УДК 519.21
https://doi.org/10.17721/1812-5409.2021/2.6

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Екстраполяція періодично корельованих стохастичних послідовностей з пропусками спостережень

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Extrapolation problem for periodically correlated stochastic sequences with missing observations

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Досліджується задача оптимального оцінювання лінійних функціоналів $A\zeta=\sum_{j=1}^{\infty}a(j)\zeta(j)$, від невідомих значень періодично корельованої стохастичної послідовності $\zeta(j)$ на основі спостережень послідовності $\zeta(j)+\theta(j)$ в точках $j\in\{\ldots,-n,\ldots,-2,-1,0\}\setminus S$, $S=\bigcup_{l=1}^{s-1}\{-M_l\cdot T+1,\ldots,-M_{l-1}\cdot T-N_l\cdot T\}$, де $\theta(j)$ - некорельована з $\zeta(j)$ періодично корельована стохастична послідовність. Отримано формули для обчислення значень середньоквадратичних похибок та спектральних характеристик оптимальних оцінок функціоналу $A\zeta$ для послідовностей з відомими спектральними щільностями. Формули, що визначають найменш сприятливі спектральні щільності та мінімаксно-робастні спектральні характеристики оптимальних лінійних оцінок функціоналів пропонуються у випадку, коли спектральні щільності послідовностей точно невідомі, а вказані множини допустимих спектральних щільностей.

Результати дослідження доповідались на Міжнародній науковій конференції "Modern Stochastics: Theory and Applications. V" (MSTA-V).

Ключові слова: Періодично корельована стохастична послідовність, мінімаксно-робастна оцінка, найменш сприятлива спектральна щільність, мінімаксно-робастна спектральна характеристика.

The problem of optimal estimation of the linear functionals $A\zeta=\sum_{j=1}^{\infty}a(j)\zeta(j)$, which depend on the unknown values of a periodically correlated stochastic sequence $\zeta(j)$ from observations of the sequence $\zeta(j)+\theta(j)$ at points $j\in\{...,-n,...,-2,-1,0\}\setminus S$, $S=\bigcup_{l=1}^{s-1}\{-M_l\cdot T+1,\ldots,-M_{l-1}\cdot T-N_l\cdot T\}$, is considered, where $\theta(j)$ is an uncorrelated with $\zeta(j)$ periodically correlated stochastic sequence. Formulas for calculation the mean square error and the spectral characteristic of the optimal estimate of the functional $A\zeta$ are proposed in the case where spectral densities of the sequences are exactly known. Formulas that determine the least favorable spectral densities and the minimax-robust spectral characteristics of the optimal estimates of functionals are proposed in the case of spectral uncertainty, where the spectral densities are not exactly known while some sets of admissible spectral densities are specified.

Key Words: Periodically correlated sequence, optimal linear estimate, mean square error, least favourable spectral density matrix, minimax spectral characteristic.

Mathematics Subject Classification – MSC2020. Primary: 60G10, 60G25, 60G35, Secondary: 62M20, 93E10, 93E11

Introduction

W.R. Bennett [1] in 1958 started to explore cyclostationarity as a phenomenon and property of the process, which describes signals in channels of communication. Studying the statistical characteristics of information transmission, he calls the group of telegraph signals the cyclostationary process, that is the process whose group of statistics changes periodically with time. W.A. Gardner [5], [6] highlights the greatest similarity of cyclostationary processes, which are a subclass of nonstationary processes, with stationary processes. He presented the bibliography of works [7] in which properties and applications of cyclostationary processes were investigated till 1992. Recent developments and applications of cyclostationary signal analysis are reviewed in the papers by A. Napolitano [30], [31]. Note, that in other sources cyclostationary processes are called periodically stationary, periodically nonstationary, periodically correlated. We will use the term periodically correlated processes. E.G. Gladyshev [8] in 1961 was the first who started the analysis of spectral properties and representation of periodically correlated sequences based on its connection with vector stationary sequences. He formulated the necessary and sufficient conditions for determining of periodically correlated sequence in terms of the correlation function. A. Makagon with coauthrs [17], [18] presented detailed spectral analysis of periodically correlated sequences. Main ideas of the research of periodically correlated sequences are outlined in the book by H.L. Hurd and A. Miamee [12].

The linear extrapolation and interpolation problems for stationary stochastic processes under the condition that spectral densities are known exactly were first introduced by A. N. Kolmogorov [15]. Solutions of the extrapolation and filtering problems for stationary processes and sequences with rational spectral densities were proposed by N. Wiener [36] and A. M. Yaglom [37]. Estimation problems for vector stationary sequences were investigated by E. J. Hannan [11] and Yu. A. Rozanov [34]. Since stochastic processes often accompanied with undesirable noise it is naturally to assume that the exact value of spectral density is unknown and the model of process is given by a set of restrictions on spectral density. K.S. Vastola and H.V. Poor [35] have demonstrated that the described procedure can result in significant increasing of the value of error. This is a reason for searching estimates which are optimal for all densities from a certain class of admissible spectral densities. These estimates are called minimax since they minimize the maximal value of the error of estimates. A survey of results in minimax (robust) methods of data processing can be found in the paper by Kassam and Poor [14].

Ulf Grenander [10] was the first who proposed the minimax approach to the extrapolation problem for stationary processes. Formulation and investigation of the problems of extrapolation, interpolation and filtering of linear functionals which depend on the unknown values of stationary sequences and processes from observations with and without noise are presented by M.P. Moklyachuk [22], [23]. Results of investigation of the problems of optimal estimation of vector-valued stationary sequences and processes are published by M.P. Moklyachuk, O.Yu. Masyutka [25], [26], [27]. In their book M.M Luz and M.P. Moklyachuk [16] presented results of investigation of the minimax estimation problems for linear functionals which depends on unknown values of stochastic sequence with stationary increments. I.I. Golichenko and M.P. Moklyachuk [2], [3], [4], [24] investigated the interpolation, extrapolation and filtering problems of linear functionals from periodically correlated stochastic sequences and processes. The interpolation and filtering problems for stationary sequences with missing values was examined by M.P. Moklyachuk, O.Yu. Masyutka and M.I.Sidei [19], [21], [28], [29]. The interpolation problem of linear functionals from periodically correlated stochastic sequences with missing observations was investigated by I.I. Golichenko and M.P. Moklyachuk in [9].

In this paper we presented results of investigation of the problem of optimal linear estimation of the functional $A\zeta = \sum_{j=1}^{\infty} a(j)\zeta(j)$, which depends on the unknown values of a periodically correlated stochastic sequence $\zeta(j)$ from observations of the sequence $\zeta(j)+\theta(j)$ at points $j \in \{\dots,-n,\dots,-2,-1,0\} \setminus S$, $S = \bigcup_{l=1}^{s-1}\{-M_l \cdot T+1,\dots,-M_{l-1}\cdot T - N_l \cdot T\}$, where $\theta(j)$ is an uncorrelated with $\zeta(j)$ periodically correlated stochastic sequence. Formulas for calculation of the mean square error and the spectral characteristic of the optimal estimate of the functional $A\zeta$ are proposed in the case where spectral densities are exactly known. Formulas that determine the least favorable spectral densities and the minimax-robust spectral characteristics of the optimal estimates of functionals are proposed in the case of spectral uncertainty, where the spectral densities are not exactly known while some sets of admissible spectral densities are specified.

1 Periodically correlated and multidimensional stationary sequences

The term *periodically correlated* process was introduced by E. G. Gladyshev [8] while W. R. Bennett [1] called random and periodic processes *cyclostationary* process.

Periodically correlated sequences are stochastic sequences that have periodic structure (see the book by H. L. Hurd and A. Miamee [12]).

Definition 1. *A complex valued stochastic sequence $\zeta(n), n \in \mathbb{Z}$ with zero mean, $\mathsf{E}\zeta(n) = 0$, and finite variance, $\mathsf{E}|\zeta(n)|^2 < +\infty$, is called cyclostationary or periodically correlated (PC) with period T (T-PC) if for every $n, m \in \mathbb{Z}$*

$$\mathsf{E}\zeta(n+T)\overline{\zeta(m+T)} = R(n+T, m+T) = R(n,m) \tag{1}$$

and there are no smaller values of $T > 0$ for which (1) holds true.

Definition 2. *A complex valued T-variate stochastic sequence $\vec{\xi}(n) = \{\xi_\nu(n)\}_{\nu=1}^{T}, n \in \mathbb{Z}$ with zero mean, $\mathsf{E}\xi_\nu(n) = 0, \nu = 1,\dots,T$, and $\mathsf{E}||\vec{\xi}(n)||^2 < \infty$ is called stationary if for all $n, m \in \mathbb{Z}$ and $\nu, \mu \in \{1,\dots,T\}$*

$$\mathsf{E}\xi_\nu(n)\overline{\xi_\mu(m)} = R_{\nu\mu}(n,m) = R_{\nu\mu}(n-m).$$

If this is the case, we denote $R(n) = \{R_{\nu\mu}(n)\}_{\nu,\mu=1}^{T}$ and call it the covariance matrix of T-variate stochastic sequence $\vec{\xi}(n)$.

