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Yaroslav ZHUK, DSc (Phys. & Math.), Prof.,  
Corresponding Member of the National Academy of Sciences of Ukraine  
ORCID ID: 0000-0002-2726-8395  
e-mail: yaroslavzhuk@knu.ua  
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Arash Soleiman FALLAH, PhD, Prof.  
ORCID ID: 0000-0002-0382-633X  
e-mail: arashsol@oslomet.no  
Oslo Metropolitan University, Oslo, Norway

## TECHNOLOGY FOR LASER THERMOFORMING OF THIN-WALLED METAL STRUCTURAL ELEMENTS

*The article investigates the thermoforming of thin-walled metal structural elements using pulsed laser irradiation with a physically nonlinear thermoviscoplastic finite element model. The research focuses on the deformation mechanisms caused by transient thermal loads and their impact on the geometric accuracy of the molded parts. Simulation shows that pulsed laser irradiation creates localized thermal stresses and residual plastic deformations, which in turn cause controlled bending of thin metal sheets without the need for external dies or mechanical tools.*

*Numerical analysis demonstrates that the temperature-dependent viscoplastic properties of alloys play a decisive role in determining the efficiency of deformation and the stability of the resulting geometry. This article proposes a numerical model for simulating the laser thermoforming (LTF) process. It is based on the thermodynamically consistent theory of coupled thermoviscoplasticity. It is shown that it is suitable for modeling LTF for thin-walled metal structural elements. Within this model, the problem formulation consists of the Cauchy equation, the equations of motion, and the heat conduction equation, as well as mechanical and thermal boundary conditions and initial conditions. A generalized model of physically nonlinear temperature-dependent thermoviscoplasticity is used to describe the behavior of the material. Spatial discretization of the axisymmetric problem of laser pulse loading of the disk is performed using the finite element method. The unsteady process of LTF of the deformed disk configuration is modeled. The final profile of the disk is obtained as a result of the thermally induced state of residual stresses and deformations caused by rapid heating and subsequent gradual cooling of the material in the laser irradiation area. The results confirm that laser thermoforming allows localized shaping of thin-walled parts with reduced material waste, lower energy consumption, and higher flexibility compared to conventional forming methods. In particular, the modeling results align with experimental trends reported in the literature, where bending angle control, curvature reversal, and multi-pass strategies are feasible for both aerospace-scale panels and micro-electromechanical system (MEMS) components.*

**Keywords:** laser thermoforming, thin-wall metal elements, thermoviscoplasticity theory, thermomechanical model, finite element simulation, residual stress-strain state.

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### Introduction

The demand for lightweight yet high-strength structural elements in mechanical engineering, aerospace, automotive, and microsystems industries continues to grow due to the superior performance of metallic materials such as aluminum, titanium, and their alloys. Traditional forming processes - deep drawing, hot stamping, and superplastic forming - remain widely used for manufacturing thin-walled components of complex geometry. However, these conventional methods suffer from several drawbacks, especially when processing materials with high elasticity and strength, such as titanium alloys (Hu et al., 2016; Langer, Spradlin, & Fitzpatrick, 2016). Laser-based processing offers an alternative tool-free approach that relies on localized, non-contact heating to achieve shape modification of metallic sheets. Among these methods, laser thermoforming (LTF) has proven particularly attractive for the precise bending and shaping of thin-walled elements. By controlled irradiation with pulsed or continuous laser beams, residual thermal stresses are generated, enabling permanent deformation without external forces or dies (Ocaña et al., 2011). Compared to conventional processes, LTF provides high design flexibility, low-cost adaptability, and the possibility to process materials that are otherwise difficult to form mechanically, such as nickel-based superalloys and refractory metals (Hu et al., 2016).

Recent advances in experimental and numerical studies have demonstrated that LTF enables smooth curvature transitions, reversal of bending direction, and multi-scale applications from large aerospace panels to micro-electromechanical systems (MEMS) (Ocaña et al., 2011). Numerical modeling based on coupled thermo-viscoplastic formulations has also become an indispensable tool for predicting deformation, stress distribution, and microstructural changes during pulsed heating and subsequent cooling (Wakchaure, Misra, & Menezes, 2024; Langer, Spradlin, & Fitzpatrick, 2020). In this paper, the interaction between a thermal pulse and a structural element is studied, making use of a dynamic problem statement involving a generalized model of physically nonlinear behavior of materials in a wide temperature range consistent with the thermodynamics of irreversible processes.

