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IMPROVEMENT OF CHARACTERIZATION INEQUALITY FOR NORMS OF φ-SUB-GAUSSIAN RANDOM VARIABLE

The paper is focused on refining the characterization inequality for the norm of φ-sub-Gaussian random variables. The core of this improvement lies in a detailed analysis of the remainder terms in Stirling's formula, which provides an approximation for $n!$. This study conducts a comparative analysis of several pairs of known remainder terms, α_n and β_n, to identify the pair that minimizes the approximation error. By calculating the absolute difference for these pairs, we demonstrate that the tightest bounds are achieved with a specific selection of higher-order terms. This optimized approximation for $n!$ is then applied to sharpen the constant in a characterization inequality for comparing different norms of a φ-sub-Gaussian random variable and also in the theorem on φ-sub-Gaussianity of a random variable.

K e y w o r d s: ***Stirling's formula, remainder, characterization inequality, norm, φ-sub-Gaussian random variable.***

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Introduction

Stirling's formula is a cornerstone of mathematical analysis, offering a powerful approximation for the factorial function, n!. The classical representation of this formula, known since the 18th century, states that as n approaches infinity, the ratio of $n!$ to $\sqrt{2\pi n}(n/e)^n$ converges to 1. This approximation is indispensable in various fields, including probability theory, statistics, and physics, where direct computation of factorials for large n is impractical.

While the limit-based formula is powerful, its practical application often requires more precise error bounds. Consequently, a significant body of research has been dedicated to refining Stirling's original approximation. The primary objective was to conduct a comparative analysis of several of these well-established remainder terms. The goal is to identify the specific pair of α_n and β_n that yields the least approximation error among the considered options.

The refined result is then applied to sharpen the characterization inequality for the norm of φ-sub-Gaussian random variables. These variables are a generalization of Gaussian random variables and have been widely researched and described in monographs (Buldygin, & Kozachenko, 2000; Vasylyk, Kozachenko, & Yamnenko, 2008).

1. Stirling's Formula Remainders

Since Stirling's time (1764), it has been known that $n! \sim C\sqrt{n}(n/e)^n$, with $C = \sqrt{2\pi}$, that is

$$\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}(n/e)^n} = 1. \tag{1}$$

Formula (1) is known as Stirling's formula, and many elementary proofs of it have been given. Most such proofs use Wallis' formula

$$\lim_{n\to\infty} \frac{2\cdot 2\cdot 4\cdot 4\cdot\dots\cdot 2n\cdot 2n}{1\cdot 3\cdot 3\cdot 5\cdot\dots\cdot(2n-1)(2n+1)} = \frac{\pi}{2}, \tag{2}$$

to show that $C = \sqrt{2\pi}$. However, one of the elementary proofs by W. Feller (Feller, 1967) avoids any appeal to Wallis' formula. Since completely elementary proofs of (2) are well known (see, for example, (Schwartz, 1967)), this does not seem to be essential. In addition, a number of upper and lower bounds for $n!$, most of which imply (1), have been obtained by various authors. One of the most elementary estimates $e^{11/12}\sqrt{n}(n/e)^n < n! < e\sqrt{n}(n/e)^n$ was obtained by Hummel (Hummel, 1940) in 1940. Most bounds are of the form

$$\sqrt{2\pi n}(n/e)^n e^{\alpha_n} < n! < \sqrt{2\pi n}(n/e)^n e^{\beta_n}, \tag{3}$$

where α_n and β_n tend to zero through positive values. For example, $\beta_n = (12n)^{-1}$ was proved in (Uspensky, 1937), while the successively better values

$$\alpha_n^{(1)} = \frac{1}{\left(12n + \frac{3}{2(2n+1)}\right)}, \quad \alpha_n^{(2)} = \frac{1}{12n} - \frac{1}{360n^3}, \quad \alpha_n^{(3)} = \frac{1}{12n} - \frac{1}{360n^3} + \frac{1}{\left(1260(1+\frac{2}{n(n+1)})n^5\right)}, \tag{4}$$

$$\begin{aligned}\beta_n^{(1)} &= \frac{1}{12n} - \frac{1}{((360+\gamma_n)n^3)}, \quad \text{where } \gamma_n = 30\frac{7n(n+1)+1}{n^2(n+1)^2},\\ \beta_n^{(2)} &= \frac{1}{12n} - \frac{1}{(360n(n+1)(n+2))},\\ \beta_n^{(3)} &= \frac{1}{12n} - \frac{1}{360n^3} + \frac{1}{1260n^5}\end{aligned} \tag{5}$$

were obtained in (Maria, 1965; Nanjundiah, 1965; Beesack, 1969; Hu, Lui, & Shi, 2006; Long, 2014; Macyz, 2021). As proofs for the mentioned remainders are given – we focus on choosing the left and right remainders that will give us the least $n!$ approximation error among those listed.