Proposition 1.1. (E. G. Gladyshev [8]). *A stochastic sequence $\zeta(n)$ is PC with period T if and only if there exists a T-variate stationary sequence $\vec{\xi}(n) = \{\xi_\nu(n)\}_{\nu=1}^{T}$ such that $\zeta(n)$ has the representation*

$$\zeta(n) = \sum_{\nu=1}^{T} e^{2\pi i n\nu/T}\xi_\nu(n), \ n \in \mathbb{Z}. \tag{2}$$

The sequence $\vec{\xi}(n)$ is called generating sequence of the sequence $\zeta(n)$.

Proposition 1.2. (E. G. Gladyshev [8]). *A complex valued stochastic sequence $\zeta(n), n \in \mathbb{Z}$ with zero mean and finite variance is PC with period T if and only if the T-variate blocked sequence $\vec{\zeta}(n)$ of the form*

$$[\vec{\zeta}(n)]_p = \zeta(nT+p), \ n \in \mathbb{Z}, p = 1,\dots,T \tag{3}$$

is stationary.

We will denote by $f^{\vec{\zeta}}(\lambda) = \left\{f^{\vec{\zeta}}_{\nu\mu}(\lambda)\right\}_{\nu,\mu=1}^{T}$ the matrix valued spectral density function of the T-variate stationary sequence $\vec{\zeta}(n) = (\zeta_1(n),\dots,\zeta_T(n))^\top$ arising from the T-blocking (3) of a univariate T-PC sequence $\zeta(n)$.

2 The classical projection method of linear extrapolation

Let $\zeta(j)$ and $\theta(j)$ be uncorrelated T-PC stochastic sequences. Consider the problem of optimal linear estimation of the functional

$$A\zeta = \sum_{j=1}^{\infty} a(j)\zeta(j),$$

that depends on the unknown values of T-PC stochastic sequence $\zeta(j)$, based on observations of the sequence $\zeta(j) + \theta(j)$ at points $j \in \{\dots, -n, \dots, -1, 0\} \setminus S$, $S = \bigcup_{l=1}^{s}\{-M_l \cdot T + 1, \dots, -M_{l-1} \cdot T - N_l \cdot T\}$, $M_l = \sum_{k=0}^{l}(N_k + K_k)$, $N_0 = K_0 = 0$,.

Let assume that the coefficients $a(j), j \geq 1$ which determine the functional $A\zeta$ satisfy condition

$$\sum_{j=1}^{\infty} |a(j)| < \infty \tag{4}$$

and are of the form

$$a(j) = a\left(\left(j - \left[\frac{j}{T}\right]T\right) + \left[\frac{j}{T}\right]T\right) = \\ = a(\nu + \tilde{j}T) = a(\tilde{j})e^{2\pi i\tilde{j}\nu/T}, \tag{5}$$

$$\nu = 1,\dots,T, \ \tilde{j} \geq 0,$$

where $\nu = T$ and $\tilde{j} = \lambda - 1$, if $j = T\cdot\lambda$, $\lambda \in \mathbb{Z}$, or $a(j) = a(T\cdot\lambda) = a(T+(\lambda-1)T) = a(\lambda-1)e^{2\pi i(\lambda-1)T/T}$.

Under the condition (4) the functional $A\zeta$ has the finite second moment.

Using Proposition 1.2, the linear functional $A\zeta$ can be written as follows

$$A\zeta = \sum_{j=1}^{\infty} a(j)\zeta(j) = \sum_{\tilde{j}=0}^{\infty} a(\tilde{j}) \sum_{\nu=1}^{T} e^{2\pi i\tilde{j}\nu/T}\zeta(\nu+\tilde{j}T) =$$

$$=\sum_{\widetilde{j}=0}^{\infty}\sum_{\nu=1}^{T}a(\widetilde{j})e^{2\pi i\widetilde{j}\nu/T}\zeta_\nu(\widetilde{j})=\sum_{\widetilde{j}=0}^{\infty}\vec{a}^\top(\widetilde{j})\vec{\zeta}(\widetilde{j})=A\vec{\zeta},$$

where

$$\vec{a}^\top(\widetilde{j})=\Big(a_1(\widetilde{j}),\dots,a_T(\widetilde{j})\Big),$$
$$a_\nu(\widetilde{j})=a(\widetilde{j})e^{2\pi i\widetilde{j}\nu/T},\ \nu=1,\dots,T, \quad (6)$$

$\vec{\zeta}(\widetilde{j})=\Big\{\zeta_\nu(\widetilde{j})\Big\}_{\nu=1}^{T}$ is T-variate stationary sequence, obtained by the T-blocking (3) of univariate T-PC sequence $\zeta(j)$, $j\geq 1$.

Let $\vec{\zeta}(j)$ and $\vec{\theta}(j)$ be uncorrelated T-variate stationary stochastic sequences with the spectral density matrices $f^{\vec{\zeta}}(\lambda)=\Big\{f^{\vec{\zeta}}_{\nu\mu}(\lambda)\Big\}_{\nu,\mu=1}^{T}$ and $f^{\vec{\theta}}(\lambda)=\Big\{f^{\vec{\theta}}_{\nu\mu}(\lambda)\Big\}_{\nu,\mu=1}^{T}$, respectively. Consider the problem of optimal linear estimation of the functional

$$A\vec{\zeta}=\sum_{\widetilde{j}=0}^{\infty}\vec{a}^\top(\widetilde{j})\vec{\zeta}(\widetilde{j}),$$

that depends on the unknown values of sequence $\vec{\zeta}(\widetilde{j})$, based on observations of the sequence $\vec{\zeta}(\widetilde{j})+\vec{\theta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$, $\widetilde{S}=\bigcup_{l=1}^{s}\{-M_l,\dots,-M_{l-1}-N_l-1\}$, $M_l=\sum_{k=0}^{l}(N_k+K_k)$, $N_0=K_0=0$,.

Let the spectral densities $f^{\vec{\zeta}}(\lambda)$ and $f^{\vec{\theta}}(\lambda)$ satisfy the minimality condition

$$\int_{-\pi}^{\pi}Tr\Big[(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}\Big]d\lambda<+\infty. \quad (7)$$

Condition (7) is necessary and sufficient in order that the error-free extrapolation of unknown values of the sequence $\vec{\zeta}(j)+\vec{\theta}(j)$ is impossible [34].

Denote by $L_2(f)$ the Hilbert space of vector valued functions $\vec{b}(\lambda)=\{b_\nu(\lambda)\}_{\nu=1}^{T}$ that are integrable with respect to a measure with the density $f(\lambda)=\{f_{\nu\mu}(\lambda)\}_{\nu,\mu=1}^{T}$:

$$\int_{-\pi}^{\pi}\vec{b}^\top(\lambda)f(\lambda)\overline{\vec{b}(\lambda)}d\lambda=$$
$$=\int_{-\pi}^{\pi}\sum_{\nu,\mu=1}^{T}b_\nu(\lambda)f_{\nu\mu}(\lambda)\overline{b_\mu(\lambda)}d\lambda<+\infty.$$

Denote by $L_2^s(f)$ the subspace in $L_2(f)$ generated by functions

$$e^{i\widetilde{j}\lambda}\delta_\nu,\delta_\nu=\{\delta_{\nu\mu}\}_{\mu=1}^{T},\ \nu=1,\dots,T,$$
$$\widetilde{j}\in\{\dots,-n,\dots,-1\}\setminus\widetilde{S},$$

where $\delta_{\nu\nu}$ is the Kronecker delta: $\delta_{\nu\nu}=1,\delta_{\nu\mu}=0$ for $\nu\neq\mu$.

Every linear estimate $\widehat{A\vec{\zeta}}$ of the functional $A\vec{\zeta}$ from observations of the sequence $\vec{\zeta}(\widetilde{j})+\vec{\theta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$ has the form

$$\widehat{A\vec{\zeta}}=\int_{-\pi}^{\pi}\vec{h}^\top(e^{i\lambda})(Z^{\vec{\zeta}}(d\lambda)+Z^{\vec{\theta}}(d\lambda))=$$
$$=\int_{-\pi}^{\pi}\sum_{\nu=1}^{T}h_\nu(e^{i\lambda})(Z_\nu^{\vec{\zeta}}(d\lambda)+Z_\nu^{\vec{\theta}}(d\lambda)), \quad (8)$$

where $Z^{\vec{\zeta}}(\Delta)=\Big\{Z_\nu^{\vec{\xi}}(\Delta)\Big\}_{\nu=1}^{T}$ and $Z^{\vec{\theta}}(\Delta)=\Big\{Z_\nu^{\vec{\eta}}(\Delta)\Big\}_{\nu=1}^{T}$ are orthogonal random measures of the sequences $\vec{\zeta}(\widetilde{j})$ and $\vec{\theta}(\widetilde{j})$, and $\vec{h}(e^{i\lambda})=\{h_\nu(e^{i\lambda})\}_{\nu=1}^{T}$ is the spectral characteristic of the estimate $\widehat{A\vec{\zeta}}$. The function $\vec{h}(e^{i\lambda})\in L_2^s(f^{\vec{\zeta}}+f^{\vec{\theta}})$.

