values of the sequence $\vec{\xi}(j)$ based on observations of the sequence $\vec{\xi}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$, are determined by the following equations, respectively,*

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\beta^2\mathrm{Tr}\,(f_0(\theta)-g_1(\theta))E, \tag{69}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\left\{\beta_k^2(f_{kk}^0(\theta)-g_{kk}^1(\theta))\delta_{kl}\right\}_{k,l=1}^{T}, \tag{70}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\beta^2\left\langle B_2, f_0(\theta)-g_1(\theta)\right\rangle I, \tag{71}$$

$$\left(f_0^{-1}(\theta)C_N(e^{i\theta})\right)^{<\frac{1}{\alpha-1}>}\left((\overline{f_0^{-1}(\theta)C_N(e^{i\theta})})^{\top}\right)^{<\frac{\alpha-1}{\alpha-1}>}=\left\{\beta_{ij}(f_{ij}^0(\theta)-g_{ij}^1(\theta))\right\}_{i,j=1}^{T} \tag{72}$$

constrained optimization problem (35) and the following restrictions on densities from the classes $D_{2\delta}^k$, $k=1,2,3,4$, respectively,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\mathrm{Tr}\,(f_0(\theta)-g_1(\theta))\right|^2 d\theta=\delta, \tag{73}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|f_{kk}^0(\theta)-g_{kk}^1(\theta)\right|^2 d\theta=\delta_k,\ k=\overline{1,T}, \tag{74}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\langle B_2, f_0(\theta)-g_1(\theta)\rangle\right|^2 d\theta=\delta, \tag{75}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|f^0_{ij}(\theta)-g^1_{ij}(\theta)\right|^2 d\theta=\delta_{ij},\ i,j=\overline{1,T}. \tag{76}$$

The minimax spectral characteristic of the optimal estimate of the functional $A_N\vec{\xi}$ is determined by the formula (27).

Discussion and conclusions

We propose methods for solving the optimal linear estimation problem for the linear functionals that depend on the unknown values $\vec{\xi}(j)$, $j=0,1,\dots,N$, of a vector-valued harmonizable symmetric α-stable random sequence $\vec{\xi}(j)= =\{\xi_k(j)\}_{k=1}^T$, from observations of the sequence $\vec{\xi}(j)+\vec{\eta}(j)$ at points $j\in\mathbb{Z}\setminus\{0,1,\dots,N\}$ where $\vec{\xi}(j)$ and $\vec{\eta}(j)$ are mutually independent harmonizable symmetric α-stable random sequences which have the spectral densities $f(\theta)$ and $g(\theta)$ satisfying the minimality condition.

The problem is investigated under the condition of spectral certainty as well as under the condition of spectral uncertainty. Formulas for calculating the value of the error and the spectral characteristic of the optimal linear estimate of the functional are derived under the condition of spectral certainty where spectral density of the sequence is exactly known. In the case where spectral density of the sequence is not exactly known, but a set of admissible spectral densities is available, relations which determine least favorable densities and the minimax-robust spectral characteristics for different classes of spectral densities are found.

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ЗАДАЧА ІНТЕРПОЛЯЦІЇ ВЕКТОРНИХ ГАРМОНІЗОВАНИХ СТІЙКИХ ПОСЛІДОВНОСТЕЙ

Задача оцінювання невідомих значень стохастичних процесів є актуальною проблемою як у теорії, так і в прикладних застосуваннях стохастичних процесів. Проблеми прогнозування майбутніх значень економічних і фізичних процесів, відновлення втраченої інформації, очищення сигналу або інших даних зі спостережень із шумом гостро постають в насиченому інформацією світі. З огляду на це розроблення методів оцінювання є одним з основних завдань сучасної теорії стохастичних процесів. У пропонованій статті розглянуто задачу оптимальної лінійної інтерполяції функціонала від невідомих значень векторної гармонізованої симетричної альфа-стійкої випадкової послідовності за спостереженнями послідовності із шумом. Використано класичний підхід для виведення формул для обчислення значень середньоквадратичної похибки та спектральної характеристики оптимальної лінійної оцінки функціонала. Основним припущенням цього підходу є те, що спектральні щільності наявних стохастичних послідовностей точно відомі. Однак на практиці отримати повну інформацію про спектральні щільності в більшості випадків неможливо. У цьому випадку знаходять параметричну або непараметричну оцінку невідомої спектральної щільності, а потім застосовують один із традиційних методів оцінювання за умови, що обрана щільність є істинною. Цей підхід може привести до значного зростання значення похибки оцінювання. Для подолання цього ефекту можна шукати оцінки, які є оптимальними для всіх щільностей з певного класу допустимих спектральних щільностей. Ці оцінки називають мінімаксними, оскільки вони мінімізують максимальні значення похибок оцінок для всіх щільностей із заданого класу. Тому у випадку спектральної невизначеності ми використовуємо мінімаксний підхід і пропонуємо формули, що визначають найменш сприятливі спектральні щільності та мінімаксні спектральні характеристики оптимальних оцінок функціонала для деяких класів допустимих спектральних щільностей.

Ключові слова: ***гармонізована стійка випадкова послідовність, періодично гармонізована стійка випадкова послідовність, оптимальна лінійна оцінка, мінімаксна (робастна) оцінка, мінімаксна спектральна характеристика, найменш сприятлива спектральна щільність.***

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