### 1. Problem statement

The formulation of the problem of irradiation of a metal structural element by a short-term thermal pulse based on a coupled model of thermo-inelastic behavior was discussed in the literature (Hu et al., 2016; Ocaña et al., 2011; Zhuk, Senchenkov, & Boichuk, 2009). In this paper, the axisymmetric version of the problem statement is used. Below, the equations of the thermo-viscoplastic model of material behavior, along with the problem equations, are briefly presented. Then, the formulation of the initial and boundary conditions usually used in modeling the LTF process is also listed to complete the problem statement.

Let a metal disk of radius and thickness  $h$  be considered. Its geometry in the cylindrical coordinate system  $Orz\phi$  is given as follows:  $|r| \leq R$ ,  $0 \leq |z| \leq h$ .

The formulation of the dynamic axisymmetric coupled problem of thermo-viscoplasticity consists of the equations of kinematics and dynamic equilibrium/motion in terms of the Cauchy stress tensor, the energy balance equation, which is reduced to the heat conduction equation, mechanical and thermal boundary conditions, as well as initial conditions that are listed below

$$\varepsilon_z = \frac{\partial u_z}{\partial z}, \quad \varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\phi = \frac{u_r}{r}, \quad \varepsilon_{rz} = \frac{1}{2} \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right), \quad (1)$$

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} (\sigma_r - \sigma_\phi) + \frac{\partial \sigma_{rz}}{\partial z} = \rho \frac{\partial^2 u_r}{\partial t^2}, \quad \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \sigma_{rz} + \frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2}, \quad (2)$$

$$c_v \dot{\theta} + 3\alpha \theta K_V (\dot{\varepsilon}_{kk} - 3\alpha \dot{\theta}) - D' - k \Delta \theta = r_s, \quad (3)$$

$$\sigma_{ij} n_j = 0 \quad \text{on } S, \quad (4)$$

$$-k \frac{\partial \theta}{\partial z} = q_s \quad \text{on } S_p \quad \text{i} \quad 0 \leq t \leq t_p; \quad \frac{\partial \theta}{\partial n} = 0 \quad \text{on } S - S_p, \quad (5)$$

$$u_r = u_z = \dot{u}_r = \dot{u}_z = 0, \quad \theta = \theta_0 \quad \text{at } t = 0 \quad (6)$$

where  $u_r$ ,  $u_z$  are the displacements along the corresponding axes;  $\varepsilon_r$ ,  $\varepsilon_z$ ,  $\varepsilon_{rz}$ , and  $\varepsilon_\phi$  are the components of the strain tensor;  $\sigma_r$ ,  $\sigma_z$ ,  $\sigma_{rz}$ , and  $\sigma_\phi$  are the components of the stress tensor;  $\theta$  is temperature;  $\alpha$ ,  $c_v$ , and  $k$  are coefficients of linear thermal expansion, heat capacity at constant volume, and thermal conductivity, respectively;  $K_V$  is volume modulus of the material;  $r_s$  is power of the given internal heat sources;  $D'$  is mechanical energy dissipation rate;  $\theta_0$  is the initial temperature;  $n_j$ 's are the component of the outward unit normal vector to the corresponding boundary surface;  $S_p$  is part of the surface ( $r \leq r_p$ ,  $z = 0$ ) subjected to the thermal pulse; and  $\dot{\varepsilon}_{kk} = \dot{\varepsilon}_r + \dot{\varepsilon}_z + \dot{\varepsilon}_\phi$  is the rate of the dilatation.

