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Про вплив анізотропії матеріалу на граничний стан ортотропної пластини з періодичною системою колінеарних тріщин під дією двовісного навантаження

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Influence of the material anisotropy on the limit state of orthotropic plate with periodic system of collinear cracks under biaxial loading

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На основі модифікованої моделі Леонова-Панасюка-Дагдейла проведено дослідження впливу анізотропії механічних властивостей матеріалу, зокрема, різниці меж міцності при розтягу та стиску, на граничний стан ортотропної пластини, послабленої періодичною системою колінеарних тріщин, в умовах дії зовнішнього двовісного навантаження. На прикладі матеріалу, що задовольняє умові міцності Гоффмана, досліджено механізм руйнування пластини з періодичною системою колінеарних тріщин. Показано вплив ступеня анізотропії матеріалу на процес руйнування та граничний стан пластини.
Ключові слова: періодична система тріщин, двовісність навантаження, ортотропія.

In the presented paper, the limiting state of the orthotropic plates weakened by the periodic system of collinear cracks under biaxial external loading is studied on the basis of the modified crack model of the Leonov-Panasyuk-Dagdale. The material of plate satisfies the strength condition of the general form. On the basis of the solution of a similar problem for an orthotropic plate with one crack, we obtain the relations for determining the basic parameters of a crack model, such as the size of the process zones, the stresses in these zones, and the opening at the top of the cracks. The criterion of critical crack opening is selected as a fracture criterion. On the example of a material satisfying Hoffman strength criterion (generalization of the Mises–Hill criterion, which takes into account the dependence of the difference between the tensile and compressive strength of unidirectional composite materials), the fracture mechanism of a plate weakened by the periodic system of collinear cracks was investigated. The influence of the degree of material anisotropy and biaxiality of external loading on the fracture process and the limiting state of the plate are shown.
Key Words: periodic system of cracks, biaxial loading, orthotropy.

Статтю представив д.ф.-м.н., проф. Жук Я.О.

In the present paper, on the basis of the general solution obtained in [1,2] and Hoffman strength criterion [3], the influence of the anisotropy of the material (the differences between moduli of elasticity along axes of orthotropy and between the ultimate tensile strength and ultimate compressive strength) on the limit state of a thin orthotropic plate weakened by a periodic system of collinear cracks under conditions of biaxial external loading is investigated.

Consider a thin orthotropic plate with periodic system of collinear cracks of length $2l$ located along the axis of orthotropy which coincides with the Ox-axis. The centers of the cracks are located at the points $x_n = 2nD \quad (x = 0, \pm 1, \pm 2, \ldots, y = 0)$. The plate is stretched by a homogeneous load applied at infinity $\sigma_y = p > 0$, $\sigma_x = q$, $\tau_{xy} = 0$ by $z \to \infty$ $(z=x+iy)$.

We replace the process zones formed under the action of the loading near the crack tips by additional cuts of length d on the continuations of the cracks whose lips are subjected to the action of

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stresses σ_x^0 , σ_y^0 . Assume that the limiting state of the material in the process zones is described by a strength criterion

$$F(\sigma_1, \sigma_2, C_i) = 0, \qquad (1)$$

where σ_1, σ_2 are the principal stresses and C_i are constants of the material.

In view of the symmetry of the problem, the directions x and y are principal. Therefore, the stresses σ_x^0 , σ_y^0 satisfy the condition of strength (1) in the process zone. These stresses are found from the solution of the system of two equations [1,2]:

$$\sigma_x^0 = \beta(\sigma_y^0 - p) + q, \quad F(\sigma_x^0, \sigma_y^0, C_i) = 0, \qquad (2)$$

$\beta = \sqrt{E_1/E_2}$; E_1, E_2 - are the elasticity moduli of the material in directions 1 and 2.

For the numerical analysis and conclusions the Hoffman criterion of strength is used. It is a generalization of the Mises–Hill criterion, which takes into account the dependence of the difference between the tensile and compressive strength of unidirectional composite materials. For the plane stressed state, this criterion has the form [3]

$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\sigma_1^c\sigma_1^t} + \frac{\sigma_2^2}{\sigma_2^c\sigma_2^t} + \frac{\sigma_1^c - \sigma_1^t}{\sigma_1^c\sigma_1^t}\sigma_1 + \frac{\sigma_2^c - \sigma_2^t}{\sigma_2^c\sigma_2^t}\sigma_2 + \frac{\tau_{12}^2}{\tau_0^2} = 1,$$

where σ_1^t , σ_2^t are ultimate strengths in tension along the axes *Ox* and *Oy*, σ_1^c , σ_2^c are ultimate strengths in compression along the axes *Ox* and *Oy*, and τ_0 is the ultimate strength in shear along the principal directions.

