Отриманий результат можна застосувати до системи $[M/M/m/m+n]^{(N)}$ з повторними викликами і чергою. У даному випадку залежність локальних характеристик від точки $(i,j)\in S$ фазового простору має простий явний вигляд і можна провести більш детальний аналіз.

Лема 1. Для $[M/M/m/m+n]^{(N)}$ - системи з повторними викликами і чергою

$$Q_{kN}=\frac{(\lambda/\mu)^k}{k!}\sum_{i=0}^{k}\binom{k}{i}\prod_{j=0}^{i-1}(j\frac{\mu}{\lambda}+N\frac{\nu}{\lambda}),$$

$$k=0,1,\ldots m-1,$$

де $\binom{k}{i}=\dfrac{k!}{i!(k-i)!}$ і за домовленістю $\prod_{i}^{j}\ldots=1$, при $j<i$.

Очевидно, формули (1)-(4) представляють собою ефективну рекурентну процедуру для обчислення стаціонарного розподілу. У випадку одного обслуговуючого приладу та одного місця в черзі векторно-матричне подання стаціонарних ймовірностей значно спрощується.

Будемо вважати $\mu=1$.

Наслідок 1. Для $[M/M/1/1+1]^{(N)}$ – системи з повторними викликами і чергою

$$\pi(0,j)=S^{-1}(N)\frac{\rho^j}{1+\rho+\lambda+j\nu}\varphi(j), \qquad (17)$$

$$\pi(1,j)=S^{-1}(N)\rho^j\frac{\lambda+j\nu}{1+\rho+\lambda+j\nu}\varphi(j),\ j=0,1,\ldots,N, \qquad (18)$$

$$\pi(2,j)=S^{-1}(N)\rho^j\frac{\Lambda(j)(1+\lambda+j\nu)}{(1+\rho+\lambda+j\nu)(1+\rho+\lambda+(j+1)\nu)}\times$$
$$\times\varphi(j),\ j=0,1,\ldots,N-1, \qquad (19)$$

$$\pi(2,N)=S^{-1}(N)\rho^N\frac{\Lambda(N)}{1+\rho+\lambda+N\nu}\varphi(N),$$

де $\rho=\dfrac{\lambda}{\mu}=\lambda$ завантаження системи первинними викликами,

$$\Lambda(j)=(\lambda+j\nu)(1+\lambda+j\nu)-\lambda,$$

$$\varphi(j)=\prod_{i=0}^{j-1}\frac{\Lambda(i)}{(i+1)\nu(1+\rho+\lambda+i\nu)},\ j=0,1,\ldots,$$

а $S(N)$ - нормуюча константа.

Щоб отримати формули для $[M/M/1/1+1]$-системи, перейдемо до границі при $N\to\infty$.

Наслідок 2. Якщо для $[M/M/1/1+1]$-системи з повторними викликами і чергою $\rho<1$, то для неї існують стаціонарні імовірності

$$\pi(0,j)=S^{-1}\frac{\rho^j}{1+\rho+\lambda+j\nu}\varphi(j), \qquad (20)$$

$$\pi(1,j)=S^{-1}\rho^j\frac{\lambda+j\nu}{1+\rho+\lambda+j\nu}\varphi(j) \qquad (21)$$

$$\pi(2,j)=S^{-1}\rho^j\frac{\Lambda(j)(1+\lambda+j\nu)}{(1+\rho+\lambda+j\nu)(1+\rho+\lambda+(j+1)\nu)}\varphi(j),$$
$$j=0,1,\ldots, \qquad (22)$$

де

$$S=\sum_{j=0}^{\infty}\rho^j\frac{1+\lambda+j\nu}{1+\rho+\lambda+j\nu}\left[1+\frac{\Lambda(j)}{1+\rho+\lambda+(j+1)\nu}\right]\varphi(j).$$

В даній роботі для обчислення стаціонарних ймовірностей отримані явні формули в векторно-матричному вигляді. Вони є зручними як для обчислення так і розв'язку оптимізаційних задач. Прості формули, подібні (17)-(19), (20)-(22) можуть бути отримані для інших типів міграцій, що визначаються вибором залежності локальних характеристик від фазової точки області S.

Список використаних джерел

1. *Falin G.I.* Retrial Queues / G.I. Falin, J.G.C. Templeton. – London Chapman & Hall, 1997. – 331 p.
2. *Artalejo J.R.* Retrial Queueing Systems. A Computational Approach / J.R. Artalejo, A. Gomes-Corral. – Springer-Verlag, 2008. – 318 p.
3. *Уолрэнд Дж.* Введение в теорию массового обслуживания.– М.:Мир, 1993. – 336 с.
4. *Anisimov V.V.* Analysis of Markov multiserver retrial queues with negative arrivals / V.V. Anisimov, J.R. Artalejo // Queuing Systems. – 2001. – Vol.39. – P. 157-182.

References

1. FALIN G.I., TEMPLETON J.G.C. (1997) *Retrial queues*. London Chapman & Hall.
2. ARTALEJO, J.R., GOMES-CORRAL, A. (2008) *Retrial Queueing Systems. A Computational Approach*, Springer-Verlag.
3. WALRAND, J. (1993) *An Introduction to queueing systems*, Moscow.
4. ANISIMOV V.V., ARTALEJO J.R. (2001) "Analysis of Markov multiserver retrial queues with negative arrivals" *J. Queuing Systems*. Vol.39., p.p. 157-182.