To do this, we need to calculate the possible error for each pair of left and right remainders. In common sense, it looks like $\frac{e^{\alpha_n} - e^{\beta_n}}{e^{\beta_n}} = e^{\alpha_n - \beta_n} - 1$. And for an equation like this – we can use the fact that for small x, $e^x - 1 \approx x$.

Let's calculate the first pair of α and β with γ_n defined in (5).

$$\beta_n^{(1)} = \frac{1}{12n} - \frac{1}{(360+\gamma_n)n^3} = \frac{1}{12n} - \frac{1}{(360 + \frac{210n^2+210n+30}{(n^2+n)^2})n^3} = \frac{33n^3+91n^2+123n+60}{60n(7n^3+19n^2+25n+12)}. \tag{6}$$

$$\alpha_n^{(1)} = \frac{1}{\left(12n + \frac{3}{2(2n+1)}\right)} = \frac{4n+2}{48n^2+24n+3}. \tag{7}$$

With the equations calculated above (6) and (7), we can compute the absolute difference between α and β.

$$\begin{aligned}\left|\alpha_n^{(1)} - \beta_n^{(1)}\right| &= \left|\frac{4n+2}{48n^2+24n+3} - \left(\frac{1}{12n} - \frac{1}{(360+\gamma_n)n^3}\right)\right|\\ &= \left|\frac{32n^5+80n^4+31n^3-75n^2-123n-60}{60n(112n^5+360n^4+559n^3+411n^2+121n+12)}\right|.\end{aligned} \tag{8}$$

Now let's do the same for the next two pairs of α and β from (4) and (5) respectively.

$$\left|\alpha_n^{(2)} - \beta_n^{(2)}\right| = \left|\frac{1}{12n} - \frac{1}{360n^3} - \left(\frac{1}{12n} - \frac{1}{360n(n+1)(n+2)}\right)\right| = \left|\frac{n^2-(n+1)(n+2)}{360n^3(n+1)(n+2)}\right| = \frac{3n+2}{360n^3(n+1)(n+2)}, \tag{9}$$

$$\left|\alpha_n^{(3)} - \beta_n^{(3)}\right| = \left|\frac{1}{1260\left(\frac{n^2+n+2}{n(n+1)}\right)n^5} - \frac{1}{1260n^5}\right| = \left|\frac{n^2+n-(n^2+n+2)}{1260(n^2+n+2)n^5}\right| = \frac{2}{1260(n^2+n+2)n^5}. \tag{10}$$

As we consider $n \geqslant 2$, it's easy to see that α and β achieve their supremum with $n=2$. So, to calculate the least possible error – let's substitute it into equations (8), (9), (10):

$$\begin{aligned}\left|\alpha_n^{(1)} - \beta_n^{(1)}\right| &= \frac{1946}{120\cdot 15714} \approx 1.03e^{-3},\\ \left|\alpha_n^{(2)} - \beta_n^{(2)}\right| &= \frac{3\cdot 2+2}{360\cdot 8\cdot 3\cdot 4} \approx 2.3e^{-4},\\ \left|\alpha_n^{(3)} - \beta_n^{(3)}\right| &= \frac{2}{1260\cdot(4+2+2)\cdot 32} \approx 6.2e^{-6}.\end{aligned} \tag{11}$$

As we convinced earlier, $\alpha_n^{(3)}$ and $\beta_n^{(3)}$ from (4) and (5) respectively give us the least error (11). That is why we will be using it further. Now let's assume that $n=2$.

Then

$$\beta_n^{(3)} = \frac{1}{12\cdot 2} - \frac{1}{360\cdot 8} + \frac{1}{1260\cdot 32} = \frac{1680-14+1}{40320} = \frac{1667}{40320}$$

and

$$\alpha_n^{(3)} = \frac{1}{12\cdot 2} - \frac{1}{360\cdot 8} + \frac{2\cdot 3}{1260\cdot(2\cdot 3+2)\cdot 32} = \frac{13334}{322560} = \frac{6667}{161280}. \tag{12}$$

2. Improvements of inequality for norm of space of φ-sub-Gaussian random variables

φ-sub-Gaussian random variables, spaces, and their properties were introduced in (Giuliano-Antonini, Kozachenko, & Nikitina, 2003). We will try to improve the accuracy of the inequality for the norms of the space of φ-sub-Gaussian random variables, referring to the lemmas from there.

