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APPROXIMATE AVERAGED BOUNDED SYNTHESIS FOR A PARABOLIC PROCESS WITH TWO SWITCHING POINTS

In this paper we analyze the problem of the optimal bounded control with semi-definite quality criterion for parabolic process with rapidly oscillating coefficients on finite time interval. The case when optimal control goes out both on lower, and on upper restriction, that is has two switch-points is considered. Our aim is to build a synthesized control which is practically preferable to program one. But after Fourier method application such a control will contain coefficients which are expressed by series and, in addition, it irregularly depends on small parameter ε. Thus, for practical application it is natural to restrict infinite series by finite sums and to get rid of dependence on a small parameter using average procedure. So, besides the law of optimal synthesis also approximate averaged feedback control, which provides control system behavior that is close to optimal one and thus has a series of advantages from the practical application point of view, is proposed and proved. The efficiency of approximate averaged control construction procedure consisted of cutting to finite sums of series in optimal control and replacing of Fourier coefficients nonregular depended on small parameter by corresponding average values is illustrated by concreate example of controlled system for parabolic process. We compare the properties of the averaged control and the sequence of optimal controls calculated at the different values of the small parameter. For comparison, we use the switching points of optimal control and averaged control, deviation between optimal control and averaged control, the difference in the values of the quality criterion on optimal control and averaged control. In considered example the precision of quality criteria value on approximate control has ε-order and for sufficiently small ε the precision is one order better then ε.

K e y w o r d s : ***optimal control, parabolic process, semi-definite quality criterion, switching point, feedback control, approximate averaged synthesis.***

AMS 2020 classification: 49N35, 49N10, 49J20.

Introduction

This paper continues the works presented in (Kapustyan, O. A., Kapustyan, O. V., & Sukretna, 2013) in which an optimal control problem with a semi-defined quality criterion for a process described by a boundary value problem for a parabolic equation with rapidly oscillating coefficients is investigated. However, in contrast to these works, in this paper we analyze the case where the optimal control has two switching points when reaching the upper and lower restrictions. Under these conditions, optimal program control and optimal control in the feedback form are built. In addition, since the obtained optimal synthesis is not convenient from the point of view of practical use (it is given by infinite series and irregularly depends on a small parameter), the law of approximate averaged synthesis, which has the necessary extreme properties, is proposed and substantiated.

The object of research is optimal control problem with a semi-defined quality criterion for a parabolic process on finite time interval.

The aim and objectives of the research is to propose alternative approximate feedback control, which provides control system behavior that is close to optimal one and in the same time can be simply realized in practice, and also to demonstrate by the example the effectivity of proposed procedure of building the approximate suboptimal synthesis.

1. Main results. Lets $\Omega \subset \mathbb{R}^n$ – bounded region with a smooth boundary and for $\varepsilon \in (0,1)$ controlled process in a cylinder $\bar{Q}_T = [0,T] \times \bar{\Omega}$ is described by a boundary value problem

$$\begin{cases} y_t^\varepsilon(x,t) = A^\varepsilon\big(y^\varepsilon(x,t)\big) + g^\varepsilon(x)v(t), \quad (t,x) \in Q_T, \\ y^\varepsilon(x,t) = 0, \quad x \in \partial\Omega, \quad t \in [0,T], \\ y^\varepsilon(x,0) = y_0^\varepsilon(x), \quad x \in \bar{\Omega}, \end{cases} \tag{1}$$

where $A^\varepsilon := \mathrm{div}\big(a^\varepsilon\vec{\nabla}\big)$, $a^\varepsilon(x) = \Big(\big(a_{ij}^\varepsilon(x)\big)\Big)_{i,j=1}^n$ – measured symmetric matrix satisfying the conditions of uniform ellipticity and boundedness, $g^\varepsilon, y_0^\varepsilon \in L_2(\Omega)$, ε – small parameter.

Control is limited by

$$v(\cdot) \in U = \{ v \in L_2(0,T) : |v(t)| \le \xi \text{ almost everywhere in } [0;T]. \tag{2}$$

The problem of optimal control is to minimize the semi-definite quality criterion

$$J^\varepsilon(v) = \left(\int_\Omega q(x) y^\varepsilon(x,T)\, dx\right)^2 + \gamma \int_0^T v^2(t)\, dt \tag{3}$$

at solutions (1) for $v(\cdot) \in U$, where $q \in L_2(\Omega)$ and $\gamma > 0$.