The mean square error $\Delta(\vec{h};f^{\vec{\zeta}},f^{\vec{\theta}})$ of the estimate $\widehat{A\vec{\zeta}}$ is calculated by the formula

$$\Delta(\vec{h};f^{\vec{\zeta}},f^{\vec{\theta}})=E|A\vec{\zeta}-\widehat{A\vec{\zeta}}|^2=$$
$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\Big[A(e^{i\lambda})-\vec{h}(e^{i\lambda})\Big]^\top f^{\vec{\zeta}}(\lambda)\overline{\Big[A(e^{i\lambda})-\vec{h}(e^{i\lambda})\Big]}d\lambda+$$
$$+\frac{1}{2\pi}\int_{-\pi}^{\pi}\vec{h}^\top(e^{i\lambda})f^{\vec{\theta}}(\lambda)\overline{\vec{h}(e^{i\lambda})}d\lambda, \quad (9)$$
$$A(e^{i\lambda})=\sum_{\widetilde{j}=0}^{\infty}\vec{a}(\widetilde{j})e^{i\widetilde{j}\lambda}.$$

The spectral characteristic $\vec{h}(f^{\vec{\zeta}},f^{\vec{\theta}})$ of the optimal linear estimate of $A\vec{\zeta}$ minimizes the mean square error

$$\Delta(f^{\vec{\zeta}},f^{\vec{\theta}})=\Delta(\vec{h}(f^{\vec{\zeta}},f^{\vec{\theta}});f^{\vec{\zeta}},f^{\vec{\theta}})=$$
$$=\min_{\vec{h}\in L_2^s(f^{\vec{\zeta}}+f^{\vec{\theta}})}\Delta(\vec{h};f^{\vec{\zeta}},f^{\vec{\theta}})=\min_{\widehat{A\vec{\zeta}}}E|A\vec{\zeta}-\widehat{A\vec{\zeta}}|^2. \quad (10)$$

With the help of the Hilbert space projection method proposed by A. N. Kolmogorov [15] we can find a solution of the optimization problem (10). The optimal linear estimate $\widehat{A\vec{\zeta}}$ is a projection of the functional $A\vec{\zeta}$ on the subspace $H^s[\vec{\zeta}+\vec{\theta}]=H^s[\zeta_\nu(\widetilde{j})+\theta_\nu(\widetilde{j}),\ \widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S},\nu=1,\dots,T]$ of the Hilbert space $H=\{\zeta:\mathsf{E}\zeta=0,\mathsf{E}|\zeta|^2<\infty\}$, generated by values $\zeta_\nu(\widetilde{j})+$

$\theta_\nu(\widetilde{j})$, $\widetilde{j} \in \{..., -n, ..., -1\}\backslash \widetilde{S}, \nu = 1, \dots, T$. The projection is characterized by following conditions

1) $\widehat{A\vec{\zeta}} \in H^s[\vec{\zeta} + \vec{\theta}]$,

2) $A\vec{\zeta} - \widehat{A\vec{\zeta}} \perp H^s[\vec{\zeta} + \vec{\theta}]$.

The condition 2) gives us the possibility to derive the formula for spectral characteristic of the estimate

$$\vec{h}^\top(f^{\vec{\zeta}}, f^{\vec{\theta}}) = \left(A^\top(e^{i\lambda})f^{\vec{\zeta}}(\lambda) - C^\top(e^{i\lambda})\right)\left[f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda)\right]^{-1} = = A^\top(e^{i\lambda}) - \left(A^\top(e^{i\lambda})f^{\vec{\theta}}(\lambda) + C^\top(e^{i\lambda})\right) \times \times \left[f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda)\right]^{-1}, \quad (11)$$

where

$$C(e^{i\lambda}) = \sum_{n\in\Gamma} \vec{c}(n)e^{in\lambda},$$

where $\Gamma = \widetilde{S} \cup \{0, 1, 2, ...\}$ and $\vec{c}(n), n \in \Gamma$, are unknown vectors of coefficients.

Condition 1) is satisfied if the system of equalities

$$\int_{-\pi}^{\pi} \vec{h}(f^{\vec{\zeta}}, f^{\vec{\theta}})e^{-im\lambda}d\lambda = 0, m \in \Gamma \quad (12)$$

holds true.

The last equalities (12) provide the following relations for all $m \in \Gamma$:

$$\sum_{\widetilde{j}=0}^{\infty} \vec{a}^\top(\widetilde{j})\frac{1}{2\pi}\int_{-\pi}^{\pi} f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(\widetilde{j}-m)}d\lambda =$$

$$\sum_{n\in\Gamma} \vec{c}^\top(n)\frac{1}{2\pi}\int_{-\pi}^{\pi} (f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(n-m)}d\lambda. \quad (13)$$

Denote the Fourier coefficients of the matrix functions $(f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda))^{-1}$ and $f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda))^{-1}$ as

$$B(m-n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} (f^{\vec{\zeta}}(\lambda) + f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(n-m)}d\lambda,$$

$$R(m-\widetilde{j}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(\widetilde{j}-m)}d\lambda,$$

$$n, m \in \Gamma, \; \widetilde{j} = 0, 1, 2....$$

Denote by $\vec{\mathbf{a}}^\top = (\vec{0}^\top, ..., \vec{0}^\top, \vec{a}^\top(0), \vec{a}^\top(1), ...)$ a vector that has first $\sum_{i=1}^{s} K_i = K_1 + ... + K_s$ zero vectors $\vec{0}^\top = (\underbrace{0, ..., 0}_{T})$, next vectors $\vec{a}(0), \vec{a}(1), ...$ are constructed from coefficients of the functional $A\zeta$ by formula (6).

Rewrite the relation (13) in the matrix form

$$\mathbf{R}\vec{\mathbf{a}} = \mathbf{B}\vec{\mathbf{c}},$$

where $\vec{\mathbf{c}}^\top = (\vec{c}^\top(k))_{k\in\Gamma}$ is a vector of the unknown coefficients. The linear operator $\mathbf{B}$ is defined by the matrix

$$\mathbf{B} = \begin{pmatrix} B_{s,s} & B_{s,s-1} & \dots & B_{s,1} & B_{s,n} \\ B_{s-1,s} & B_{s-1,s-1} & \dots & B_{s-1,1} & B_{s-1,n} \\ \dots & \dots & \dots & \dots & \\ B_{1,s} & B_{1,s-1} & \dots & B_{1,1} & B_{1,n} \\ B_{n,s} & B_{n,s-1} & \dots & B_{n,1} & B_{n,n} \end{pmatrix},$$

constructed with the help of the block-matrices

$$B_{lm} = \{B_{lm}(k,j)\}_{k=-M_{l-1}-N_l-1}^{-M_l}\,{}_{j=-M_{m-1}-N_m-1}^{-M_m},$$
$$B_{lm}(k,j) = B(k-j), \; l, m = 1, ..., s,$$

$$B_{ln}(k,j) = \{B_{ln}(k,j)\}_{k=-M_{l-1}-N_l-1}^{-M_l}\,{}_{j=0}^{\infty},$$
$$B_{ln}(k,j) = B(k-j), \; l = 1, ..., s,$$

$$B_{nl}(k,j) = \{B_{nl}(k,j)\}_{k=0}^{\infty}\,{}_{j=-M_{m-1}-N_m-1}^{-M_m},$$
$$B_{nl}(k,j) = B(k-j), \; m = 1, ..., s,$$

$$B_{nn}(k,j) = \{B_{nn}(k,j)\}_{k=0}^{\infty}\,{}_{j=0}^{\infty},$$
$$B_{nn}(k,j) = B(k-j).$$

The linear operator $\mathbf{R}$ is defined by the corresponding matrix, which is constructed in the same manner as matrix $\mathbf{B}$.

The unknown coefficients $\vec{c}(k), k \in \Gamma$ are determined from the equation

$$\vec{\mathbf{c}} = \mathbf{B}^{-1}\mathbf{R}\vec{\mathbf{a}}, \quad (14)$$

where the k-th component of the vector $\vec{\mathbf{c}}$ is the k-th component of vector $\mathbf{B}^{-1}\mathbf{R}\vec{\mathbf{a}}$:

$$\vec{c}(k) = (\mathbf{B}^{-1}\mathbf{R}\vec{\mathbf{a}})(k), \; k \in \Gamma.$$

We will suppose that the operator $\mathbf{B}$ has the inverse matrix.