Lemma 1. *Let $\xi \in Sub_\varphi(\Omega)$. Then for every $a > 0$ the next inequality holds true:*

$$\mathsf{E}\,|\xi|^a \leqslant 2\left(\frac{a}{e}\right)^a (\tau_\varphi(\xi))^a \inf_{t>0} \exp\left(\varphi(t - a\ln t)\right) \leqslant 2\,(\tau_\varphi(\xi))^a \left(\frac{a}{\varphi^{-1}(a)}\right)^a.$$

Then, from 1, we can obtain the following result.

Corollary 1. *Let $\xi \in Sub_\varphi(\Omega)$. Then the next inequality holds true*

$$\tau_\varphi(\xi) \geqslant \theta_\varphi(\xi)/\sqrt{2},$$

where $\theta_\varphi(\xi) = \sup\limits_{n\geqslant 2} \mathsf{E}\left(|\xi|^n\right)^{\frac{1}{n}} \frac{\varphi^{-1}(n)}{n}$, and $\theta_\varphi(\xi)$ is a norm for $Sub_\varphi(\Omega)$.

Lemma 2. *Let $\xi \in Sub_\varphi(\Omega)$. Then for $k = 1, 2, \ldots$ the following inequalities hold true:*

$$\left|\mathsf{E}\,\xi^k\right| \leqslant \mathsf{E}\,|\xi|^k \leqslant 2\left(\tau_\varphi(\xi)\right)^k \frac{e^k}{\left(\varphi^{-1}(k)\right)^k} k!.$$

Corollary 2. *If $\xi \in Sub_\varphi(\Omega)$, then the next inequality holds true*

$$\tau_\varphi(\xi) \geqslant \frac{1}{e\sqrt{2}} \nu_\varphi(\xi),$$

where $\nu_\varphi(\xi) = \sup_{n\geqslant 2} |\mathsf{E}\,\xi^n|^{\frac{1}{n}} \frac{\varphi^{-1}(n)}{(n!)^{\frac{1}{n}}}$.

Norms $\theta_\varphi(\cdot)$ and $\nu_\varphi(\cdot)$ are less popular than $\tau_\varphi(\cdot)$, yet they are used in applications of φ-sub-Gaussian processes, see, for example (Troshki, 2017).

Lemma 3. *Let $\xi \in Sub_\varphi(\Omega)$ and $\nu_\varphi(\xi) \geqslant \theta_\varphi(\xi)$. Then the next inequality holds true:*

$$\nu_\varphi(\xi) \leqslant e^{\frac{329227}{322560}} \theta_\varphi(\xi). \tag{13}$$

Proof. We can see that this inequality holds true according to proposition 1, lemma 3, and Stirling's formula (3), stating that $n! = n^n e^{-n} (2\pi n)^{1/2} e^{\theta_n}$ for $n \geqslant 2$, where $|\theta_n| \leqslant \frac{1}{12n} - \frac{1}{360n^3} + \frac{1}{\left(1260(1+\frac{2}{n(n+1)})n^5\right)}$. Using the value calculated in (12), we derive the following inequality

$$(n!)^{-\frac{1}{n}} = \frac{1}{n} \frac{e^{1+\frac{\theta_n}{n}}}{(2\pi n)^{\frac{1}{2n}}} \leqslant \frac{1}{n} e^{1+\frac{\theta_n}{n}} = \frac{1}{n} e^{\frac{329227}{322560}}.$$

□

Lemma 3 improves the following result from (Giuliano-Antonini, Kozachenko, & Nikitina, 2003, Corollary 4.4):

$$\nu_\varphi(\xi) \leqslant \exp\left\{\frac{49}{48}\right\} \theta_\varphi(\xi).$$

