Вісник Київського національного університету імені Тараса Шевченка
Серія фізико-математичні науки

2019, №1

Bulletin of Taras Shevchenko National University of Kyiv
Series Physics & Mathematics

УДК 519.9

Харитонов О. М.[1], к.ф.-м.н., доц.,
Савченко С. Р.[2], студент,
Міранда Н.[3], студент.

Оптимізація швидких міжпланетних траєкторій з використанням ядерного дворежимного двигуна

[1] Київський національний університет імені Тараса Шевченка, 83000, м. Київ, пр-т. Глушкова 4д,
e-mail: kharytonov@univ.kiev.ua
[2] Київський національний університет імені Тараса Шевченка, 83000, м. Київ, пр-т. Глушкова 4д,
e-mail: serg.savchenko1999@gmail.com
[3] Центральна школа Ліону, 69130, м. Ліон, авеню Гай де Колонь 36.
e-mail: nicolas.miranda@ecl17.ec-lyon.fr

O. M. Kharytonov, associate professor, PhD [1]
S. R. Savchenko, three-years student [2]
N.Miranda, four-years student [3]

Optimization of sprint interplanetary trajectories with nuclear bimodal thermal propulsion

[1] Taras Shevchenko National University of Kyiv, 83000, Kyiv, Glushkova st., 4d,
e-mail: kharytonov@univ.kiev.ua
[2] Taras Shevchenko National University of Kyiv, 83000, Kyiv, Glushkova st., 4d,
e-mail: serg.savchenko1999@gmail.com
[3] Ecole Central Lyon, France, 36 Avenue Guy de Collongue, 69130, Ecully
e-mail: nicolas.miranda@ecl17.ec-lyon.fr

Міжпланетні місії вимагають швидких і економічних трансферів. З цією метою поєднання короткотривалих перельотів з використанням великої та малої тяги може забезпечити хороший компроміс. Збереження певної кількості палива при початковому імпульсі з великою тягою і його використання для корекції траєкторії дозволяє зекономити паливо і збільшити корисне навантаження. Досліджено оптимальний шлях, для отримання переваг від двигунів як великої, так і малої тяги з метою збільшення корисного навантаження. З використанням простої моделі ідеального двигуна обмеженої потужності і методу транспортуючої траєкторії знайдено аналітичний вираз для знаходження остаточного корисного навантаження. Розв'язання задачі оптимізації дозволило відшукати оптимальний розподіл палива між маневрами з використанням двигунів великої та малої тяги. Оскільки маса системи з малою тягою залежить від електричної потужності, ми використали її, щоб визначити оптимальну електричну потужність для перельоту за заданий час. У результаті було встановлено інтервал часу перельоту, для якого комбінація двигунів великої та низької тяги стає оптимальною.

Ключові слова: швидкий переліт, дворежимний ядерний двигун, метод транспортуючої траєкторії

Interplanetary missions require fast and fuel-efficient transfers. Combining small times transfers of high-thrust and efficiency of low-thrust propulsion can provide a good compromise. Saving an amount of fuel from the initial high-thrust burn and using it to correct the trajectory could lead to an economy of fuel. We investigated the optimal way to take advantages of both high and low-thrust propulsion benefits in order to maximize the payload mass of the mission. Using a simple model of ideal engine of limited power and the transporting trajectory method, we determined the analytical expression of final payload mass. The solution of the optimization problem gave us the optimal repartition of fuel between high and low-thrust maneuvers for a given thrust of thermal propulsion and electrical power of low-thrust propulsion system. As the mass of the low-thrust propulsion system depends on the electrical power, we took it into account to determine the optimal electrical power for a sprint trajectory in a given time. As a result, we could obtain the interval of transfer time for which the combination of high and low thrust becomes optimal.

Key Words: sprint trajectory, bimodal nuclear propulsion, transporting trajectory method

Статтю представив д.ф.-м.н., проф. Жук Я.О.

The problem of sprint interplanetary trajectories of the space vehicle with nuclear bi-modal propulsion is investigated. The optimization problem is formulated as follows: to distribute the total Δv budget between high- and low-thrust arcs to provide the maximum of payload. The high-thrust maneuver is modeled using impulse approximation and the low-thrust arc is modeled using the conception of ideal engine of limited power [1]. The engine parameters from [2] are used.