The mean-square error of the optimal estimate $\widehat{A\vec{\zeta}}$ is calculated by the formula (9) and is of the form

$$\Delta(\vec{h}, f^{\vec{\zeta}}, f^{\vec{\theta}}) = E|A\vec{\zeta} - \widehat{A\vec{\zeta}}|^2 =$$

$$= \sum_{\widetilde{j}=0}^{\infty}\sum_{\widetilde{k}=0}^{\infty} \vec{a}^{\top}(\widetilde{j})\frac{1}{2\pi}\int_{-\pi}^{\pi} f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}\times$$
$$\times f^{\vec{\theta}}(\lambda)e^{-i\lambda(\widetilde{j}-\widetilde{k})}d\lambda\cdot\overline{\vec{a}(\widetilde{k})}+$$

$$+\sum_{n\in\Gamma}\sum_{k\in\Gamma}\vec{c}^{\top}(\widetilde{j})\frac{1}{2\pi}\int_{-\pi}^{\pi}(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}\times$$
$$\times e^{-i\lambda(n-k)}d\lambda\cdot\overline{\vec{c}(k)} =$$

$$= \langle \mathbf{D}\vec{\mathbf{a}},\vec{\mathbf{a}}\rangle + \langle \mathbf{B}\vec{\mathbf{c}},\vec{\mathbf{c}}\rangle, \qquad (15)$$

where $\langle a, b\rangle$ denotes the scalar product, $\mathbf{D}$ is defined by the corresponding matrix, which is constructed in the same manner as matrix $\mathbf{B}$, with elements

$$D(\widetilde{k}-\widetilde{j}) =$$
$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}f^{\vec{\theta}}(\lambda)\right]^{\top}e^{i(\widetilde{j}-\widetilde{k})\lambda}d\lambda,$$
$$\widetilde{k}\ge 0,\ \widetilde{j}\ge 0.$$

See [27] for more details.

Theorem 1. *Let $\zeta(j)$ and $\theta(j)$ be uncorrelated T-PC stochastic sequences with the spectral density matrices $f^{\vec{\zeta}}(\lambda)$ and $f^{\vec{\theta}}(\lambda)$ of T-variate stationary sequences $\vec{\zeta}(\widetilde{j})$ and $\vec{\theta}(\widetilde{j})$, respectively. Assume that $f^{\vec{\zeta}}(\lambda)$ and $f^{\vec{\theta}}(\lambda)$ satisfy the minimality condition (7). Assume that condition (4) is satisfied and operator $\boldsymbol{B}$ is invertible. The spectral characteristic $\vec{h}(f^{\vec{\zeta}}, f^{\vec{\theta}})$ and the mean square error $\Delta(f^{\vec{\zeta}}, f^{\vec{\theta}})$ of the optimal linear estimate of the functional $A\vec{\zeta}$ based on observations of the sequence $\vec{\zeta}(\widetilde{j})+\vec{\theta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$, are calculated by formulas (11) and (15).*

Consider the mean-square estimation problem of $A\vec{\zeta}$ based on observations of the sequence $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$. In this case the spectral density $f^{\vec{\theta}}(\lambda)=0$. The spectral characteristic $\vec{h}(f^{\vec{\zeta}})$ of the estimate $\widehat{A\vec{\zeta}}$ is of the form

$$\vec{h}^{\top}(f^{\vec{\zeta}}) = A^{\top}(e^{i\lambda}) - C^{\top}(e^{i\lambda})\left[f^{\vec{\zeta}}(\lambda)\right]^{-1}, \qquad (16)$$

where unknown coefficients $\vec{c}(k)$, $k\in\Gamma$ are determined from the relation

$$\mathbf{B}\vec{\mathbf{c}} = \vec{\mathbf{a}} \qquad (17)$$

or

$$\vec{\mathbf{c}} = \mathbf{B}^{-1}\vec{\mathbf{a}},$$

where the linear operator $\mathbf{B}$ is defined by the matrix

$$\mathbf{B} = \begin{pmatrix} B_{s,s} & B_{s,s-1} & \dots & B_{s,1} & B_{s,n}\\ B_{s-1,s} & B_{s-1,s-1} & \dots & B_{s-1,1} & B_{s-1,n}\\ \dots & \dots & \dots & \dots & \\ B_{1,s} & B_{1,s-1} & \dots & B_{1,1} & B_{1,n}\\ B_{n,s} & B_{n,s-1} & \dots & B_{n,1} & B_{n,n}\end{pmatrix},$$

constructed with the help of the block-matrices

$$B_{lm} = \{B_{lm}(k,j)\}_{k=-M_{l-1}-N_{l}-1}^{-M_l}\,{}_{j=-M_{m-1}-N_m-1}^{-M_m},$$
$$B_{lm}(k,j) = B(k-j),\ l,m=1,...,s,$$

$$B_{ln}(k,j) = \{B_{ln}(k,j)\}_{k=-M_{l-1}-N_{l}-1}^{-M_l}\,{}_{j=0}^{\infty},$$
$$B_{ln}(k,j) = B(k-j),\ l=1,...,s,$$

$$B_{nl}(k,j) = \{B_{nl}(k,j)\}_{k=0}^{\infty}\,{}_{j=-M_{m-1}-N_m-1}^{-M_m},$$
$$B_{nl}(k,j) = B(k-j),\ m=1,...,s,$$

$$B_{nn}(k,j) = \{B_{nn}(k,j)\}_{k=0}^{\infty}\,{}_{j=0}^{\infty},$$
$$B_{nn}(k,j) = B(k-j),$$

with elements

$$B(k-j) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[(f^{\vec{\zeta}}(\lambda))^{-1}\right]^{\top}e^{i(j-k)\lambda}d\lambda,$$
$$k\in\Gamma,\ j\in\Gamma.$$

The mean square error $\Delta(f^{\vec{\zeta}})$ is defined by the formula

$$\Delta(f^{\vec{\zeta}}) = \langle\vec{\mathbf{c}},\vec{\mathbf{a}}\rangle. \qquad (18)$$

Thus, in the case without noise we have the following result.

Corollary 1. *Let $\zeta(j)$ be a T-PC stochastic sequence with the spectral density matrix $f^{\vec{\zeta}}(\lambda)$ of T-variate stationary sequence $\vec{\zeta}(j)$. Assume that $f^{\vec{\zeta}}(\lambda)$ satisfies the minimality condition*

$$\int_{-\pi}^{\pi}Tr\left[(f^{\vec{\zeta}}(\lambda))^{-1}\right]d\lambda < +\infty. \qquad (19)$$

Assume that condition (4) is satisfied and operator $\boldsymbol{B}$ is invertible. Then the optimal linear estimate *of $A\vec{\zeta}$ based on observations of $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$, is given by the formula*

$$\widehat{A\vec{\zeta}}=\int_{-\pi}^{\pi}\vec{h}^{\top}(f^{\vec{\zeta}})Z^{\vec{\zeta}}(d\lambda)=\int_{-\pi}^{\pi}\sum_{\nu=1}^{T}h_{\nu}(f^{\vec{\zeta}})Z^{\vec{\zeta}}_{\nu}(d\lambda).$$

The spectral characteristic $\vec{h}(f^{\vec{\zeta}})$ and the mean square error $\Delta(f^{\vec{\zeta}})$ of $\widehat{A\vec{\zeta}}$ are calculated by formulas (16) and (18).

Let us consider the mean-square estimation problem of functional

$$A_N\zeta=\sum_{j=1}^{N\cdot T}a(j)\zeta(j)$$

that depends on unknown values of T-PC stochastic sequence $\zeta(j)$, based on observations of the sequence $\zeta(j)+\theta(j)$ at points $j\in\{...,-n,...,-1,0\}\setminus S$. $\theta(j)$ is uncorrelated with $\zeta(j)$ T-PC stochastic sequence.

Using Proposition 1.2, the linear functional $A_N\zeta$ can be written as follows

$$\begin{aligned}A_N\zeta&=\sum_{j=1}^{N\cdot T}a(j)\zeta(j)=\\&=\sum_{\widetilde{j}=0}^{N-1}a(\widetilde{j})\sum_{\nu=1}^{T}e^{2\pi i\widetilde{j}\nu/T}\zeta(\nu+\widetilde{j}T)=\\&=\sum_{\widetilde{j}=0}^{N-1}\sum_{\nu=1}^{T}a(\widetilde{j})e^{2\pi i\widetilde{j}\nu/T}\zeta_{\nu}(j)=\\&=\sum_{\widetilde{j}=0}^{N-1}\vec{a}^{\top}(\widetilde{j})\vec{\zeta}(\widetilde{j})=A_N\vec{\zeta},\end{aligned}$$

where $\vec{a}^{\top}(\widetilde{j})$ is defined by relation (6), $\vec{\zeta}(\widetilde{j})=\left\{\zeta_{\nu}(\widetilde{j})\right\}_{\nu=1}^{T}$ is T-variate stationary sequence, obtained by the T-blocking (3) of univariate T-PC sequence $\zeta(j)$, $j\geq 1$.