Lemma 4. *Let ξ be a random variable with $\mathsf{E}\,\xi = 0$, and let φ be a N-function satisfying the condition Q. Let $\lambda_0 > 0$ be some number and*

$$c_0 = \inf_{0<|\lambda|\leqslant\lambda_0} \varphi(\lambda)/\lambda^2.$$

Let also $\nu_\varphi(\xi) < \infty$. Then $\xi \in \mathrm{Sub}_\varphi(\Omega)$ and the following inequality holds:

$$\tau_\varphi(\xi) \leqslant S_\varphi \nu_\varphi(\xi), \tag{14}$$

where $S_\varphi = \max\limits_{i=\overline{1,3}} \gamma_i^{-1}$, and γ_1 is the root of the equation $\gamma = \lambda_0\sqrt{c_0(1-\gamma)}$, γ_2 is the root of the equation $\gamma^3 - 2(1-\gamma) = 0$, and γ_3 is the root of the equation $\gamma = \varphi^{-1}(2)\sqrt{c_0(1-\gamma)}$.

The proof can be found in (Giuliano-Antonini, Kozachenko, & Nikitina, 2003).

Also, note that Lemmas 3 and 4 establish the equivalence of the norms $\nu_\varphi(\xi)$ and $\tau_\varphi(\xi)$. Furthermore, a random variable ξ belongs to $Sub_\varphi(\Omega)$ if and only if $\mathsf{E}\,\xi = 0$ and $\nu_\varphi(\xi) < \infty$.

3. Orlicz spaces of exponential type

Let ψ be an arbitrary N-function. The Orlicz space generated by the N-function $U(x) = \{\exp\{\psi(x)\} - 1, x \in \mathbb{R}$, is called an Orlicz space of exponential type. We shall be interested in the Orlicz space of exponential type $Exp_{\varphi^*}(\Omega)$ generated by the Young-Fenchel transform φ^* of an N-function φ. The Luxemburg norm in $Exp_{\varphi^*}(\Omega)$ is denoted by σ_{φ^*}, i.e. for a random variable $\xi \in Exp_{\varphi^*}(\Omega)$ we have

$$\sigma_{\varphi^*}(\xi) = \inf\left\{a > 0\colon\ \mathsf{E}\left\{\exp \varphi^*(\xi/a)\right\} \leqslant 2\right\}.$$

The space $Exp_{\varphi^*}(\Omega)$ is a Banach space with respect to the norm $\sigma_{\varphi^*}(\cdot)$.

Lemma 5. *Let φ be an N-function and $\xi \in Exp_{\varphi^*}(\Omega)$. Then, for any $p \geqslant 1$, the following inequality holds:*

$$\left(\mathsf{E}\,|\xi|^p\right)^{\frac{1}{p}} \leqslant 2^{\frac{1}{p}} \frac{p}{\varphi^{-1}(p)} \sigma_{\varphi^*}(\xi). \tag{15}$$

The proof can be found in (Giuliano-Antonini, Kozachenko, & Nikitina, 2003).

As a consequence of (13), the following inequalities are also improved.

Lemma 6. *Let φ be a N-function satisfying the condition Q, and let ξ be a random variable such that $\xi \in$ Sub$_\varphi(\Omega)$ and* $\mathsf{E}\,\xi = 0$. *Then*

$$\tau_\varphi(\xi) \leqslant S_\varphi e^{\frac{329227}{322560}} \sigma_{\varphi^*}(\xi),$$

where S_φ is defined in (14).

Lemma 7. *Let ξ be a random variable such that*

$$P\{|\xi| \geqslant x\} \leqslant C \exp\left\{-\psi\left(\frac{x}{D}\right)\right\},$$

where $\psi(x)$ is an N-function. Then $\xi \in$ Exp$_\psi(\Omega)$ and $\sigma_\psi(\xi) \leqslant (1+C)D$.

Lemma 8. *Let $\xi \in$* Sub$_\varphi(\Omega)$*; then $\xi \in Exp_{\varphi^*}(\Omega)$ and the following inequality holds:*

$$\sigma_{\psi^*}(\xi) \leqslant 3\tau_\varphi(\xi). \tag{16}$$

Considering the lemmas, propositions, and equation (12) mentioned above, we can arrive at the following theorem.

Theorem 1. *A random variable ξ belongs to the space Sub$_\varphi(\Omega)$ only if* $\mathsf{E}\,\xi = 0$ *and there exist constants $C > 0$ and $D > 0$ such that*

$$P\{|\xi| \geqslant x\} \leqslant C \exp\left\{-\varphi^*\left(\frac{x}{D}\right)\right\} \tag{17}$$

for any $x > 0$. If (17) *holds, then*

$$\tau_\varphi(\xi) \leqslant S_\varphi e^{\frac{329227}{322560}} (1+C)D,$$

where S_φ is defined in (14).