We will assume the following convergences of the coefficients of the optimal control problem (1) – (3):

$$g^\varepsilon \to g^0, \quad y_0^\varepsilon \to y_0 \quad \text{weakly in } L_2(\Omega) \text{ as } \varepsilon \to 0;$$
$$a^\varepsilon \to a^0 \text{ in sense of G-convergence of matrices as } \varepsilon \to 0. \tag{4}$$

For definitions and properties of G-convergence of matrices see (Jikov, Kozlov, & Oleinik, 1994).

Further using the limit functions from (4) we can assume that the optimal control problem (1) – (3) is defined also for $\varepsilon = 0$ (so-called averaged problem).

It is known that for each fixed control $v \in U$ problem (1) has a unique solution in the class $C\big([0,T]; L_2(\Omega)\big)$ (Ladyzhenskaya, 2013). In addition, for an arbitrary $\varepsilon \in [0,1)$ the optimal control problem (1) – (3) also has a unique solution $v^\varepsilon(\cdot) \in U$ (Lions, 1971).

2. Construction of optimal synthesis. Applying the Fourier method and decomposing the data of the initial optimal control problem by the eigenbasis $\{X_i^\varepsilon(x)\}_{i=1}^\infty$ of the operator A^ε: $A^\varepsilon X_i^\varepsilon + (\lambda_i^\varepsilon)^2 X_i^\varepsilon = 0,\ i = \overline{1,\infty}$, the problem (1) – (3) is split and reduced to an equivalent optimal control problems for the first order ordinary differential equation (Temam, 1997).

In addition, we will assume that the spectrum of the averaged operator A^0 is simple, i.e

$$0 < (\lambda_1^0)^2 < (\lambda_2^0)^2 < \cdots. \tag{5}$$

Let g_i^ε, q_i^ε be the Fourier coefficients of the corresponding functions according to the system $\{X_i^\varepsilon(x)\}_{i=1}^\infty$. Let us denote

$$f^\varepsilon(t) = \sum_{i=1}^\infty e^{(\lambda_i^\varepsilon)^2(t-T)} q_i^\varepsilon\, g_i^\varepsilon,\quad a_0^\varepsilon = \sum_{i=1}^\infty e^{-(\lambda_i^\varepsilon)^2 T} q_i^\varepsilon\, y_{0,i}^\varepsilon,$$

$$\Re^\varepsilon(x,t) = \sum_{i=1}^\infty q_i^\varepsilon\, e^{(\lambda_i^\varepsilon)^2(t-T)} X_i^\varepsilon(x).$$

The resulting optimal control problem for an ordinary differential equation has already been studied in (Kapustyan, O. A., Kapustyan, O. V., & Sukretna, 2013) under the condition that $f^\varepsilon(t)$ is a positive and strictly monotonically increasing function.

We consider another condition and assume that for each $\varepsilon \in [0,1)$ the function $f^\varepsilon(t)$ remains strictly monotonically increasing and changes sign, i. e., there exists such $t_0^\varepsilon \in (0,T)$ that

$$f^\varepsilon(0) < 0,\quad f^\varepsilon(t_0) = 0,\quad f^\varepsilon(T) > 0,$$
$$f^\varepsilon(t) \text{ strictly monotonically increases on } [0;T]. \tag{6}$$

Refusion of the requirement of function $f^\varepsilon(t)$ positiveness allows us to observe a more complex behavior of the optimal control. In particular, it becomes possible for the optimal control to have two switching points when reaching both the lower $v = -\xi$ and upper $v = \xi$ restrictions. This qualitatively new case we will pay our further attention to.