Let $\vec{\zeta}(j)$ and $\vec{\theta}(j)$ be uncorrelated T-variate stationary stochastic sequences with the spectral density matrices $f^{\vec{\zeta}}(\lambda)=\left\{f^{\vec{\zeta}}_{\nu\mu}(\lambda)\right\}_{\nu,\mu=1}^{T}$ and $f^{\vec{\theta}}(\lambda)=\left\{f^{\vec{\theta}}_{\nu\mu}(\lambda)\right\}_{\nu,\mu=1}^{T}$, respectively. Consider the problem of optimal linear estimation of the functional

$$A_N\vec{\zeta}=\sum_{\widetilde{j}=0}^{N-1}\vec{a}^{\top}(\widetilde{j})\vec{\zeta}(\widetilde{j}),\tag{20}$$

that depends on the unknown values of sequence $\vec{\zeta}(\widetilde{j})$, based on observations of the sequence $\vec{\zeta}(\widetilde{j})+\vec{\theta}(\widetilde{j})$ at points $\widetilde{j}\in\{...,-n,...,-1\}\setminus\widetilde{S}$, $\widetilde{S}=\bigcup_{l=1}^{s}\{-M_l,\ldots,-M_{l-1}-N_l-1\}$, $M_l=\sum_{k=0}^{l}(N_k+K_k)$, $N_0=K_0=0$.

The estimate

$$\widehat{A_N\vec{\zeta}}=\int_{-\pi}^{\pi}\vec{h}_N^{\top}(e^{i\lambda})Z^{\vec{\zeta}}(d\lambda)\tag{21}$$

of the functional $A_N\vec{\zeta}$ is defined by the spectral characteristic $\vec{h}_N(e^{i\lambda})\in L_2^s(f^{\vec{\zeta}}+f^{\vec{\theta}})$.

Denote by $\mathbf{a}_N^{\vec{}\,\top}=(\vec{0}^{\top},...,\vec{0}^{\top},\vec{a}^{\top}(0),...,\vec{a}^{\top}(N-1),\vec{0}^{\top},\vec{0}^{\top},...)$ a vector that has first $\sum_{i=1}^{s}K_i$ zero vectors $\vec{0}^{\top}$, next N vectors $\vec{a}(0),...,\vec{a}(N-1)$ are constructed from coefficients of the functional $A_N\zeta$ by formula (6).

With the help of Hilbert space projection method we can derive the following relations for all $m\in\Gamma$:

$$\sum_{\widetilde{j}=0}^{N-1}\vec{a}^{\top}(\widetilde{j})\frac{1}{2\pi}\int_{-\pi}^{\pi}f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(\widetilde{j}-m)}d\lambda=$$
$$\sum_{n\in\Gamma}\vec{c}^{\top}(n)\frac{1}{2\pi}\int_{-\pi}^{\pi}(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1}e^{i\lambda(n-m)}d\lambda.\tag{22}$$

Denote by $\mathbf{R}_N$ the linear operator which is defined as follows: $\mathbf{R}_N(k,j)=\mathbf{R}(k,j)$, $j\leq N-1$, $\mathbf{R}_N(k,j)=0$, $j>N-1$. Then we can rewrite the relations (22) in the matrix form

$$\mathbf{R}_N\mathbf{a}_N^{\vec{}}=\mathbf{B}\vec{\mathbf{c}}.$$

The unknown vectors $\vec{c}(k)$, $k\in\Gamma$, are determined from the equation

$$\vec{\mathbf{c}}=\mathbf{B}^{-1}\mathbf{R}_N\mathbf{a}_N^{\vec{}}.$$

The spectral characteristic of the optimal estimate $\widehat{A_N\vec{\zeta}}$ is calculated by formula

$$\begin{aligned}&\vec{h}_N^{\top}(e^{i\lambda})=\\&=\left(A_N^{\top}(e^{i\lambda})f^{\vec{\zeta}}(\lambda)-C^{\top}(e^{i\lambda})\right)\left[f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda)\right]^{-1},\end{aligned}\tag{23}$$

where

$$A_N(e^{i\lambda})=\sum_{\widetilde{j}=0}^{N-1}\vec{a}(\widetilde{j})e^{i\widetilde{j}\lambda}.$$

The mean-square error of the optimal estimate $\widehat{A_N\vec{\zeta}}$ is calculated by formula

$$\Delta(\vec{h}_N,f^{\vec{\zeta}},f^{\vec{\theta}})=E|A_N\vec{\zeta}-\widehat{A_N\vec{\zeta}}|^2=$$

$$= \sum_{\widetilde{j}=0}^{N-1} \sum_{\widetilde{k}=0}^{N-1} \vec{a}^{\top}(\widetilde{j}) \frac{1}{2\pi} \int_{-\pi}^{\pi} f^{\vec{\zeta}}(\lambda)(f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1} \times$$
$$\times f^{\vec{\theta}}(\lambda) e^{-i\lambda(\widetilde{j}-\widetilde{k})} d\lambda \cdot \overline{\vec{a}(\widetilde{k})} +$$

$$+ \sum_{n\in\Gamma} \sum_{k\in\Gamma} \vec{c}^{\top}(\widetilde{j}) \frac{1}{2\pi} \int_{-\pi}^{\pi} (f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda))^{-1} \times$$
$$\times e^{-i\lambda(n-k)} d\lambda \cdot \overline{\vec{c}(k)} =$$
$$= \langle \mathbf{D}_N \vec{\mathbf{a}}_N, \vec{\mathbf{a}}_N \rangle + \langle \mathbf{B}\vec{\mathbf{c}}, \vec{\mathbf{c}} \rangle, \quad (24)$$

where linear operator $\mathbf{D}$ is defined as follows: $\mathbf{D}_N(k,j) = \mathbf{D}(k,j)$, $k,j \le N-1$, $\mathbf{D}_N(k,j) = 0$ if $k > N-1$ or $j > N-1$.

Theorem 2. *Let $\zeta(j)$ and $\theta(j)$ be uncorrelated T-PC stochastic sequences with the spectral density matrices $f^{\vec{\zeta}}(\lambda)$ and $f^{\vec{\theta}}(\lambda)$ of T-variate stationary sequences $\vec{\zeta}(\widetilde{j})$ and $\vec{\theta}(\widetilde{j})$, respectively. Assume that $f^{\vec{\zeta}}(\lambda)$ and $f^{\vec{\theta}}(\lambda)$ satisfy the minimality condition (7). Assume that operator $\boldsymbol{B}$ is invertible. The spectral characteristic $\vec{h}_N(e^{i\lambda})$ and the mean square error $\Delta(\vec{h}_N; f^{\vec{\zeta}}, f^{\vec{\theta}})$ of the optimal linear estimate of the functional $A_N\vec{\zeta}$ based on observations of the sequence $\vec{\zeta}(\widetilde{j}) + \vec{\theta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$, are calculated by formulas (23) and (24).*

In the case of observation without noise we have the following result.

Corollary 2. *Let $\zeta(j)$ be a T-PC stochastic sequence with the spectral density matrix $f^{\vec{\zeta}}(\lambda)$ of T-variate stationary sequence $\vec{\zeta}(j)$. Assume that $f^{\vec{\zeta}}(\lambda)$ satisfies the minimality condition (19). Assume that operator $\boldsymbol{B}$ is invertible. The spectral characteristic $\vec{h}_N(e^{i\lambda})$ and the mean square error $\Delta(f^{\vec{\zeta}})$ of $\widehat{A_N\vec{\zeta}}$ are calculated by formulas*

$$\vec{h}_N^{\top}(e^{i\lambda}) = A_N^{\top}(e^{i\lambda}) - C^{\top}(e^{i\lambda}) \left[f^{\vec{\zeta}}(\lambda)\right]^{-1}, \quad (25)$$

$$\Delta(f^{\vec{\zeta}}) = \langle \vec{\mathbf{c}}, \vec{\mathbf{a}}_N \rangle. \quad (26)$$

The linear operator $\boldsymbol{B}$ is defined in Corollary 1, vector $\vec{\mathbf{c}}$ is defined by the equation $\vec{\mathbf{c}} = \boldsymbol{B}^{-1}\vec{\mathbf{a}}_N$.