The aim of maneuver is fly over the Mars in a given time T. Using the Tsiolkowski formula and an ideal model of low-thrust propulsion, we could find an analytical expression (1) of the final payload mass M_π for this bimodal transfer:

$$M_\pi = \frac{M_0 e^{-\frac{\sqrt{ve_\infty^2 + 2\frac{\mu_e}{R_0}} - \sqrt{\frac{\mu_e}{R_0}}}{V}}}{1 + \frac{M_0}{2N} e^{-\frac{\sqrt{ve_\infty^2 + 2\frac{\mu_e}{R_0}} - \sqrt{\frac{\mu_e}{R_0}}}{V}} J} - M_x - M_t \quad (1)$$

where M_π is the initial mass of the spacecraft, M_χ is the propulsion system mass, M_t is the tank mass, N is the electrical power of the low-thrust propulsion system, V is the exhaust speed of high-thrust propulsion, μ_e is the standard gravitational parameter of the Earth, R_0 is the height of initial circular orbit of the spacecraft around the Earth, $V_{\square}^{e}$ is the speed vector of the spacecraft on a hyperbolic trajectory at an infinite distance from the Earth, J is the following functional :

$$J = \int_0^T a^2 dt \quad (2)$$

where $\boldsymbol{a}$ is the jet acceleration due to the low-thrust propulsion.

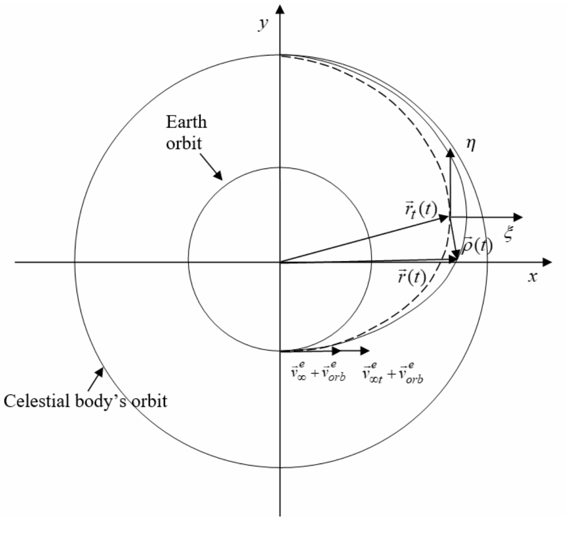

Fig 1. Bimodal trajectory (full line) and transporting trajectory (dotted line)

In accordance with (1) maximizing M_π requires to minimize J. To find the analytical expression for functional (2), we used the transporting trajectory method [3]. We used Keplerian arc corresponding to the fully high-thrust maneuver for a given time T as a transporting trajectory (Fig. 1).

In the chosen transporting coordinates system, the

$$\begin{aligned} &\dot{\xi} = \upsilon_\xi; \dot{\eta} = \upsilon_\eta; \dot{\upsilon}_\eta = a\cos(\theta); \\ &\dot{\upsilon}_\xi = a\sin(\theta); \dot{J} = a^2; J \to \min; \\ &\xi(0) = 0; \xi(T) = 0; \eta(0) = 0; \eta(T) = 0; \\ &\upsilon_\eta(0) = V_{\infty,x}^e - V_{\infty,t,x}^e = \Delta V_{\infty,x}^e \\ &\upsilon_\xi(0) = V_{\infty,y}^e - V_{\infty,t,y}^e = \Delta V_{\infty,y}^e \\ &\upsilon_\eta(T) = \upsilon_\xi(T) = 0 \end{aligned} \quad (3)$$