Надійшла до редколегії 23.11.2018

РАДІОФІЗИКА

УДК 616-073.757.1

Даник А.Ю., аспірант[1]

Проблема впливу розсіяного випромінювання у рентгенівській діагностиці

[1]Факультет радіофізики, електроніки та ком'ютерних систем
Київський національний університет імені Тараса Шевченка, м. Київ, пр-т. Акад. Глушкова 4-е
email:AntonDanik@gmail.com

Danyk A.Y., PhD student[1]

The problem of scattered radiation in X-ray imaging

[1]Faculty of Radiophysics, Electronics and Computer Systems
National Taras Shevchenko University of Kyiv
Ukraine, Kyiv, Acad. Glushkova av., 4-e

email:AntonDanik@gmail.com

Розглянуто та порівняно різні методи рішення проблеми впливу розсіяного випромінювання на рентгенівські зображення: простота та ефективність методів пригнічення розсіяного випромінювання протиставляються гнучкості та безпечності (зменшення дозового навантаження) методів оцінки та компенсації впливу рентгенівського випромінювання. Рішення в основі яких лежить використання результатів Монте-Карло симуляції у поєднанні із методом суперпозиції ядер розсіювання відзначено як найбільш перспективні.

Ключові слова: рентгенівське зображення; розсіяне рентгенівське випромінювання; протирозсіюючий растр; суперпозиція ядер розсіювання.

Different solutions of the scattered radiation problem in X-ray imaging are reviewed and compared: simplicity and efficiency of the scatter suppression techniques (anti-scatter grid, air gap, slit scheme) are contrasted to the flexibility and safety (dose reduction) of scatter estimation/compensation techniques (hardware methods: beam stoppers application, dark field; software methods: analytical approximation, Monte-Carlo simulation based approaches, scattering kernels superposition technique). Large enough air gap filters scattered radiation as good as high-quality anti-scatter grid while providing dose reduction in cost of X-ray apparatus size increasing. Scatter estimation/compensation software techniques could resolve the problem requiring less radiation dose than hardware techniques do but in cost of greater computing resources. Monte-Carlo based approaches in conjunction with kernels superposition idea is admitted as the most promising.

Key Words: X-ray image; scattered X-ray radiation; anti-scatter grid; kernels superposition.

Статтю представив д.ф.-м.н., проф. Анісімов І.О.

1. Introduction

X-ray imaging is based on the simple idea of getting the shadow pictures of the object in X-rays. Generally, X-ray apparatus should include such important parts as the source of the X-ray radiation and the detector sensible to the radiation emitted by the source. Object is placed between the source and the detector such way that radiation emitted by the source passes through the object's volume of interest and reaches the detector. Due to the interaction with the object's substance, the X-ray radiation photons may undergo the absorption, scattering, or freely pass the object as presented on fig.1.

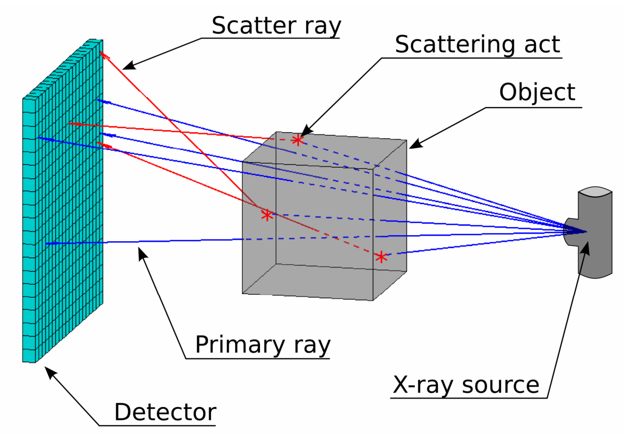

Figure 1 X-ray imaging scheme

Primary radiation is a part of the initial X-ray source radiation that has passed the object without interactions with object's substance. This radiation produces a shadow picture of the object's volume of interest. Radiation of scattered X-ray photons mainly caused by Compton and Rayleigh processes is almost non-collinear to the primary radiation [1]. So scattering distorts the resulting X-ray image contrast and sharpness adding an extra noise and uneven background to the resulting X-ray image. The Hounsfield Scale of image is also become distorted. Thus scattered radiation compensation is an important task especially for medical imaging.

Solutions of the "scattered radiation" problem in X-ray imaging can be divided into two groups: "scattered radiation suppression" - methods, whose main goal is reduction of the amount of the scattered radiation that takes part in the X-ray image formation; "scattered radiation estimation and compensation" – methods, whose main goal is correction of the X-ray image that was formed under scattered radiation influence. In this paper different approaches for scattering compensation in X-ray imaging are compared and the most optimal approach for scatter suppression, flexibility and safety is evaluated. The feasible ways of this approach improvement are suggested.

2. Scattered radiation suppression

The general principle for suppression of the scattered X-ray radiation is in the fact that the main part of the scattered X-ray radiation is non-collinear to the primary X-ray radiation. The most common methods using this idea are "air-gap variation", "slit-scheme" and anti-scatter grid.

To compare the methods of this section let's use "selectivity" (ξ) parameter [2]. This parameter is defined by relations between scattered and primary radiation with (δ') and without (δ) grid as follows: $\xi = \delta / \delta'$.