Example 1. Let $\zeta(n)$, $n \in \mathbb{Z}$, be a 2-PC stochastic sequence such that $\zeta(2n) = \eta(n)$ is a univariate white noise with the spectral density $f(\lambda) = 1$ and $\zeta(2n+1) = \gamma(n)$ is an uncorrelated with $\eta(n)$ univariate stationary Ornstein-Uhlenbeck sequence with the spectral density $g(\lambda) = \frac{1}{|1-e^{i\lambda}|^2}$. Consider the problem of estimation of the functional

$$A_1\zeta = \zeta(1) + \zeta(2)$$

based on observations of $\zeta(n), n \in \{..., -1, 0\} \setminus \{-3, -2\} = \{..., -5, -4, -1, 0\}$. Here $S = \{-3, -2\}$, $N_1 = K_1 = 1$, $M_1 = 2$.

Rewrite functional $A_1\zeta$ in the form (20)

$$A_1\zeta = \zeta(1) + \zeta(2) =$$
$$= (1,1) \cdot \begin{pmatrix} \zeta_1(0) \\ \zeta_2(0) \end{pmatrix} = \vec{a}^{\top}(0)\vec{\zeta}(0) = A_1\vec{\zeta},$$

where $\vec{a}(0) = (a(1 + 0 \cdot 2)e^{2\pi i 1\cdot 0/2}, a(2 + 0 \cdot 2)e^{2\pi i 2\cdot 0/2})^{\top} = (1,1)^{\top}$, $\vec{\zeta}(0) = (\zeta(1+0\cdot 2), \zeta(2+0\cdot 2))^{\top} = (\zeta_1(0), \zeta_2(0))^{\top}$, $\widetilde{S} = \{-2\}$. The spectral density matrix of 2-variate stationary sequence $\vec{\zeta}(n)$ is of the form

$$f^{\vec{\zeta}}(\lambda) = \begin{pmatrix} f(\lambda) & 0 \\ 0 & g(\lambda) \end{pmatrix}$$

The matrix $[f^{\vec{\zeta}}(\lambda)]^{-1}$ is of the form

$$[f^{\vec{\zeta}}(\lambda)]^{-1} =$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} e^{-i\lambda} + \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} e^{i\lambda} =$$
$$= B(0) + B(-1)e^{-i\lambda} + B(1)e^{i\lambda}$$

and satisfies the minimality condition (19). In the last equality matrices

$$B(0) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \, B(-1) = B(1) = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

are the Fourier coefficients of the function $[f^{\vec{\zeta}}(\lambda)]^{-1}$. In order to find the spectral characteristic $\vec{h}_1(e^{i\lambda})$ and the mean-square error $\Delta(f^{\vec{\zeta}})$ of the estimate $\widehat{A_1\zeta}$ let us use the Corollary 2. To find the unknown coefficients

$$\vec{c}(k) = (\mathbf{B}^{-1}\vec{\mathbf{a}}_N)(k),$$
$$k \in \Gamma = \widetilde{S} \cup \{0, 1, ...\} = \{-2, 0, 1, ...\}$$

we use the equation (17), where vectors $\vec{\mathbf{c}}^{\top} = (\vec{c}^{\top}(-2), \vec{c}^{\top}(0), \vec{c}^{\top}(1), ...)$, $\vec{\mathbf{a}}_1^{\top} = (\vec{0}^{\top}, \vec{a}^{\top}(0), \vec{0}^{\top}, ...)$. The operator $\mathbf{B}$ is defined by matrix

$$\mathbf{B} = \begin{pmatrix} B_{11} & B_{1n} \\ B_{n1} & B_{nn} \end{pmatrix},$$

with block-matrices

$$B_{11} = \{B_{11}(k,j)\}_{k=-2\, j=-2} = B(0),$$

$$B_{1n} = \{B_{1n}(k,j)\}_{k=-2\, j=0}^{\infty} = \\ = (B(-2)\, B(-3)\, B(-4)\, ...) = (O_2\, O_2\, O_2\, ...),$$

$$B_{n1} = \{B_{n1}(k,j)\}_{k=0\, j=-2} = \\ = (B(2)\, B(3)\, B(4)\, ...)^\top = (O_2\, O_2\, O_2\, ...)^\top,$$

$$B_{nn} = \{B_{nn}(k,j)\}_{k=0}^{\infty}{}_{j=0}^{\infty} = \\ = \begin{pmatrix} B(0) & B(-1) & O_2 & ... \\ B(1) & B(0) & B(-1) & ... \\ O_2 & B(1) & B(0) & ... \\ . & . & . & ... \end{pmatrix},$$

where $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

The inverse matrix $\mathbf{B}^{-1}$ can be represented in the form

$$\mathbf{B}^{-1} = \begin{pmatrix} B_{11}^{-1} & 0 \\ 0 & B_{nn}^{-1} \end{pmatrix},$$

where $B_{11}^{-1} = (B(0))^{-1}$, B_{nn}^{-1} is the inverse matrix to B_{nn}. To find B_{nn}^{-1} we use that matrix $[f^{\vec{\zeta}}(\lambda)]^{-1}$ admits factorization

$$[f^{\vec{\zeta}}(\lambda)]^{-1} = \sum_{j=-\infty}^{\infty} B(j) e^{ij\lambda} = \\ = \left(\sum_{k=0}^{\infty} \psi(k) e^{-ik\lambda}\right) \left(\sum_{k=0}^{\infty} \psi(k) e^{-ik\lambda}\right)^* = \\ = \left(\left(\sum_{k=0}^{\infty} \varphi(k) e^{-ik\lambda}\right)^* \left(\sum_{k=0}^{\infty} \varphi(k) e^{-ik\lambda}\right)\right)^{-1}.$$

where $\psi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\psi(1) = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$, $\psi(k) = O_2, k \geq 2$ and $\varphi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\varphi(k) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, k \geq 1$.

If we denote by Ψ and Φ linear operators determined by matrices with elements $\Psi(i,j) = \psi(j-i)$, $\Phi(i,j) = \varphi(j-i)$, for $0 \leq i \leq j$, $\Psi(i,j) = 0$, $\Phi(i,j) = 0$, for $0 \leq j < i$. Then elements of the matrix B_{nn} can be represented in the form $B_{nn}(i,j) = (\Psi\Psi^*)(i,j)$. It is not hard to verify that $\Psi\Phi = \Phi\Psi = I$. This makes possible to write elements of B_{nn}^{-1} in the form $B_{nn}^{-1}(i,j) = (\Phi^*\Phi)(i,j) = \sum_{l=0}^{\min(i,j)} (\varphi(i-l))^* \varphi(j-l)$.

Using equation $\vec{\mathbf{c}} = \mathbf{B}^{-1}\vec{\mathbf{a}}_N$ we can represent the unknown coefficients $\vec{c}(k), k \in \Gamma$ in the form

$$\vec{c}(-2) = \vec{0},$$
$$\vec{c}(0) = B_{nn}^{-1}(0,0)\vec{a}(0),$$
$$\vec{c}(1) = B_{nn}^{-1}(1,0)\vec{a}(0),$$
$$...$$
$$\vec{c}(i) = B_{nn}^{-1}(i,0)\vec{a}(0),\ i \geq 2.$$

The spectral characteristic $\vec{h}_1(e^{i\lambda})$ is determined by the formula (25)

$$\vec{h}_1^\top(e^{i\lambda}) = -\vec{c}^\top(0) B(-1) e^{-i\lambda} = \\ = -B_{nn}^{-1}(0,0)\vec{a}(0) B(-1) e^{-i\lambda}.$$

Since $B_{nn}^{-1} = \varphi^*(0)\varphi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the spectral characteristic is of the form

$$\vec{h}_1^\top(e^{i\lambda}) = -(0,-1)e^{-i\lambda}.$$

The optimal linear estimate $\widehat{A_1\vec{\zeta}}$ can be calculated by the formula (21)

$$\widehat{A_1\vec{\zeta}} = \zeta_2(-1) = \zeta(0).$$

The mean-square error of the estimate $\widehat{A_1\vec{\zeta}}$ determined by (26) equals

$$\Delta(f^{\vec{\zeta}}) = \langle \vec{\mathbf{c}}^\top, \vec{\mathbf{a}}_1 \rangle = 2.$$

3 Minimax (robust) method of linear extrapolation problem

Let $f(\lambda)$ and $g(\lambda)$ be the spectral density matrices of T-variate stationary sequences $\vec{\zeta}(j)$ and $\vec{\theta}(j)$, obtained by T-blocking (3) of T-PC sequences $\zeta(j)$ and $\theta(j)$, respectively.

The obtained formulas may be applied for finding the spectral characteristic and the mean square error of the optimal linear estimate of the functionals $A\vec{\zeta}$ and $A_N\vec{\zeta}$ only under the condition that the spectral density matrices $f(\lambda)$ and $g(\lambda)$ are exactly known. If the density matrices are not known exactly while a set $D = D_f \times D_g$ of possible spectral densities is given, the minimax (robust) approach to estimation of functionals from unknown values of stationary sequences is reasonable. In this case we find the estimate which minimizes the mean square error for all spectral densities from the given set simultaneously.