Example. Consider the Orlicz N-function $\varphi_p(x) = \frac{|x|^p}{p}$, $1 < p < 2$, or, for $p > 2$, the piecewise definition $\varphi_p(x) = \frac{|x|^p}{p}$, $|x| \geqslant 1$, and $\varphi_p(x) = \frac{|x|^2}{p}$, $|x| < 1$, and examine inequality (16) in this particular setting.

Put $\tau_{\varphi_p}(\xi) = \tau$. Let's apply Lemma 1. Since for $s > 1$

$$\inf_{t \geqslant 1} \exp\{\varphi_p(t) - s \ln t\} = \left(\frac{e}{s}\right)^{\frac{s}{p}},$$

then

$$\mathsf{E}\,|\xi|^s \leqslant 2\left(\frac{s}{e}\right)^{\frac{s}{q}} \tau^s. \tag{18}$$

Since $\varphi_p^*(x) = \frac{|x|^q}{q}$ for the case $1 < p < 2$, and $\varphi_p^*(x) \leqslant \frac{|x|^q}{q}$ (as $q < 2$) we have by (18) for $a > 0$

$$\mathsf{E}\exp\{\varphi^*(\xi/a)\} \leqslant \mathsf{E}\exp\left\{\frac{|\xi|^q}{qa^q}\right\} = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\mathsf{E}\,|\xi|^k q}{(qa^q)^k} \leqslant 1 + 2\sum_{k=1}^{\infty}\left(\frac{k^k}{k!e^k}\right)\left(\frac{\tau}{a}\right)^q k. \tag{19}$$

It follows from Stirling's formula and (12) that

$$\frac{k^k}{k!e^k} \leqslant \frac{1}{\sqrt{2\pi}} e^{\frac{13334}{322560}}.$$

Therefore

$$\mathsf{E}\exp\left\{\varphi^*\left(\frac{\xi}{a}\right)\right\} \leqslant 1 + \frac{2e^{\frac{13334}{322560}}}{\sqrt{2\pi}} \sum_{k=1}^{\infty}\left(\frac{\tau}{a}\right)^{kq}.$$

The above geometric series converges if $\tau/a < 1$ and in such case its sum is equal to

$$1 + \frac{2e^{\frac{13334}{322560}}}{\sqrt{2\pi}} \frac{(\tau/a)^q}{1-(\tau/a)^q}.$$

Put

$$L = \left(\frac{2e^{\frac{13334}{322560}}}{\sqrt{2\pi}} + 1\right)^{1/q},$$

then the last quantity in (19) is not greater than 2 if $a/\tau > L > 1$. This gives us the following result

$$\sigma_{\varphi^*}(\xi) \leqslant L\tau_{\varphi_p}(\xi).$$

Discussions and conclusion

By refining Stirling's formula through an optimal choice of remainder terms, we obtained sharper constants in the characterization inequality for norms of φ-sub-Gaussian random variables. This illustrates how classical analytic improvements translate into tighter probabilistic bounds, with potential applications to concentration inequalities and stochastic modeling.

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УТОЧНЕННЯ ХАРАКТЕРИЗАЦІЙНОЇ НЕРІВНОСТІ ДЛЯ НОРМ φ-СУБГАУССОВОЇ ВИПАДКОВОЇ ВЕЛИЧИНИ

Здійснено уточнення характеризаційної нерівності між нормами φ-субгауссової випадкової величини. Суть цього вдосконалення полягає в більш детальному аналізі залишкових членів у формулі Стірлінга, що використовується для підрахунку наближеного значення для $n!$. У пропонованому дослідженні проведено порівняльний аналіз декількох пар відомих залишкових членів, α_n і β_n, щоб визначити пару, яка мінімізує похибку наближення. Обчислюючи абсолютну різницю для цих пар, продемонстровано, що найтісніші межі досягаються у разі конкретного вибору членів вищого порядку. Це поліпшене наближення для $n!$ потім застосовується для уточнення сталої в характеризаційній нерівності для порівняння різних норм φ-субгауссової випадкової величини та теоремі про φ-субгауссовість випадкової величини.

Ключові слова: ***формула Стірлінга, залишок, характеризаційна нерівність, норма, φ-субгауссова випадкова величина.***

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