Anti-scatter grid

Anti-scatter grid is the most common solution of the scattered radiation problem. Anti-scatter grid is a grid of metal plates (for example lead) oriented to the focus of the X-ray source as shown on fig.2.

Anti-scatter grid's properties are mostly determined by grid's materials, focal distance F, grid frequency N, grid ratio r, plates' thickness D and gaps' thickness d (fig.2).

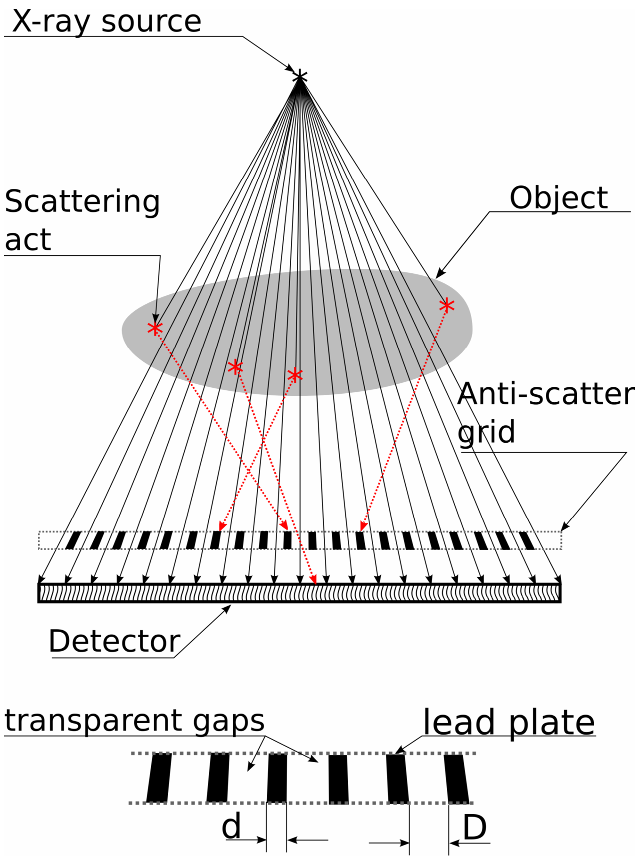

Figure 2 Anti-scatter grid scheme

Anti-scatter grid frequency N (the number of plates per 1 cm) and grid ratio r are defined as

$$N = \frac{1}{d + D}$$

$$r = \frac{h}{D}$$

All mentioned grid's parameters (especially grid ratio), X-ray apparatus configuration, X-ray apparatus and grid collimation (grid should be focused on the X-ray source focus) as while as studied object may influence grid transparency for both scattered and primary radiation. The transparency of the grid is determined by its transmission coefficients TP and TS for primary and scattered radiation appropriately. In terms of transmission coefficients, grid selectivity is equal to:

$$\xi = \frac{\delta}{\delta'} = \frac{T_P}{T_S}$$

Anti-scatter grid greatly suppresses scattered radiation but in the same time primary radiation is also suppressed. To describe how much radiation in total passed through the grid the transmission coefficient for total radiation (sum of the primary and the scattered radiation) is used. This parameter is highly depended on the studied object - in some cases the primary radiation prevails, in other cases scattered radiation prevails. The loss in intensity of radiation sensed by the detector should be compensated by increasing of X-ray source intensity that leads to proportional increasing of the absorbed dose. X-ray source intensity increasing or dose increasing is described by the Bucky factor: $B=1/T_t$.

Typical grid parameters for 90 keV X-ray tube and 20 cm acrylic phantom [3] are presented in the tab.1

Table 1

Typical grid parameters

r	T_S	T_P	ξ	T_t	B
6:1	0.194	0.623	3.21	0.310	3.23
12:1	0.078	0.533	6.83	0.197	5.07
16:1	0.050	0.506	10.12	0.154	6.50

Despite the strong scattered radiation suppression, anti-scatter grid has a few disadvantages, prompting in many cases to seek an alternative. The main disadvantages are as follows.

Application of anti-scatter grid (see Bucky factor in tab.1) leads to significant signal decrease and in turn to contrast and signal-to-noise decrease. Bucky factor compensation requires increasing of the X-ray exposure that leads to proportional increasing of the radiation dose absorbed by studied object, that is strongly unwanted in case of medical imaging.

Anti-scatter grid, just like the studied object, participate in image formation and resulting X-ray image contains the image of anti-scatter grid.

Anti-scatter grid requires very good collimation of the X-ray system. Focal points of the X-ray source and the anti-scattered grid should coincide, otherwise anti-scatter grid transparency deteriorates and artifacts occur.

Anti-scatter grid increases the mass and the size of the system that in some cases is strongly unwanted (for example in computed tomography, CT).

In case of latest CT scanners' generation, it is hard to construct the detector with anti-scatter grid (detector in such systems should be sensitive to X-rays in a wide angles' range)

Air-gap

Another way to reduce the amount of the scattered radiation in the detector plane is increasing the distance (air gap size) between the studied object and the detector. This scheme, presented on the fig.3, requires "point" X-ray source (with small focus). The method is based on the idea that the radiation intensity from such source will decrease inversely proportional to the square of the distance from the source.