Applying the maximum principle, under the condition that the system of inequalities

$$\frac{\mp a_0^\varepsilon f^\varepsilon(0)}{\gamma+\int_0^T (f^\varepsilon(s))^2 ds} < -\xi,\qquad \frac{\mp a_0^\varepsilon f^\varepsilon(T)}{\gamma+\int_0^T (f^\varepsilon(s))^2 ds} > \xi,\qquad \xi\int_0^T |f^\varepsilon(s)|\, ds < \mp a_0^\varepsilon \tag{7}$$

is fulfilled for all $\varepsilon \in [0,1)$, we obtain the optimal control in the feedback form for optimal control problem (1) – (3)

$$u^\varepsilon[t, y^\varepsilon(t)] = \begin{cases} \mp\xi, & t \in [0, t_-^\varepsilon], \\ -\dfrac{(\Re^\varepsilon(t), y^\varepsilon(t)) \pm \xi\int_{t_+^\varepsilon}^T f^\varepsilon(s)ds}{\gamma + \int_{t_-^\varepsilon}^{t_+^\varepsilon} (f^\varepsilon(s))^2 ds} f^\varepsilon(t), & t \in [t_-^\varepsilon, t_+^\varepsilon], \\ \pm\xi, & t \in [t_+^\varepsilon, T], \end{cases} \tag{8}$$

where $y^\varepsilon(x,t)$ – solution of the boundary value problem (1) with control (8), and the switching points t_-^ε and t_+^ε are determined from the system

$$\begin{cases} \dfrac{a_0^\varepsilon \mp \xi\int_0^{t_-^\varepsilon} f^\varepsilon(s)ds \pm \xi\int_{t_+^\varepsilon}^T f^\varepsilon(s)ds}{\gamma + \int_{t_-^\varepsilon}^{t_+^\varepsilon} (f^\varepsilon(s))^2 ds} f^\varepsilon(t_-^\varepsilon) = \pm\xi, \\ \dfrac{a_0^\varepsilon \mp \xi\int_0^{t_-^\varepsilon} f^\varepsilon(s)ds \pm \xi\int_{t_+^\varepsilon}^T f^\varepsilon(s)ds}{\gamma + \int_{t_-^\varepsilon}^{t_+^\varepsilon} (f^\varepsilon(s))^2 ds} f^\varepsilon(t_+^\varepsilon) = \mp\xi. \end{cases} \tag{9}$$

The existence and uniqueness of solutions of system (9) follows from the method of finding the control. In addition, the uniqueness of the solution of the system of equations (9) can also be obtained from the conditions (6) and (7).

3. Approximate averaged synthesis. For an arbitrary $\varepsilon \in [0;1)$ and $N \in \mathbb{N}$ we define

$$a_{0\,N}^\varepsilon = \sum_{i=1}^N e^{-(\lambda_i^\varepsilon)^2 T} q_i^\varepsilon\, y_i^{0\,\varepsilon},\qquad f_N^\varepsilon(t) = \sum_{i=1}^N e^{(\lambda_i^\varepsilon)^2(t-T)}\, q_i^\varepsilon\, g_i^\varepsilon,$$

$$\Re_N^\varepsilon(x,t) = \sum_{i=1}^N q_i^\varepsilon\, e^{(\lambda_i^\varepsilon)^2(t-T)}\, X_i^\varepsilon(x).$$

Let $(8)_N$ and $(9)_N$ be the formulas (8) and (9) respectively in which all occurrences of a_0^ε, $f^\varepsilon(t)$ and $\Re^\varepsilon(x,t)$ are replaced with $a_{0\,N}^\varepsilon$, $f_N^\varepsilon(t)$ and $\Re_N^\varepsilon(x,t)$ respectively, and t_{-N}^ε, t_{+N}^ε are the corresponding switching points.

It can be proved that the formally constructed law $(8)_N$, $(9)_N$ is the optimal control law for the problem (1) – (3) with the functional J^ε in which the function $q(x)$ is replaced by the function $q_N(x) = \sum_{i=1}^N q_i^\varepsilon X_i^\varepsilon(x)$ and for each $N \in \mathbb{N}$ the points t_{-N}^ε, t_{+N}^ε are determined uniquely, and for an arbitrary $\varepsilon \in [0;1)$ the convergences $t_{-N}^\varepsilon \to t_-^\varepsilon$, $t_{+N}^\varepsilon \to t_+^\varepsilon$ as $N \to \infty$ are fulfilled.

We will construct the law of approximate averaged synthesis for the problem (1) – (3), and make sure of the closeness of the values of the quality criterion (3) on the optimal control and constructed approximate control.