To describe the physically nonlinear thermo-viscoplastic behavior of the disk material, a generalized model developed in (Zhuk et al., 2001; Zhuk, Senchenkov, & Boichuk, 2009) is used. It is consistent with the thermodynamics of irreversible processes and consists of a representation of the total strain as a sum of elastic, inelastic, and thermal components,

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p + \varepsilon_{ij}^\theta, \quad ij \leftrightarrow r, z, rz, \phi, \quad (7)$$

where thermal deformation is represented as:

$$\varepsilon_{ij}^\theta = \delta_{ij} \int_{\theta_0}^{\theta} \alpha(\theta') d\theta'; \quad (8)$$

- Hooke's law written for the deviatoric and volumetric parts of the stress tensor

$$s_{ij} = 2G(e_{ij} - \varepsilon_{ij}^p), \quad \sigma_{kk} = 3K_V(\varepsilon_{kk} - \varepsilon_{kk}^p), \quad (9)$$

where  $s_{ij}$  and  $e_{ij}$  are the deviators of the stress and strain tensors, respectively;  $G$  is the shear modulus; the summation over the repeating indices is assumed;

- the flow rule accompanied by the condition of plastic incompressibility

$$\dot{\varepsilon}_{ij}^p = \lambda s_{ij}, \quad \dot{\varepsilon}_{kk}^p = 0; \quad (10)$$

- kinetic equation

$$D_2^p = D_0^2 \exp \left[ - \left( \frac{Z^2}{3J_2} \right)^n \right], \quad Z = K + D, \quad J_2 = s_{ij}s_{ij}/2, \quad D_2^p = \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p / 2, \quad \lambda^2 = D_2^p / J_2 \quad (11)$$

where  $Z = K + D$ ,  $J_2 = s_{ij}s_{ij}/2$ ,  $D_2^p = \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p / 2$ ,  $\lambda^2 = D_2^p / J_2$ ;

- and the evolution equations for the internal variables of isotropic and kinematic hardening

$$\dot{K} = m_1(K_1 - K)\dot{W}_p, \quad K(0) = K_0; \quad \dot{\beta}_{ij} = m_2(D_1 u_{ij} - \beta_{ij})\dot{W}_p, \quad \beta_{ij}(0) = 0; \quad D = \beta_{ij} u_{ij}, \quad u_{ij} = \sigma_{ij} / (\sigma_{mn} \sigma_{mn})^{1/2}, \quad \dot{W}_p = \sigma_{ij} \dot{\varepsilon}_{ij}^p. \quad (12)$$

The values  $D_0$ ,  $D_1$ ,  $K_0$ ,  $K_1$ ,  $m_1$ ,  $m_2$  and  $n$  are constants of the constitutive model. The expression for the dissipative function  $D'$  used was derived in (Zhuk, Senchenkov, & Boichuk, 2009) in the frame of thermodynamics of irreversible processes

$$D' = \sigma_{ij} \dot{\varepsilon}_{ij}^p - K \dot{\delta} - \beta_{ij} \dot{\alpha}_{ij} = \dot{W}_p - \dot{W}_{sk} - \dot{W}_{sp}. \quad (13)$$

The values of  $\delta$  and  $\alpha_{ij}$  are some internal variables conjugate to the thermodynamic forces  $K$  and  $\beta_{ij}$  (parameters of isotropic and kinematic hardening, respectively).

We assume that a laser pulse irradiates the center of the surface  $z = 0$  over a circular spot of radius  $r_p$ . This single thermal pulse is modeled by a specified heat flux  $q_s$  of duration  $t_p$  across the disk boundary. The time-space law describing the pulse is given as follows:

$$q_s = \begin{cases} q_0 \cos \frac{\pi r}{2r_p} \sin \frac{\pi t}{t_p}; & r \leq r_p, \quad t \leq t_p, \\ 0; & r > r_p, \quad t > t_p. \end{cases} \quad (14)$$

A thermal insulation condition is assumed on the rest of the disk surface. After the pulse has ceased, the irradiated part of the surface is also considered to be thermally insulated. Equations (1)–(13) with initial and boundary conditions (14) constitute the formulation of a coupled problem of thermomechanics of physically nonlinear solids under thermal loading.

**2. Material properties and solution technique**

Steel 35CrMo was chosen as the disk material. The physical and mechanical properties for this material and their dependencies on temperature were taken from (Senchenkov, & Tabieva, 1996; Zhuk, Senchenkov, & Boichuk, 2009).

The problem statement, as formulated by equations (1)–(14), represents an essentially nonlinear problem which may only be solved numerically. Here, the same approach as developed in (Zhuk, Senchenkov, & Boichuk, 2009; Zhuk et al., 2001) for solving dynamic plane and axisymmetric problems of thermo-viscoplasticity is adopted.