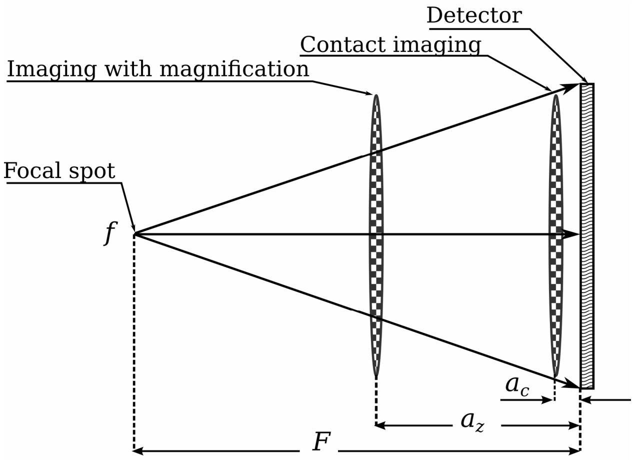

Figure 3 Air-gap scheme

In case of small "air gap" (contact imaging), relation between scattered and primary radiation is given by

$$\delta = C\left(\frac{F}{a_c}\right)^2,$$

where, C - proportionality factor, a_c - object-detector distance and F – focal spot – detector distance.

In case of greater "air gap" (imaging with magnification)

$$\delta' = C\left(\frac{F}{a_z}\right)^2,$$

where, a_z - object-detector distance.

Then, "air gap" selectivity is given by simple expression

$$\xi = \frac{\delta}{\delta'} = \left(\frac{a_z}{a_c}\right)^2$$

As it evident from the scheme, air gap leads to magnification of the object image that is defined by magnification coefficients as follows

$$m_c = \frac{F}{F-a_c}\,;\, m_z = \frac{F}{F-a_z}$$

Then selectivity in terms of magnification coefficients is

$$\xi = \left(\frac{(m_z - 1)m_c}{(m_c - 1)m_z} \right)^2$$

Assume "contact imaging" with small magnification m_c *=1.4*, then for greater "air gap" selectivity will depend on magnification as shown on fig.4

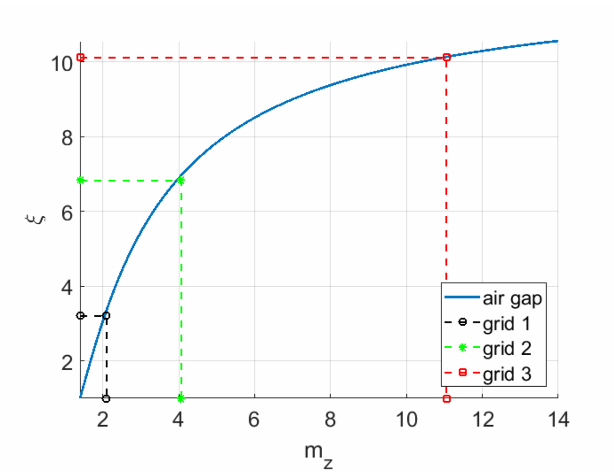

Figure 4 Air-gap selectivity as function of magnification

Comparing of "anti-scatter grid" and "air gap" methods it is easy to see that "air gap" with fivefold magnification and anti-scatter grid with ratio 16:1 have similar selectivity. It means that air gap technique with fivefold magnification filters scattered radiation as good as high-quality anti-scatter grid. In the same time the air gap method is deprived of the most anti-scatter grid disadvantages. There is no primary radiation suppression, and thus a dose is lower. There is no anti-scatter grid image presented on the resulting X-ray image. There is no need in precise collimation or anti-scatter grid (cheaper).

Nevertheless, there are few disadvantages present.

In most cases it is hard to make a large enough air gap (especially for CT). Due to magnification the detector size should be adjusted.

In general, the literature sources state that when it's possible to make a large enough air gap it gives better results than application of an anti-scatter grid. In chest radiography studies [4] air gap comparing to anti-scatter grid leads to increasing of contrast with the same dose for a posteroanterior projection and to decreasing of the dose with slightly decreased contrast in case of lateral projection. In cephalometric radiography studies [5] an air gap comparing to an anti-scatter grid leads to better selectivity and signal-to-noise ratio improvement. The dose was reduced more than twice.

Slit-scheme

The main idea of this method is to replace imaging in wide beam by scanning the studied object with highly collimated narrow spot beam (pencil beam), as shown on fig.5.

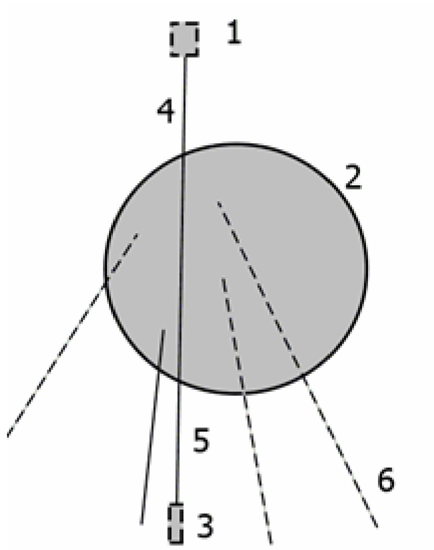

Figure 5 slit scheme: 1)X-ray source; 2) studied object; 3) detector; 4) incident ray; 5) primary ray; 6) scattered ray

Amount of scattered radiation is reduced proportionally to changes in irradiated area. Smaller irradiated area leads to less amount of scattering centers. With or without highly collimated detector, slit scheme dramatically suppresses scattered radiation and usually gives results that don't require further scattering corrections.