Further, the main object of our research is the approximate averaged control

$$v_N[t, z_N^\varepsilon(t)] = \begin{cases} \mp\xi, & t \in [0, t_{-N}^0], \\ -\dfrac{(\Re_N^0(t), z_N^\varepsilon(t)) \pm \xi\int_{t_{+N}^0}^T f_N^0(s)\, ds}{\gamma + \int_{t_{-N}^0}^{t_{+N}^0} (f_N^0(s))^2 ds} f_N^0(t) & t \in [t_{-N}^0, t_{+N}^0], \\ \pm\xi & t \in [t_{+N}^0, T], \end{cases} \tag{10}$$

where $z_N^\varepsilon(x,t)$ – solution of the problem (1) with control (10), t_{-N}^0, t_{+N}^0 – switching points that are defined from the system $(9)_N$ at $\varepsilon = 0$.

Using the ideas from (Kapustyan, O. A., Kapustyan, O. V., & Sukretna, 2013), it is possible to prove the correctness of the proposed approximate averaged synthesis (10), that is, the control in the feedback form (10) provides close to the optimal value of the objective functional (3). Namely, we have.

Theorem. Let $g^\varepsilon, y_0^\varepsilon, q \in L_2(\Omega)$ and the assumptions (4) – (7) be fulfilled. Then for an arbitrary small $\eta > 0$ there exist $N_0 \in \mathbb{N}$ and $\varepsilon_0 > 0$ such that for any $N \geq N_0$ and $\varepsilon \in (0, \varepsilon_0)$ for the controls (8), (10) we have the inequalities

$$\| u^\varepsilon[\,\cdot\,, y^\varepsilon] - v_n[\,\cdot\,, z_n^\varepsilon] \|_{L_2(0,T)} < \eta,$$
$$\| y^\varepsilon(\,\cdot\,, t) - z_n^\varepsilon(\,\cdot\,, t)] \| < \eta \quad \text{for all} \quad t \in [0,T],$$
$$| J^\varepsilon(u^\varepsilon[t, y^\varepsilon]) - J^\varepsilon(u_n[t, z_n^\varepsilon]) | < \eta. \tag{11}$$

4. Numeric illustration. Let the controlled process in the rectangle $\bar{Q} = [0,1] \times [0,\pi]$ be described by a boundary value problem

$$\begin{cases} y_t^\varepsilon(x,t) = y_{xx}^\varepsilon(x,t) + \sqrt{\frac{2}{\pi}}(\sin x + 3\sin 2x)v(t), \quad (t,x) \in Q, \\ y^\varepsilon(0,t) = y^\varepsilon(\pi,t) = 0, \quad t \in [0,1], \\ y^\varepsilon(x,0) = 25\sqrt{2\pi}\sin^2\frac{x}{\varepsilon}, \quad x \in [0,\pi], \end{cases} \tag{12}$$

where $\varepsilon > 0$ – small parameter. Control is limited by

$$v(\cdot) \in U = \{ v \in L_2(0,1) : |v(t)| \le 1 \text{ almost everywhere in } [0,1]\}. \tag{13}$$

The problem of optimal control is to minimize the quality criterion on solutions (12) for $v(\cdot) \in U$

$$J^\varepsilon(v) = \left(\int_0^\pi \sqrt{\frac{2}{\pi}}\,(2\sin 2x - \sin x)\, y^\varepsilon(x,1)dx\right)^2 + \int_0^1 v^2(t)\,dt. \tag{14}$$

The optimal control problem (12) – (14) can be defined at $\varepsilon = 0$ if the following limit function is used

$$y^\varepsilon(x,0) = 25\sqrt{2\pi}\sin^2\frac{x}{\varepsilon} \to y^0 = 25\sqrt{\frac{\pi}{2}} \quad \text{weakly in} \quad L_2(0,\pi) \quad as \quad \varepsilon \to 0. \tag{15}$$

Applying the results presented above, we obtain formulas for optimal and approximate control for problem (12) – (14).