Definition 3. *For a given class of pairs of spectral densities $D = D_f \times D_g$ the spectral density matrices $f^0(\lambda) \in D_f$, $g^0(\lambda) \in D_g$ are called the least favorable in D for the optimal linear estimation of the functional $A\vec{\zeta}$ if*

$$\Delta(f^0, g^0) = \Delta(\vec{h}(f^0, g^0); f^0, g^0) = \\ = \max_{(f,g)\in D} \Delta(\vec{h}(f, g); f, g).$$

Definition 4. *For a given class of pairs of spectral densities $D = D_f \times D_g$ the spectral characteristic $\vec{h}^0(\lambda)$ of the optimal linear estimate of the functional $A\vec{\zeta}$ is called minimax (robust) if*

$$\vec{h}^0(\lambda) \in H_D = \bigcap_{(f,g)\in D} L_2^s(f + g),$$

$$\min_{\vec{h}\in H_D} \max_{(f,g)\in D} \Delta(\vec{h}; f, g) = \max_{(f,g)\in D} \Delta(\vec{h}^0; f, g).$$

Taking into consideration these definitions and the obtained relations we can verify that the following lemmas hold true.

Lemma 1. *The spectral density matrices $f^0(\lambda) \in D_f$, $g^0(\lambda) \in D_g$, that satisfy the minimality condition (7), are the least favorable in the class D for the optimal linear estimation of $A\vec{\zeta}$, if the Fourier coefficients of the matrix functions*

$$(f^0(\lambda) + g^0(\lambda))^{-1}, \quad f^0(\lambda)(f^0(\lambda) + g^0(\lambda))^{-1}, \\ f^0(\lambda)(f^0(\lambda) + g^0(\lambda))^{-1}g^0(\lambda)$$

define matrices $\boldsymbol{B}^0, \boldsymbol{R}^0, \boldsymbol{D}^0$, that determine a solution of the constrained optimization problem

$$\max_{(f,g)\in D}(\langle \boldsymbol{R}\vec{a}, \boldsymbol{B}^{-1}\boldsymbol{R}\vec{a}\rangle) + \langle \boldsymbol{D}\vec{a}, \vec{a}\rangle) = \\ = \langle \boldsymbol{R}^0\vec{a}, (\boldsymbol{B}^0)^{-1}\boldsymbol{R}^0\vec{a}\rangle) + \langle \boldsymbol{D}^0\vec{a}, \vec{a}\rangle.$$

The minimax spectral characteristic $\vec{h}^0 = \vec{h}(f^0, g^0)$ is given by (11), if $\vec{h}(f^0, g^0) \in H_D$.

In the case of observations of the sequence without noise the following corollary holds true.

Corollary 3. *The spectral density matrix $f^0(\lambda) \in D_f$, that satisfies the minimality condition (19), is the least favorable in the class D_f for the optimal linear estimation of $A\vec{\zeta}$ based on observations of $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$, if the Fourier coefficients of the matrix function $(f^0(\lambda))^{-1}$ define the matrix $\boldsymbol{B}^0$, that determine a solution of the constrained optimization problem*

$$\max_{f\in D_f} \langle \boldsymbol{B}^{-1}\vec{a}, \vec{a}\rangle = \langle (\boldsymbol{B}^0)^{-1}\vec{a}, \vec{a}\rangle.$$

The minimax spectral characteristic $\vec{h}^0 = \vec{h}(f^0)$ is given by (16), if $\vec{h}(f^0) \in H_D$.

The least favorable spectral densities $f^0(\lambda) \in D_f$, $g^0(\lambda) \in D_g$ and the minimax spectral characteristic $\vec{h}^0 = \vec{h}(f^0, g^0)$ form a saddle point of the function $\Delta(\vec{h}; f, g)$ on the set $H_D \times D$. The saddle point inequalities

$$\Delta(\vec{h}^0; f, g) \leq \Delta(\vec{h}^0; f^0, g^0) \leq \Delta(\vec{h}; f^0, g^0), \\ \forall \vec{h} \in H_D, \forall f \in D_f, \forall g \in D_g$$

hold true when $\vec{h}^0 = \vec{h}(f^0, g^0)$, $\vec{h}(f^0, g^0) \in H_D$ and (f^0, g^0) is a solution of the constrained optimization problem

$$\Delta\left(\vec{h}(f^0, g^0); f, g\right) \to \sup, \\ (f, g) \in D_f \times D_g. \quad (27)$$

The linear functional $\Delta(\vec{h}(f^0, g^0); f, g)$ is calculated by the formula

$$\Delta(\vec{h}(f^0, g^0); f, g) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(A^\top(e^{i\lambda})g^0(\lambda) + (C^0(e^{i\lambda}))^\top\right)(f^0(\lambda) + g^0(\lambda))^{-1}f(\lambda)(f^0(\lambda) + g^0(\lambda))^{-1}\times \\ \times \left(A^\top(e^{i\lambda})g^0(\lambda) + (C^0(e^{i\lambda}))^\top\right)^* d\lambda + \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(A^\top(e^{i\lambda})f^0(\lambda) - (C^0(e^{i\lambda}))^\top\right)(f^0(\lambda) + g^0(\lambda))^{-1}\times \\ \times g(\lambda)(f^0(\lambda) + g^0(\lambda))^{-1}\left(A^\top(e^{i\lambda})f^0(\lambda) - (C^0(e^{i\lambda}))^\top\right)^* d\lambda,$$

where $C^0(e^{i\lambda}) = \sum_{n\in\Gamma} \vec{c}^{\,0}(n)e^{in\lambda}$, column vectors $\vec{c}^{\,0}(n) = ((\mathbf{B}^0)^{-1}\mathbf{R}^0\vec{\mathbf{a}})(n)$.

The constrained optimization problem (27) is equivalent to the unconstrained optimization problem [**?**]:

$$\Delta_D(f, g) = -\Delta(\vec{h}(f^0, g^0); f, g) + \\ + \delta((f, g)\,|D_f \times D_g) \to \inf, \quad (28)$$

where $\delta((f,g)|D_f \times D_g)$ is the indicator function of the set $D = D_f \times D_g$. A solution of the problem (28) is characterized by the condition $0 \in \partial\Delta_D(f^0,g^0)$, where $\partial\Delta_D(f^0,g^0)$ is the subdifferential of the convex functional $\Delta_D(f,g)$ at point (f^0,g^0) [33].

The form of the functional $\Delta(\vec{h}(f^0,g^0);f,g)$ admits finding the derivatives and differentials of the functional in the space $L_1 \times L_1$. Therefore the complexity of the optimization problem (28) is determined by the complexity of calculating of subdifferentials of the indicator functions $\delta((f,g)|D_f \times D_g)$ of the sets $D_f \times D_g$ [13].

Taking into consideration the introduced definitions and the derived relations we can verify that the following lemma holds true.

Lemma 2. *Let (f^0,g^0) be a solution to the optimization problem (28). The spectral densities $f^0(\lambda)$, $g^0(\lambda)$ are the least favorable in the class $D = D_f \times D_g$ and the spectral characteristic $\vec{h}^0 = \vec{h}(f^0,g^0)$ is the minimax of the optimal linear estimate of the functional $A\vec{\zeta}$ if $\vec{h}(f^0,g^0) \in H_D$.*

In the case of estimation of the functional based on observations without noise we have the following statement.

Lemma 3. *Let $f^0(\lambda)$ satisfies the condition (19) and be a solution of the constrained optimization problem*

$$\Delta(\vec{h}(f^0);f) \to sup, f(\lambda) \in D_f, \qquad (29)$$

$$\Delta(\vec{h}(f^0);f) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(C^0(e^{i\lambda})\right)^{\top} \times \\ \times (f^0(\lambda))^{-1} f(\lambda) (f^0(\lambda))^{-1} \overline{(C^0(e^{i\lambda}))} d\lambda,$$

where $C^0(e^{i\lambda}) = \sum_{n\in\Gamma} \vec{c}^{\,0}(n)e^{in\lambda}$, column vectors $\vec{c}^{\,0}(n) = ((\boldsymbol{B}^0)^{-1}\vec{\mathbf{a}})(n)$.

Then $f^0(\lambda)$ is the least favorable spectral density matrix for the optimal linear estimation of $A\vec{\zeta}$ based on observations of $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$. The minimax spectral characteristic $\vec{h}^0 = \vec{h}(f^0)$ is given by (16), if $\vec{h}(f^0) \in H_D$.

4 The least favorable spectral densities in the class $D = D_0 \times D_V^U$

Let $f(\lambda)$ and $g(\lambda)$ be the spectral density matrices of T-variate stationary sequences $\vec{\zeta}(j)$ and $\vec{\theta}(j)$, obtained by T-blocking (3) of T-PC sequences $\zeta(j)$ and $\theta(j)$, respectively.