Despite the fact of good scattered radiation suppression, this method has serious disadvantages that strongly limit its application. Method requires a lot of time to make an image. It does not allow to scan moving objects (chest for example). Collimation causes a large thermal load on equipment.

3. Scattered radiation estimation and compensation

Instead of reducing the amount of the scattered radiation incident on the detector, methods of this category deal with the influence of the scattered radiation on the X-ray image after its registration. X-ray radiation sensed by the detector (*M*) is a sum of the primary radiation (*P*) and the scattered radiation (*S*). In case of known scattered radiation the primary radiation can be restored from the total radiation sensed by the detector using a simple subtraction (compensation) as follows: $P = M - S$.

Method of this group can be split into two subgroups: hardware and software methods. The main goal for both is to provide an estimation of the scattered radiation, but the way how they approach the result is different. The first one mostly relies on physical measurements of studied object. The later relies on numerical calculations.

Hardware methods

Hardware methods estimate scattered radiation by registering it during the experiment with the same X-ray apparatus and phantom similar to the studied object. Estimation may be even performed simultaneously with the main X-ray examination using several techniques: beam-stop schemes and dark field measurements.

Hardware methods of scattered radiation estimation could be used not only for estimation\compensation procedure but also for study or validation of suppression techniques [6], where air gap and anti-scatter grid effectiveness were studied and compared using beam stop technique.

Beam-stop

Schematically the beam stop scheme principle is shown on fig.6.

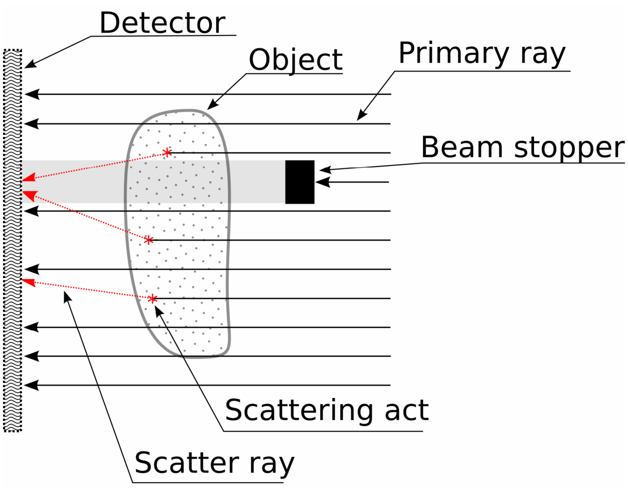

Figure 6 Beam stop scheme

The main idea is to place a stopper between the X-ray source and the studied object such way that stopper absorbs all the primary radiation. Thus the radiation detected in the area of stopper's projection shadow consists of the scattered radiation only. Stoppers array placed between the X-ray source and the studied object may be applied for estimation of scattered radiation distribution by interpolation. This technique could be used during the conventional X-ray examination but the resulting X-ray image will contain image of the stoppers. In order to avoid presence of stoppers on the resulting image it is necessary to perform multiple examinations: with and without stoppers or with different stoppers locations.

Despite the fact that each mentioned configuration has been admitted by their authors as a working solution, all they have the same problem: resulting X-ray image contains the image of beam stoppers. This problem can be solved by additional examination, as it was mentioned above. When a patient needs a series of examinations the images with beam stoppers can be scanned only once to save the resulting scatter radiation estimation in a library [7, 8]. For all next X-ray examinations, the scattered radiation is recovered from the library.

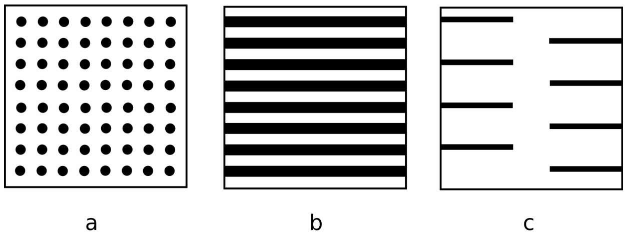

Figure 7 Beam stop arrays configurations

It may be concluded, that despite the fact of good scattered radiation estimation the beam stop technique has a few disadvantages.vElimination of beam stoppers presence in the resulting X-ray image requires dose increase. Array of beam stoppers make the X-ray apparatus more complicated and heavy that may cause problems with CT.

Dark field

Generally, X-ray apparatus is focused to examine some object's area by collimator. Collimator parts can be used as beam stopper in order to obtain areas with blocked primary radiation. Radiation detected in the collimator's shadow contains only the scattered radiation. This scheme, as shown on fig.8, requires a larger detector to register not only the object's region of interest but also the collimator shadow region.

This method works well only the in cases of homogenous objects with symmetry (breast examination for example) [9]. Such objects produce scatter radiation that can be restored with a good accuracy from data registered in the collimator shadow. Obviously in other cases the data sampled only on the borders of field of view is not enough for recovering of scatter radiation distribution.