The corresponding spectral problem for our example does not depend on ε, has the form

$$\begin{cases} X'' + \mu^2 X = 0, \quad x \in (0,\pi), \\ X(0) = X(\pi) = 0, \end{cases} \tag{16}$$

and has eigenvalues $\lambda_n^2 = n^2$ and eigenfunctions $X_n(x) = \sqrt{\frac{2}{\pi}}\sin nx$. Therefore, taking into account the form of the function $q(x) = \sqrt{\frac{2}{\pi}}(2\sin 2x - \sin x)$, instead of infinite series, we get sums of two terms.

Problem (12) – (14) is equivalent to the optimal control problem for an ordinary differential equation of the first order. Let's denote

$$a_0^\varepsilon = e^{-1}\frac{2\varepsilon^2\sin^2\frac{\pi}{\varepsilon} - 8}{4 - \varepsilon^2} + 2e^{-4}\frac{2\varepsilon^2\sin^2\frac{\pi}{\varepsilon}}{2 - 2\varepsilon^2}.$$
$$f(t) = -e^{t-1} + 6e^{4(t-1)}. \tag{17}$$

The function $f(t)$ is strictly monotonically increasing on $[0; 1]$ and changes its sign at the point $t_0 = 1 - \frac{\ln 6}{3}$ (see Figure 1).

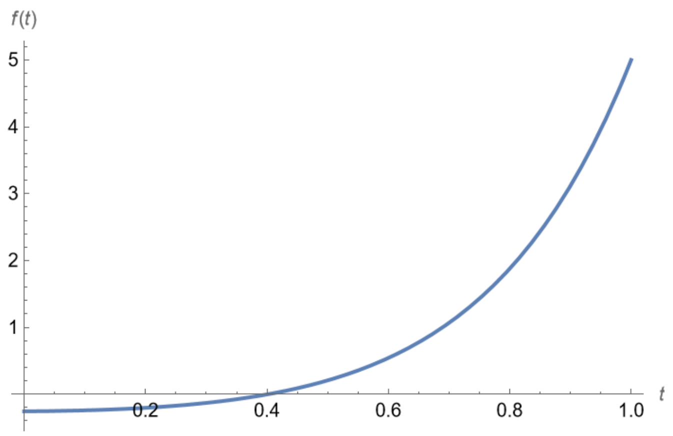

Fig. 1. Graph of function $f(t)$

Investigating the dependence of the initial value a_0^ε on the small parameter ε (see Figure 2), it is obvious that the following inequalities are satisfied for $\varepsilon \in (0; 0.8]$

$$\frac{-a_0^\varepsilon f(0)}{1 + \int_0^1 f^2(s)ds} < -1, \quad \frac{-a_0^\varepsilon f(1)}{1 + \int_0^1 f^2(s)ds} > 1, \quad \int_0^1 |f(s)|\,ds < a_0^\varepsilon. \tag{18}$$

The optimal control of the problem (12) – (14) is given in the form

$$u^\varepsilon(t) = \begin{cases} -1, & t \in [0, t_-^\varepsilon], \\ -\dfrac{a_0^\varepsilon - \int_0^{t_-^\varepsilon} f(s)\,ds + \int_{t_+^\varepsilon}^1 f(s)ds}{1 + \int_{t_-^\varepsilon}^{t_+^\varepsilon} f^2(s)ds} f(t), & t \in [t_-^\varepsilon\,,\, t_+^\varepsilon], \\ 1, & t \in [t_+^\varepsilon\,, 1], \end{cases} \tag{19}$$

where switching points t_-^ε, t_+^ε are determined from the system

$$\begin{cases} \dfrac{a_0^\varepsilon - \int_0^{t_-^\varepsilon} f(s)ds + \int_{t_+^\varepsilon}^1 f(s)ds}{1 + \int_{t_-^\varepsilon}^{t_+^\varepsilon} f^2(s)ds} f(t_-^\varepsilon) = 1, \\ \dfrac{a_0^\varepsilon - \int_0^{t_-^\varepsilon} f(s)ds + \int_{t_+^\varepsilon}^1 f(s)ds}{1 + \int_{t_-^\varepsilon}^{t_+^\varepsilon} f^2(s)ds} f(t_+^\varepsilon) = -1. \end{cases} \tag{20}$$