The crack opening displacement $\delta(x, l, L)$ at a point x from the segments $|x - x_n| \le L, y = 0$ is given by the formula [1,2]:

$$\delta(x) = \frac{2T_0\sigma_y^0 l}{\pi}\frac{\sin\alpha}{\alpha}\int_{\omega}^{\sec\rho}\ln\left(\frac{1 + \xi\cos^2\rho + \sin\rho\sqrt{1 - \xi^2\cos^2\rho}}{1 - \xi\cos^2\rho + \sin\rho\sqrt{1 - \xi^2\cos^2\rho}}\frac{\xi - 1}{\xi + 1}\right)\frac{d\xi}{\sqrt{1 - \xi^2\sin^2\alpha}}, \qquad (3)$$

where

$$T_0 = \frac{1}{\sqrt{E_1E_2}}\sqrt{2\left(\sqrt{\frac{E_1}{E_2}} - \nu_{12}\right) + \frac{E_1}{G_{12}}}, \quad \rho = \frac{\pi p}{2\sigma_y^0},$$

$$\alpha = \sin\frac{\pi l}{2D}, \quad \omega = \sin\frac{\pi x}{2D}\Big/\sin\alpha .$$

It is clear that the integral in relation (3) is computed in the finite form as $D \to \infty$, which corresponds to the case of a single crack [1].

The size of process zone is determined by the ratio [1,2]

$$\sin\frac{\pi l}{2D}\Big/\sin\frac{\pi L}{2D} = \cos\frac{\pi p}{2\sigma_y^0}. \qquad (4)$$

We choose the critical crack opening displacement criterion as a fracture criterion, then the start of the crack occurs at the moment the crack tip opening displacement attains a certain limit value δ_c , i.e. $\delta(l) = \delta_c$.

Then, basing on (3), the field of ultimate loads p_* can be defined by

$$\sigma_y^0(p_*, q_*)\frac{\sin\alpha}{\alpha}\int_1^{\sec\rho_*}\ln\left(\frac{1 + \xi\cos^2\rho_* + \sin\rho_*\sqrt{1 - \xi^2\cos^2\rho_*}}{1 - \xi\cos^2\rho_* + \sin\rho_*\sqrt{1 - \xi^2\cos^2\rho_*}}\frac{\xi - 1}{\xi + 1}\right)\frac{d\xi}{\sqrt{1 - \xi^2\sin^2\alpha}} = 2\sigma_y^0(p_*^{(1)}, 0)\ln\sec\frac{\pi p_*^{(1)}}{2\sigma_y^0(p_*^{(1)}, 0)}, \qquad (5)$$

where $\rho_* = \dfrac{\pi p_*}{2\sigma_y^0(p_*, q_*)}$, $p_*^{(1)}$ is the limit load in uniaxial loading of the plate with a single crack. The change in $p_*^{(1)}$ from zero to the ultimate strength of the material in the direction y corresponds to the change in the length of the crack from infinity to zero, i.e., includes the whole range of change in the length of the crack.

Since the stresses σ_y^0, which depend on the constants that characterize the mechanical properties of the material, enter into the determining relations (6), it is clear that the limit state of the plate depends also on these constants.

However, it should be noted that the limit state of the plane weakened by a system of cracks is not always determined by the fracture criteria of type (6). If the external load is such that the condition $d = D - l$ is satisfied, then the emergence of prefracture zones of two neighboring cracks occurs, which can also be thought to be the condition of limit state. In view of (4), this condition takes the form

$$\frac{D}{l}=\frac{\pi}{2\arcsin\left(\cos\rho^{(d)}\right)},\ \rho^{(d)}=\frac{\pi\, p_*^{(d)}}{2\sigma_y^0\left(p_*^{(d)}\right)}.\quad(6)$$

The relation (6) determines the load $p_*^{(d)}$ at which the areas of process zones of neighboring cracks occur.

Fig.1 shows limit fracture curves obtained on the basis of (5) (solid curves), curves of coalescence of process zones obtained on the basis of (6) (dashed curves) for $\beta = 0.1, 0.5, 0.9$ and $p_*^{(0)}/\sigma_2^t = 0.5$, and a limit strength curve for a defect-free material (dash-dot curve) for $\sigma_1^t/\sigma_2^t = 0.8$, $\sigma_2^c/\sigma_2^t = 0.4$, $\sigma_2^c/\sigma_2^t = 0.5$; $D/l = 5$.

Fig. 2 shows dependences of the limit load p_*/σ_2^t on the ratio of the moduli of elasticity E_1/E_2 for $q/\sigma_2^t = -0.2, 0.5$ for material with the same ultimate strengths. As can be seen from the obtained results, an increase in the degree of anisotropy of the material leads to a decrease in the limit load p acting perpendicularly to the line of the cracks in the region of tensile loads q acting along the line of the cracks and to an increase in the limit load $p$in the region of compressive loads q.All calculations were performed for $D/l = 5$.