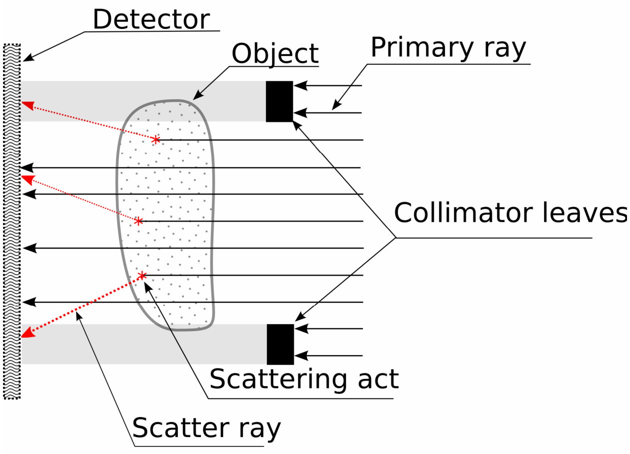

Figure 8 "Dark field" scheme

In contrast to the beam stop method this one doesn't offer dose increasing. Nevertheless, in most cases it provides significantly worse scatter radiation estimation accuracy and can be used only as first approximation for software methods.

Software methods

Software methods use information about the X-ray apparatus and the object (such as geometry, intensity, spectrum, kind of materials etc.) to make a prediction of the scatter radiation distribution. Analytical approximation methods are proposed to estimate scatter radiation analytically using numerous simplifications including simplified model of the object and simplified physics equations. Realistic simulation techniques were proposed to simulate interaction of the X-ray radiation using phantom similar to the studied object in order to obtain scatter radiation estimation. Scattering kernels superposition methods were proposed to split the studied object into sub-regions with quasi-homogenous scattering properties and describe the resulting scatter radiation as the algebraic sum of the scatter radiation given by each sub-region.

Analytical approximation

Usually analytical methods are applicable only to simple homogenous or heterogeneous smooth geometry objects. Even in such cases, the problem still remains too complex to find the exact solution and to simplify it. In the most works only the Compton process is accounted neglecting Rayleigh scattering as while as multiple scattering processes.

One of the simplest scatter radiation estimations is made for model presented on the fig.9 [10]. Object assumed to be an infinite plate of constant thickness t located on the distant g from the detector (g is the size of the air gap) and irradiated by mono-energetic "pencil" beam (beam with very small width that can be described by Dirac delta function) that falls normally to the surface of the object.

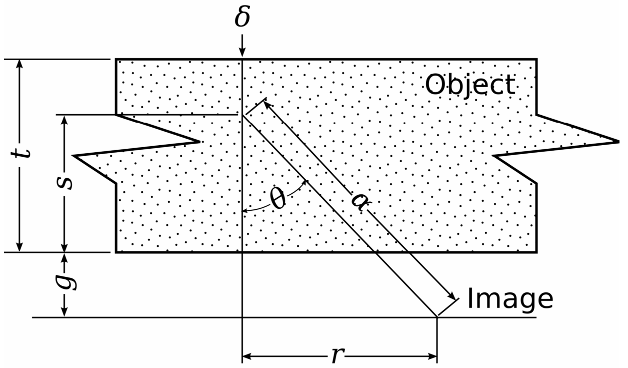

Figure 9 Simple homogenous model

After applying the simplified Klein-Nishina formula for Compton scattering (differential cross section) and Berr-Lambert law for primary radiation attenuation the scattered radiation estimator looks like expression (1), where, $\theta = tan^{-1}[r/(s+g)]$; $\alpha = [s/(s+g)]\ [r^2+(s+g)^2]^{1/2}$; μ_p, μ_s - effective linear attenuation coefficients of the primary beam and scattered radiation respectively; K is the experimental constant dependent on the object thickness, beam energy and objects material. This constant eliminates the dependence on the X-ray source intensity by normalization of this source intensity such that total area integral I_S become equal to the scatter-to-primary ratio $SPR = \int_{-\infty}^{\infty} I_S(r)dr$ for given object thickness and source spectrum. Value of SPR can be experimentally measured or numerically estimated [11].

The analytical model is closely correlated to Monte-Carlo simulation results with respect to mentioned constant K [11].

Significant improvements of this scheme were proposed in [12]. Authors introduce up to two materials (heterogeneous model) the more complicated but still smooth geometry (tested on the slabs and cylinder), poly-energetic spectrum of X-ray source.

Using Monte-Carlo simulation of X-ray interaction with simple phantoms, like slabs, cylinders with one layer of material (homogeneous case) or two layers of material (heterogeneous case) authors proved that this model predicts the shape of the scatter radiation distribution with high accuracy (from 2% error for mono-energetic case and up to 8% error in poly-energetic case).

Analytical models are useful for estimation of the Point Spread Functions or scattering kernels (distribution of the scatter radiation for irradiation of the homogeneous object by a "pencil" beam). Independent analytical methods are week because the studied object is strictly non-homogeneous object unknown density distribution.

$$I_S(r) = K\int_0^{\infty} \exp[-(t-s)]\exp(-\mu_S\alpha)(2\sin\theta - \sin^3\theta)\left(\frac{1}{s+g}\right)\left[1+\left(\frac{r}{s+g}\right)^2\right]dr \quad (1)$$

Realistic simulation

Simulation of X-rays interaction with realistic phantoms is the numerical solution of the problem of X-ray propagation through the nonhomogeneous medium including: X-rays tube, collimators, air, model of the studied object or phantom and the detector. The common simulation technique is the Monte-Carlo simulation.

Model consists of different objects described by attributes, like coordinates, phase, velocity, energy, object class etc.

Object class defines object attributes limits and possible interactions with other objects (for example, absorbing of the photon by the water phantom) or phenomenon like birth or radioactive decay that are defined by modelled physics.