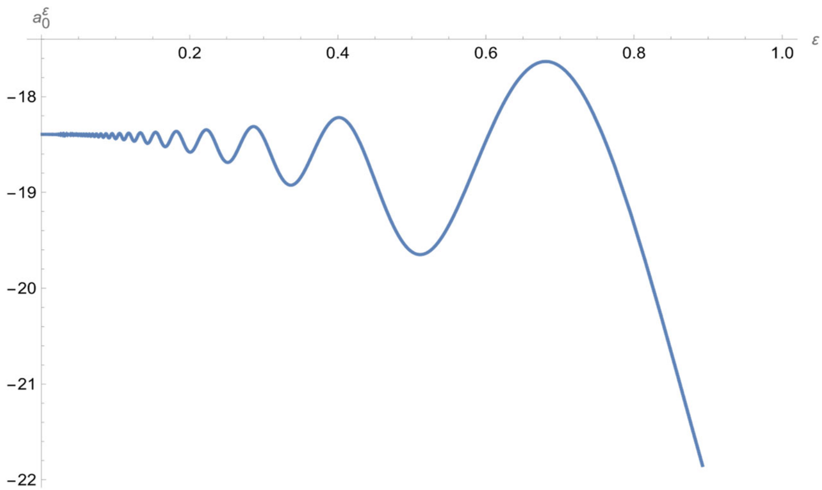

Fig. 2. Graph of a_0^ε depend of ε

Replacing a_0^ε by the limiting value $a_0 = -\frac{50}{e}$ at $\varepsilon \to +0$, we have the corresponding averaged control

$$v(t) = \begin{cases} -1, & t \in [0, t_-^0], \\ -\dfrac{-\frac{50}{e} - \int_0^{t_-^0} f(s)ds + \int_{t_+^0}^1 f(s)ds}{1 + \int_{t_-^0}^{t_+^0} f^2(s)ds} f(t), & t \in [t_-^0, t_+^0], \\ 1, & t \in [t_+^0, 1]. \end{cases} \tag{21}$$

where switching points t_-^0, t_+^0, are determined from the system

$$\begin{cases} \dfrac{-\frac{50}{e} - \int_0^{t_-^0} f(s)ds + \int_{t_+^0}^1 f(s)ds}{1 + \int_{t_-^0}^{t_+^0} f^2(s)ds} f(t_-^0) = 1, \\ \dfrac{-\frac{50}{e} - \int_0^{t_-^0} f(s)ds + \int_{t_+^0}^1 f(s)ds}{1 + \int_{t_-^0}^{t_+^0} f^2(s)ds} f(t_+^0) = -1. \end{cases} \tag{22}$$

Further we compare the properties of the averaged control (21) and the sequence of optimal controls (19) calculated at the values of the parameter ε which form a decreasing to zero sequence. As such a sequence of values of the ε parameter, we choose the geometric progression $\varepsilon_k = \frac{0.8}{2^k}$, $k = 0, 1, 2, \dots$. For comparison, we use the following characteristics of the controlled system:

1) switching points t_-^ε, t_+^ε of optimal control when reaching the lower and upper limits, respectively;
2) deviation $\Delta u^\varepsilon = \|u^\varepsilon - v\|_{L_2(0,1)}$ in the space $L_2(0,1)$ between optimal control and averaged control;
3) the difference in the values of the quality criteria $\Delta J^\varepsilon = |J^\varepsilon(u^\varepsilon) - J^\varepsilon(v)|$.

The results of numerical calculations are given in Table 1.

Table 1

Numerical calculations for comparison of averaged control and sequence of optimal controls

ε_k	t_-^ε	t_+^ε	Δu^ε	ΔJ^ε
0,8	0,3666316467	0,4333202569	0,00785654	33,5323
0,4	0,3640556501	0,4351436479	0,00154902	6,04324
0,2	0,3649294736	0,4345296526	0,00162396	6,50383
0,1	0,3645953528	0,4347649688	0,000406207	1,60729
0,05	0,3645116882	0,4348237867	0,000101566	0,400672
0,025	0,3644907636	0,4348384904	0,0000253932	0,100096
0,0125	0,3644855320	0,4348421663	$6{,}34819 \cdot 10^{-6}$	0,0250196
0,00625	0,3644842240	0,4348430853	$1{,}58847 \cdot 10^{-6}$	0,00625461
0,003125	0,3644838970	0,4348433151	$3{,}86031 \cdot 10^{-7}$	0,00156363
0,0015625	0,3644838153	0,4348433725	$9{,}16799 \cdot 10^{-8}$	0,000390908
0,00078125	0,3644837948	0,4348433868	$3{,}62806 \cdot 10^{-8}$	0,0000977268
0,00039062	0,3644837897	0,4348433904	$7{,}24372 \cdot 10^{-9}$	0,0000244317
0,00019531	0,3644837885	0,4348433913	$5{,}12177 \cdot 10^{-9}$	0,00000610792