Consider the problem of minimax estimation of the functional $A\vec{\zeta}$ based on observations of the sequence $\vec{\zeta}(\widetilde{j})+\vec{\theta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$, under the condition that the spectral density matrices $f(\lambda)$ and $g(\lambda)$ belong to the class $D = D_0 \times D_V^U$, where

$$D_0^1 = \left\{ f(\lambda) | \, \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\lambda)d\lambda = P \right\},$$

$$D_V^{U1} = \left\{ g(\lambda) | \, V(\lambda) \le g(\lambda) \le U(\lambda), \right. \\ \left. \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\lambda)d\lambda = Q \right\},$$

$$D_0^2 = \left\{ f(\lambda) | \, \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{Tr}\; f(\lambda)d\lambda = p \right\},$$

$$D_V^{U2} = \left\{ g(\lambda) | \, \mathrm{Tr}\; V(\lambda) \le \mathrm{Tr}\; g(\lambda) \le \mathrm{Tr}\; U(\lambda), \right. \\ \left. \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{Tr}\; g(\lambda)d\lambda = q \right\},$$

where P, Q are known positive definite Hermitian matrices, spectral densities $V(\lambda), U(\lambda)$ are known and fixed, p, q are known and fixed numbers.

With the help of the method of Lagrange multipliers we can find that solution $(f^0(\lambda), g^0(\lambda))$ of the constrained optimization problem (27) satisfy the following relations for these sets of admissible spectral densities.

For the pair $D_0^1 \times D_V^{U1}$ we have relations

$$(g^0(\lambda)\overline{A(e^{i\lambda})}+\overline{C^0(e^{i\lambda})})((g^0(\lambda))^{\top}A(e^{i\lambda})+C^0(e^{i\lambda}))^{\top} \\ = (f^0(\lambda)+g^0(\lambda))\overline{\vec{\alpha}}\vec{\alpha}^{\top}(f^0(\lambda)+g^0(\lambda)), \quad (30)$$

$$(f^0(\lambda)\overline{A(e^{i\lambda})}-\overline{C^0(e^{i\lambda})})((f^0(\lambda))^{\top}A(e^{i\lambda})-C^0(e^{i\lambda}))^{\top} \\ = (f^0(\lambda)+g^0(\lambda))(\overline{\vec{\beta}}\vec{\beta}^{\top}+\psi_1(\lambda)+\psi_2(\lambda))(f^0(\lambda)+g^0(\lambda)), \quad (31)$$

where $\vec{\alpha}, \vec{\beta}$ are Lagrange multipliers, $\psi_1(\lambda) \le 0$ and $\psi_1(\lambda) = 0$ if $g^0(\lambda) \ge V(\lambda)$, $\psi_2(\lambda) \ge 0$ and $\psi_2(\lambda) = 0$ if $g^0(\lambda) \le U(\lambda)$.

For the pair $D_0^2 \times D_V^{U2}$ we have relations

$$(g^0(\lambda)\overline{A(e^{i\lambda})}+\overline{C^0(e^{i\lambda})})((g^0(\lambda))^{\top}A(e^{i\lambda})+C^0(e^{i\lambda}))^{\top} \\ = \alpha^2(f^0(\lambda)+g^0(\lambda))^2, \quad (32)$$

$$(f^0(\lambda)\overline{A(e^{i\lambda})}-\overline{C^0(e^{i\lambda})})((f^0(\lambda))^\top A(e^{i\lambda})-C^0(e^{i\lambda}))^\top$$
$$=(\beta^2+\varphi_1(\lambda)+\varphi_2(\lambda))(f^0(\lambda)+g^0(\lambda))^2, \quad (33)$$

where α^2, β^2 are Lagrange multipliers, $\varphi_1(\lambda) \leq 0$ and $\varphi_1(\lambda)=0$ if Tr $g^0(\lambda) \geq$ Tr $V(\lambda)$, $\varphi_2(\lambda) \geq 0$ and $\varphi_2(\lambda)=0$ if Tr $g^0(\lambda) \leq$ Tr $U(\lambda)$.

Hence the following theorem holds true.

Theorem 3. *Let the spectral densities $f^0(\lambda)$ and $g^0(\lambda)$ satisfy the minimality condition (7). The least favorable spectral densities $f^0(\lambda)$, $g^0(\lambda)$ in the class $D_0^1 \times D_V^{U1}$ for the optimal linear extrapolation of the functional $A\vec{\zeta}$ are determined by relations (30), (31). The least favorable spectral densities $f^0(\lambda)$, $g^0(\lambda)$ in the class $D_0^2 \times D_V^{U2}$ for the optimal linear extrapolation of the functional $A\vec{\zeta}$ are determined by relations (32), (33). The minimax spectral characteristic of the optimal estimate of the functional $A\vec{\zeta}$ is determined by the formula (11).*

In the case of observations of the sequence without noise the following corollaries hold true.

Corollary 4. *Let the spectral density $f^0(\lambda)$ satisfies the minimality condition (19). The least favorable spectral density $f^0(\lambda)$ in the class D_0^1 or D_0^2 for the optimal linear extrapolation of the functional $A\vec{\zeta}$ based on observations of $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$ is determined by relations, respectively*

$$(\overline{C^0(e^{i\lambda})})(C^0(e^{i\lambda}))^\top = f^0(\lambda)\overline{\vec{\alpha}}\vec{\alpha}^\top f^0(\lambda), \quad (34)$$

$$(\overline{C^0(e^{i\lambda})})(C^0(e^{i\lambda}))^\top = \alpha^2(f^0(\lambda))^2, \quad (35)$$

by the constrained optimization problem (29) and restrictions on the density from the corresponding class D_0^1 or D_0^2. The minimax spectral characteristic of the optimal estimate of the functional $A\vec{\zeta}$ is determined by the formula (16).

Corollary 5. *Let the spectral density $f^0(\lambda)$ satisfies the minimality condition (19). The least favorable spectral density $f^0(\lambda)$ in the class D_V^{U1} or D_V^{U2} for the optimal linear extrapolation of the functional $A\vec{\zeta}$ based on observations of $\vec{\zeta}(\widetilde{j})$ at points $\widetilde{j} \in \{..., -n, ..., -1\} \setminus \widetilde{S}$ is determined by relations, respectively*

$$(\overline{C^0(e^{i\lambda})})(C^0(e^{i\lambda}))^\top =$$
$$= f^0(\lambda)(\overline{\vec{\beta}}\vec{\beta}^\top + \psi_1(\lambda)+\psi_2(\lambda))f^0(\lambda), \quad (36)$$

$$(\overline{C^0(e^{i\lambda})})(C^0(e^{i\lambda}))^\top =$$
$$= (\beta^2+\varphi_1(\lambda)+\varphi_2(\lambda))(f^0(\lambda))^2, \quad (37)$$

by the constrained optimization problem (29) and restrictions on the density from the corresponding class D_V^{U1} or D_V^{U2}. The minimax spectral characteristic of the optimal estimate of the functional $A\vec{\zeta}$ is determined by the formula (16).

5 Conclusions

In this article we study the extrapolation of the functionals $A\zeta$ and $A_N\zeta$ which depend on the unobserved values of a periodically correlated stochastic sequence $\zeta(j)$. Estimates are based on observations of a periodically correlated stochastic sequence $\zeta(j)+\theta(j)$ with missing observations, that means that observations of $\zeta(j)+\theta(j)$ are known at points $j \in \mathbb{Z} \setminus S$, $j \in \{..., -n, ..., -2, -1, 0\} \setminus S$, $S = \bigcup_{l=1}^{s-1}\{-M_l \cdot T + 1, \ldots, -M_{l-1} \cdot T - N_l \cdot T\}$. The sequence $\theta(j)$ is an uncorrelated with $\zeta(j)$ additive noise.

The extrapolation problem is considered under the condition of spectral certainty and under the condition of spectral uncertainty. In the first case of spectral certainty the spectral density matrices $f(\lambda)$ and $g(\lambda)$ of the T-variate stationary sequences $\vec{\zeta}(n)$ and $\vec{\theta}(n)$, obtained by T-blocking of T-PC sequences $\zeta(j)$ and $\theta(j)$, respectively, are suppose to be known exactly. With the help of Hilbert space projection method formulas for calculating the spectral characteristic and the mean-square error of the optimal estimate of the functionals are proposed. In the second case of spectral uncertainty the spectral density matrices are not exactly known while a class $D = D_f \times D_g$ of admissible spectral densities is given. Using the minimax (robust) estimation method we derived relations that determine the least favorable spectral densities and the minimax spectral characteristic of the optimal estimate of the functional $A\zeta$. The problem is investigated in details for two special classes of admissible spectral densities. In each of cases of spectral certainty and uncertainty the case of observations of the sequence without noise $\theta(j)$ are presented.

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Received: 06.06.2021