Each process describes interaction between objects like photon birth inside the X-ray source, Compton scattering, photoelectric effect, etc. Processes may lead to some combination of changing of the interacted objects attributes (velocity and energy of the photon due to Compton scattering), death of interacted object or objects (absorbing of the photon during photoelectric effect), birth of some new objects (emission of the photon by X-ray source).

Result of each mentioned "process" are defined by the probability distribution, for example photon birth inside the X-ray source can be defined by Poisson distribution, it's energy distribution is defined by the spectrum of the X-ray source, it's initial coordinate and velocity distribution are defined by geometry and X-ray source emission indicatrix etc. The parameters' values are estimating through the randomization.

Time is discrete and computation is performed by time steps. At first all processes except motion are computed, changing of coordinates is computed after that.

In contrast to analytical models which are based on lots of simplifications simulation can theoretically "compute everything", and the quality of the scatter radiation prediction depends on amount of computer resources or simulation time. The more physics is taken into account for the simulation, the more reliable the result should be. Increasing of computational accuracy (decrease the time step, increase the "digital size" of attributes etc.) also should lead to better result, but may dramatically increase the computational time.

There are lots of programs for simulating interactions of the matter with radiation using Monte-Carlo method are available today. The most commonly used are Geant4 [13] and GATE [14] (Geant4 Application for Tomographic Emission) that encapsulates the Geant4 libraries in the simulation toolkit adapted to the field of nuclear medicine – CERN open source project that is being developed and maintained by an international collaboration.

EGS4 [15] (Electron Gamma Shower), originally developed at Stanford Linear Accelerator Centre and now is an open source project maintained by the National Research Council of Canada - EGSnrc [16].

ITS [17] is an open source project maintained by Sandia National Laboratories.

MCNP (stands for Monte Carlo N-Particle) [18] is a project of Los Alamos National Laboratory. It has common roots with ITS.

PENELOPE (Penetration and ENErgy LOss of Positrons and Electrons) [19] is an open source project.

PARTRAC (PARticle TRACks) [20, 21].

LEPTS (Low-Energy Particle Track Simulation, [22]) has been integrated into the Geant4 [23]

EPOTRAN (Electron and POsitron TRANsport) [24].

Mentioned programs are compared in [25]. They mostly differ by physical models (models of interactions), databases (their origin and nature), output data format and user interface. Two projects that most frequently advised are: Geant4 - free open source project that requires some programming skills; MCNP – more user-friendly then previous one but has some using restrictions.

Simulation can recreate the scatter radiation obtained during the X-ray examination. The accuracy depends on how the simulated object's structure is close to the studied object. Object structure, in some cases, could be well predicted from additional information like kind of object (breast, head, chest etc.), geometrical sizes and shape, anamnesis, CT or MRI images etc. In this case studied object is simulated in accordance with its structure prediction and then result of the simulation is used for compensation procedure. In the case of breast imaging scatter radiation could be estimated by Monte Carlo simulation with a good accuracy [26]. In [27] authors proposed to use the result of planning CT for prediction of the studied object structure (to construct the object model) and shown that Monte Carlo simulation with such object's model gives good results by adequate time that was achieved by GPU parallelization. When the accuracy of the object's structure prediction is not enough it is possible to construct iterative procedure, like one presented on the fig.9, which simply updates the object's structure prediction in order to minimize the difference between X-ray images obtained during the simulation and real X-ray examination. The efficiency of Monte Carlo based iterative procedure that resembles the scheme shown on the fig. 10 has been demonstrated [28, 29].

Monte Carlo simulation is efficient instrument used, mostly, in iterative schemes to improve the results of CT imaging. It gives good estimation for scatter radiation and doesn't require additional scanning or dose increasing but in the same time may require a lot of computational resources (a lot of time for providing a series of MC simulations). Using of GPU parallelization with different strategies such as projection de-noising, CT image down-sampling, and interpolation along the angular direction may improve the quality to processing time ratio [27]. In planar X-ray imaging (radiography), in contrast to CT, Monte Carlo simulation may be less efficient due to the lack of information about the studied object structure (it is hard to reconstruct the object from only one projection).

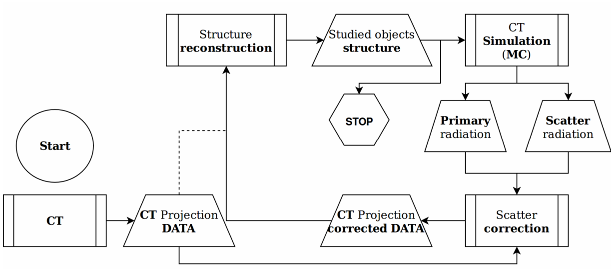

Figure 10 Monte Carlo based iterative scheme for scatter compensation

Image oriented methods

In many cases the scatter radiation could be represented as low-frequency smooth signal. Thus it can be obtained by filtering of the X-ray images using standard methods of image filtration, for example high-pass filtering [30] or segmentation. For homogenous simple objects filtering may be useful but in case of most medical applications the object is strongly non-homogeneous and these techniques could lose the information about small objects decreasing the resolution.

Deconvolution could be improved by taking into account more physical aspects – for example, the convolution kernels could be computed from analytical considerations or Monte Carlo simulation. In [31] scatter radiation assumed to be related with registered radiation by Gauss-like function composed using one of the analytical models with a few free parameters that are approximated through the X-ray image segmentation.