θ is the angle between the nozzle axis and the x axis. The vector $V_{\square,t}^{e}$ is the speed vector, on Earth position, on the unique Keplerian arc that links the Earth and Mars in a given time T, which is the transporting trajectory. The vector $V_{\square}^{e}$ is the speed vector, on Earth position, on the bimodal trajectory. To set this speed vector means allocating a certain amount of fuel to the high-thrust burn, and the remaining fuel for low-thrust. The maximum of fuel allocated to high-thrust corresponds to the Keplerian trajectory where no fuel is used for low-thrust. It implies $| V_{\square}^{e} | \leq | V_{\square,t}^{e} |$ .The solution of variational problem

$$J = \frac{4}{T}((V_{\infty,x}^e - V_{\infty,t,x}^e)^2 + (V_{\infty,y}^e - V_{\infty,t,y}^e)^2) \quad (4)$$

(3) provides an analytical expression for J:

Taking into account that M_π depends on $V^e_{\square}{}^2 = V^e_{\square,x}{}^2 + V^e_{\square,y}{}^2$, the selection of ($V^e_{\square,x}$, $V^e_{\square,y}$) must satisfy the condition of J constrained minimum:

$$\begin{cases} J = \frac{4}{T}((V_{\infty,x}^e - V_{\infty,t,x}^e)^2 + (V_{\infty,y}^e - V_{\infty,t,y}^e)^2) \\ V_{\infty,x}^{e\;2} + V_{\infty,x}^{e\;2} = V_\infty^{e\;2} \end{cases} \quad (5)$$

Its resolution gives another relation (6) between the two speed components:

$$\frac{V_{\infty,y}^e}{V_{\infty,t,y}^e} = \frac{V_{\infty,x}^e}{V_{\infty,t,x}^e} \quad (6)$$

This formula shows that the vectors are colinear.

3. Results

In (1), the propulsion system mass M_χ is an empirical function of the electrical power, shown in (7), empirical constants A, B, C and D are exposed in [1]:

$$M_\chi(N) = 0.7(\frac{A}{BN + C} + D)N \quad (7)$$

Both M_χ and the low-thrust efficiency increase when N increases. For bimodal propulsion to be efficient, this additional mass has to be overcome by the fuel mass saved with the increasing of thrust. A too small electrical power would imply a dead weight, and a too big one would imply a too heavy propulsion system: for a given travel time T, this benefit is present in an interval of values of N and reach a maximum, as shown in the Fig 2.

For several values of time of transfer and electrical power, we calculate the optimal distribution of fuel between high and low-thrust by minimizing the expression (1). It allows us to draw the maximum payload mass according to the electrical power for several values of T. The plot is illustrated in the Fig.2 :

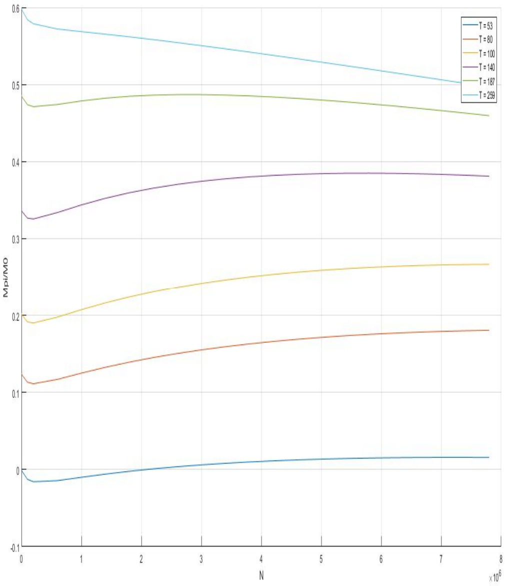

Fig 2. Maximum payload mass according to the electrical power of the low-thrust propulsion system, for T = 259, 187, 140, 100, 80, 53 days (top to bottom)

Bimodal propulsion is beneficial when the maximum of the previous curves is not for $N = 0$, otherwise the classical approach is better. It happens for time under 187 days. It means that payload mass can be increased for travel time under 187 days by using bimodal propulsion. To compare the efficiency between the bimodal and high-thrust only approaches, we plot the surfaces showing the maximal payload masses according to $V^e_{\square,x}$ and $V^e_{\square,y}$. The flat surface corresponds to the payload mass for the high-thrust only approach.