While $\varepsilon = 0$ we obtain the following values of the switching points of the averaged control $t_-^0 = 0{,}364483788033\ldots$, $t_+^0 = 0{,}434843391635\ldots$, the norm in space $L_2(0,1)$ of the averaged control $\|v\|_{L_{2(0,1)}} = 0{,}976372248559\ldots$ and the value of the quality criterion on the averaged control $J^0(v) = 304{,}046396\ldots$.

The proposed procedure for constructing an approximate control by replacing the rapidly oscillating functions included in the problem formulation with the corresponding averaged ones gives sufficient accuracy.

Discussion and conclusions

In the paper we analyze the problem of optimal bounded control of a parabolic process with a semi-definite quality criterion in the case the optimal control reaches both lower and upper limits that is it has two switching points. In addition to the law of optimal synthesis, an approximate averaged control in feedback form is proposed and substantiated. Proposed averaged control ensures the behavior of the controlled system in a close to optimal mode and at the same time has a number of advantages from the point of view of practical application. The example of a specific controlled system for a parabolic process illustrates the effectiveness of the procedure for constructing an approximate averaged control which consists in "cutting" the infinite series included in the expressions for optimal control to finite sums and replacing the Fourier coefficients which irregularly depend on a small parameter with the corresponding averaged values.

Authors' contribution: Yuriy Loveikin – conceptualization; empirical data collection and validation; empirical research; Anna Sukretna – methodology; analysis of sources, preparation of literature review or theoretical foundations of research.

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НАБЛИЖЕНИЙ УСЕРЕДНЕНИЙ ОБМЕЖЕНИЙ СИНТЕЗ ДЛЯ ПАРАБОЛІЧНОГО ПРОЦЕСУ З ДВОМА ТОЧКАМИ ПЕРЕМИКАННЯ

Проаналізовано задачу оптимального обмеженого керування з напіввизначеним критерієм якості для параболічного процесу зі швидко осцилюючими коефіцієнтами на скінченному інтервалі часу. Розглянуто випадок, коли оптимальне керування виходить як на нижнє, так і на верхнє обмеження, тобто має дві точки перемикання. Наша мета полягає в тому, щоб побудувати синтезоване керування, що є практично кращим, ніж програмне. Проте після застосування методу Фур'є таке керування буде містити коефіцієнти, які виражаються рядами, і, крім того, воно нерегулярно залежить від малого параметра ε. Отже, для практичного застосування природно обмежити нескінченні ряди скінченними сумами і позбутися залежності від малого параметра за допомогою процедури усереднення. Тож, окрім закону оптимального синтезу, також запропоновано та доведено наближене усереднене керування зі зворотним зв'язком, що забезпечує поведінку системи керування, наближену до оптимальної, а отже, має переваги з позиції практичного застосування. На конкретному прикладі керованої системи для параболічного процесу проілюстровано ефективність процедури побудови наближеного усередненого керування, яка полягає в заміні нескінченних рядів скінченними сумами за оптимального керування та у разі заміни нерегулярно залежних від малого параметра коефіцієнтів Фур'є на відповідні середні значення. Ми порівнюємо властивості усередненого керування та послідовності оптимальних керувань, розрахованих за різних значень малого параметра. Для порівняння використовуємо точки перемикання оптимального та усередненого керувань, відхилення між оптимальним й усередненим керуванням, різницю значень критерію якості на оптимальному та усередненому керуваннях. У розглянутому прикладі точність значення критерію якості на наближеному управлінні має ε-порядок і для достатньо малого ε точність на порядок вища за ε.

К л ю ч о в і с л о в а : ***оптимальне керування, параболічний процес, напіввизначений критерій якості, точка перемикання, керування зі зворотним зв'язком, наближений усереднений синтез.***

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