Further improvement of the deconvolution technique is called as "scattering kernels superposition". The main idea here is to represent the scatter radiation S as linear combination of the primary radiation P and scattering kernels K (point spread functions).

$$S(x,y) = \iint P(\tilde{x},\tilde{y}) \cdot K_{\tilde{x},\tilde{y}}(x-\tilde{x}, y-\tilde{y}) d\tilde{x} d\tilde{y}$$

Scattering kernel $K_{\tilde{x},\tilde{y}}(x-\tilde{x}, y-\tilde{y})$ for given image point $\tilde{x},\tilde{y}$ is the scatter radiation registered by the detector while irradiating the object by a pencil beam at corresponding point:

$$P(\tilde{x},\tilde{y}) = \delta(\tilde{x}-X, \tilde{y}-Y)$$

$$S(x,y) = K_{X,Y}(x-X, y-Y)$$

In case of homogeneous object, where each point of the image could be described by the same scattering kernel the linear combination is transformed into the convolution:

$$K_{\tilde{x},\tilde{y}}(x-\tilde{x}, y-\tilde{y}) = K_0(x-\tilde{x}, y-\tilde{y})$$

$$S(x,y) = \{P * K_0\}(x,y)$$

In general case, studied object can be approximated by series of quasi-homogeneous regions i and scatter radiation is approximated as follow:

$$S(x,y) = \sum_i \{P_i * K_i\}(x,y)$$

Finally, the radiation registered by the detector M is a sum of the primary radiation and the scattered radiation:

$$M(x,y) = P(x,y) + \sum_i \{P_i * K_i\}(x,y)$$

Knowing the partitioning of the object into the quasi-homogeneous regions and corresponding scattering kernels the primary radiation can be recovered iteratively or by deconvolution [32].

Scattering kernels strongly depend on X-ray source spectrum, kind of matter distributed inside the object, it's structure and geometrical parameters. etc. In [32] partitioning of the object into the quasi-homogeneous regions and scattering kernels construction has been proposed to be done by support vector machine from the results of the Monte Carlo simulation. Interactions of the X-rays with phantoms similar to the studied object are simulated with different observation parameters (angles, air-gap, spectrum etc.) and different object parameters (thickness, width etc.). Scattering kernels are obtained by simulation of the object irradiation at specific point by a pencil beam. Such image is the sum of primary and scattered radiation. Optionally obtained library of scattered radiation kernels may be shrunk (keep only most characteristic ones) through the clustering analysis. Support vector machine is trained to identify which scattering kernel from the library should be applied to a specific point of the X-ray image.

A convolutional neuronal network could be used for the purposes [33].

Compared to other methods that requires a Monte Carlo simulation scattering kernels superposition require the simulation to be performed once beforehand for construction of the library and to train the classifier. The scatter radiation compensation is faster in the second case, but at the cost of accuracy.

Conclusions

The problem of the scatter radiation influence on the X-ray images is still relevant, especially in medical context. To resolve this problem suppression techniques are used, such as anti-scatter grid with air-gapping. Suppression techniques greatly raise the image contrast suppressing significant part of the scattered radiation on the detector but in cost of radiation dose increasing. Modern medicine believes in "smaller the dose - healthier the patient" thesis (ALARA or As Low As ReAsonable principle). Moreover, some X-ray apparatus configurations greatly complicate the use of suppression methods (especially in CT, where large air gaps and heavy collimators and anti-scatter grids are unwanted). In contrast to suppression techniques, scatter estimation\compensation techniques could help to lower the dose and to avoid the mentioned hardware incompatibility problems. Among all of the solutions, scattering kernels superposition enhanced by neural network and Monte Carlo based iterative procedure seems to be the most efficient methods – the first method works faster but in general is expected to be less accurate then second, but second method requires lots of computing resources and so is slower than first one.

Until present many methods are already have been proposed and validated but some promising ones are still being actively developed by commercial organizations and therefore they are the commercial secrets that actualizes further research in this field.

It may be concluded that now the most promising approaches for scatter compensation should include Monte-Carlo based simulation as while as modern data analysis techniques like neural networks. This combination provides the possibility to take into account as much known physical properties as possible and to predict the unknown physical properties like underline object structure influence of noise, etc.

Список використаних джерел

[1] *Monnin P.* A comprehensive model for x-ray projection imaging system efficiency and image quality characterization in the presence of scattered radiation. / P. Monnin, F. Verdun, H. Bosmans, S. R. Perez, N. Marshall // Physics in Medicine and Biology. – 2017. – 62(14). – P. 5691–5722.

[2] *Александр М.* Влияние рассеянного рентгеновского излучения на качество изображения и методы его подавления. / М. Александр, П. Николай // Биотехносфера. – 2012. –(3-4 (21-22)). – P. 10-14.

References

[1] MONNIN, P. et al. (2017) A comprehensive model for x-ray projection imaging system efficiency and image quality characterization in the presence of scattered radiation. *Physics in Medicine and Biology.* 62(14). p.5691–5722. https://doi.org/10.1088/1361-6560/aa75bc

[2] ALEKSANDR, M. & NIKOLAY, P. (2012) Vliyaniye rasseyannogo rentgenovskogo izlucheniya na kachestvo izobrazheniya i metody ego podavleniya. *Biotekhnosfera.* (3-4 (21-22)). p.10-14.