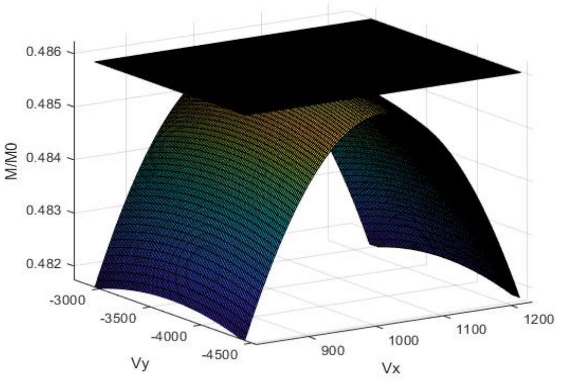

Fig 3. Maximum payload mass for 187 days

Fig 3 show the maximum payload mass and the speed vector components of the high-thrust initial burn which allows us to reach this maximum. This surface is calculated with the electrical power that maximize the payload mass on the 187 days curve on Fig 2, respectively 2,2MW.

Fig shows this surface for a travel time of 100 days. In this case, the optimal electrical power is 7,8MW :

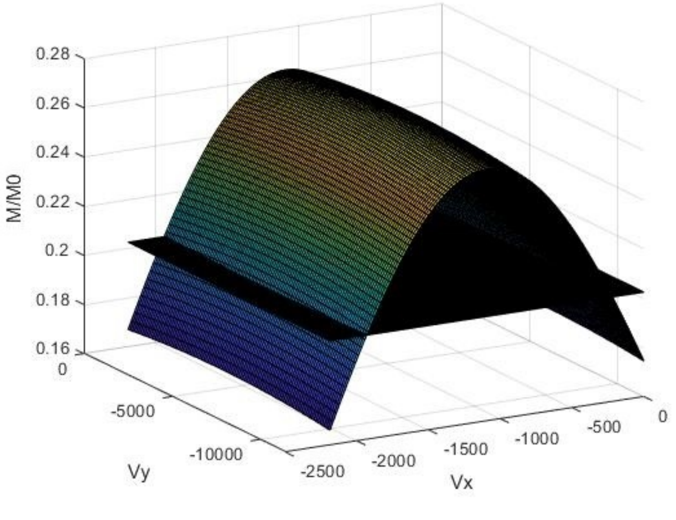

Fig 4. Maximum payload mass for 100 days

The relative gain between the high-thrust only approach (the flat surface) and the bimodal approach in this case is 32%.

Conclusion

In conclusion, we proposed a simple algorithm to determine the benefit of bimodal propulsion for interplanetary missions. Using an analytical expression of payload mass makes it possible to obtain the optimal fuel repartition between high and low-thrust maneuvers. The next step could be to pursue this study for a two-burn case, with a circularization around Mars.

Список використаних джерел

1. *Харитонов О.М.* Дослідження ефективності комбінації дуг високої та низької тяги для міжпланетних перельотів / О. М. Харитонов // Вісник Київського національного університету. (ISSN 1728-3817). – 2011. – 26. – С. 27-31.
2. Borowski S. K. Nasa Technical Report TM-1998-208834 “*Vehicle and Mission Design Options for Humans Exploration of Mars/Phobos Using “Bimodal” NTR and LANTR Propulsion”*/ S. K. Borowski, L. A. Dudzinski – 1998. – Available from : tps://ntrs.nasa.gov/
3. Beletsky V. Essays in the Motion of Celestial Bodies / V. Beletsky. – 2001.– Available from: http://www.springer.com/

References

1. KHARYTONOV, A. (2011) The investigation of the efficiency of the combination of high-and-low thrust arcs for interplanetary transfers. *Vysnyk of Taras Shevchenko University of Kyiv* (ISSN 1728-3817).

2. BOROWSKI, S. & DUDZINSKI, L. (1998) NASA Technical Report TM-1998-208834: *Vehicle and Mission Design Options for Humans Exploration of Mars/Phobos Using “Bimodal” NTR and LANTR Propulsion*. Available from : tps://ntrs.nasa.gov/

3. BELETSKY, V. (2001) Essays in the Motion of Celestial Bodies. Available from: http://www.springer.com/

Надійшла до редколегії 01.07.19