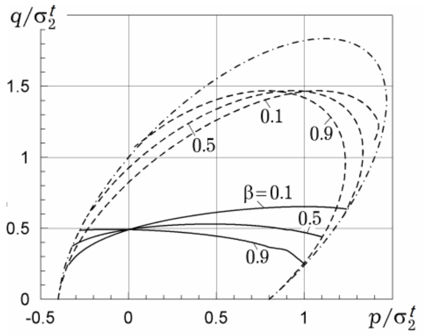

Fig.1. The critical load and coalescence of process zones for different E_1/E_2

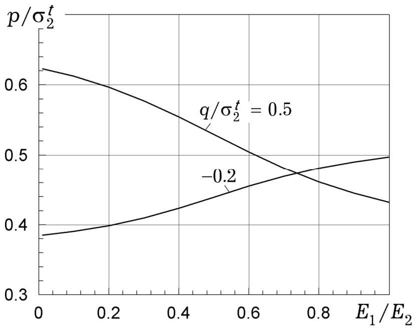

Fig.2. Dependence of critical load on the ratio of the moduli of elasticity E_1/E_2

Fig.3 shows limit fracture curves obtained on the basis of (5) (solid curves), curves of coalescence of process zones obtained on the basis of (6) (dashed curves), and the corresponding limit strength curves (dash-dot curve)for $\sigma_1^c/\sigma_1^t = 0.5, 1.0, 2.0$, and $\beta = 0.5$, $p_*^{(0)}/\sigma_2^t = 0.5$, $\sigma_1^t/\sigma_2^t = 1.0$.

Fig.4 shows limit fracture curves obtained on the basis of (5) (solid curves), curves of coalescence of process zones obtained on the basis of (6) (dashed curves), and the corresponding limit strength curves (dash-dot curve) for $\sigma_2^c/\sigma_2^t = 0.5, 1.0, 2.0$, and $\beta = 0.5$, $p_*^{(0)}/\sigma_2^t = 0.5$, $\sigma_1^t/\sigma_2^t = 1.0$.

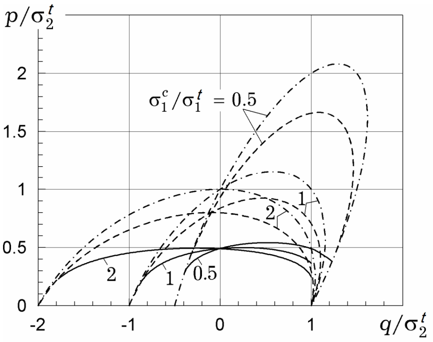

Fig.3. Dependence of critical load on the ratio of tensile and compressive strengths for different σ_1^c/σ_1^t

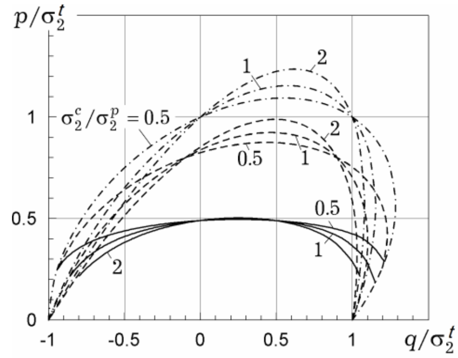

Fig.4. Dependence of critical load on the ratio of tensile and compressive strengths for different σ_2^c/σ_2^t

Fig.5 shows dependences of the dimensionless limit load p_*/σ_2^t of the ultimate strength in compression along the axis of orthotropy$Oy$to the ultimate strength in tension along the same axis for different values of the load acting along the line of the crack $q/\sigma_2^t = -0.8, -0.5, -0.1, 0.5, 0{,}8$ and

$\sigma_1^t / \sigma_2^t = 1.0$. Fig. 6 shows the dependences of the dimensionless limit load p_* / σ_2^t on the ratio of the ultimate strength in compression along the axis of orthotropyOx to the ultimate strength in tension along the axis of orthotropyOy σ_1^c / σ_2^t for different values of the load acting along the line of the crack $q / \sigma_2^t = -0.8, -0.5, -0.1, 0.5, 0{,}8$ and $\sigma_1^t / \sigma_2^t = 1.0$.

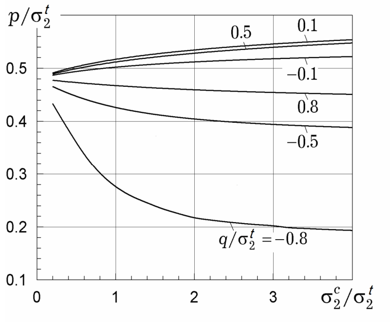

Fig.5. Dependence of critical load on the ratioof tensile and compressive strengths σ_2^c / σ_2^t

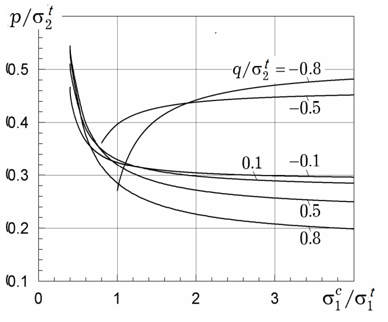

Fig.6. Dependence of critical load on the ratio of tensile and compressive strengths σ_1^c / σ_1^t

The proposed modification of the Leonov-Panasyuk-Dagdale crack model to the case of orthotropic materials allows to effectively solve problems of the destruction of orthotropic bodies with cracks, the material of which satisfies the condition of the strength of the general form.

As it follows from the presented results, the difference between the ultimate tensile strength and ultimatecompressive strength along each direction influences substantially the limit state of the orthotropic plate weakened by a periodic system of collinear cracks.

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