The numerical implementation of this technique is carried out as a double iterative process. The spatial discretization of the problem is carried out by making use of the finite element method (FEM). The calculations were performed for a fine mesh, especially in the irradiated area, to correctly simulate the thermomechanical response caused by large temperature gradients. The application of FEM within the framework of the external iterative process leads to the problem of the dynamics of viscoelastic solids. The second time derivatives in the equations of motion for an arbitrary moment of time are represented by Newmark’s formulas. At each time increment, the problem is solved by the iteration method, while each iteration consists of solving a linearized problem of motion.

**3. Calculation results and analysis**

Calculations were performed for a disk of radius  $R=5 \cdot 10^{-3}$  m and thickness  $h=10^{-4}$  m, which are typical for micromechanical systems. The radius of the irradiation zone was  $r_p=1.5 \cdot 10^{-3}$  m, the pulse durations  $t_p$  have been varied from  $10^{-8}$  s to  $10^{-7}$  s, and the heat flux parameter  $q_0$  was also varied from  $6 \cdot 10^7$  to  $2 \cdot 10^8$  kW/m<sup>2</sup>. The initial temperature  $\theta_0$  of the disk was assumed to be the room temperature of 20 °C.

The mechanism of laser thermoforming of thin-walled parts is to create laser-induced thermal residual strain without the use of tools or external forces. When a disk is irradiated, a region of high temperature gradients occurs on the surface and in the near-surface zone, which changes its configuration over time. After the end of the laser pulse, in the absence of heat exchange with the environment, the temperature gradually equalizes over the disk volume. As a result of the fast expansion of the material over the irradiated zone, significant compressive stresses occur, which form a quasi-static component of the stress field. After a significant time has elapsed compared to the pulse duration (for the considered configuration and conditions, calculations showed that the stabilization time can be chosen as  $t=0.4 \cdot 10^{-4}$  s), the compressive stresses disappear, and a region of quasi-static residual tensile stresses is formed in the center of the disk, which can subsequently compromise the strength and durability of the structure.

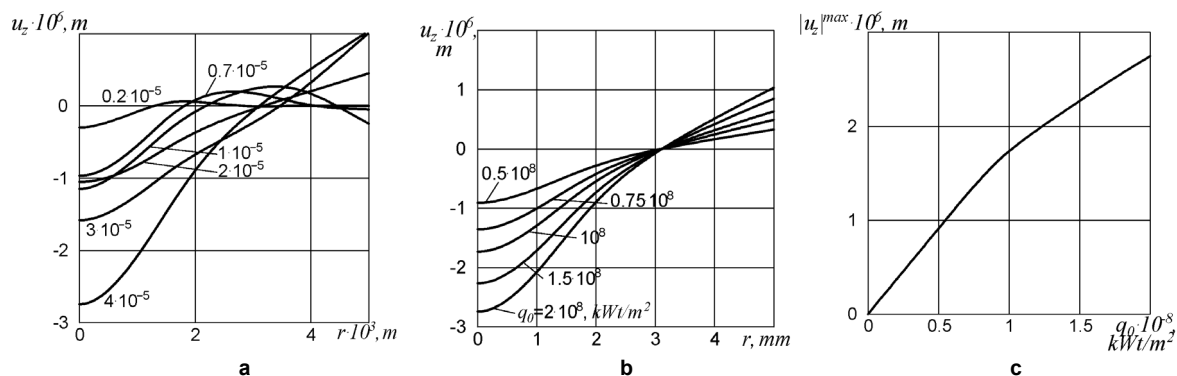


Fig. 1. Results of disk forming simulation

The developed problem statement, the method of its solution, and the obtained modeling results were used to study the applicability of the LTF technology to treat the structural elements by short thermal pulses for the purpose of stamping or forming the desired profile (configuration). The presence of significant inelastic strains induced by thermal processing, which are asymmetric relative to the median surface of the disk, leads to a modification of the surface curvature of metal plates, shells, and structural elements without the use of an external force load. The results of modeling this process are illustrated in Fig. 1.

Figure 1 a shows the evolution of displacements of the surface points of a disk irradiated by a laser pulse for the case of a free edge. The time moments for which the surface profiles were drawn are marked with numbers. The maximum deflections are achieved in the center of the disk. Fig. 1 b shows the residual displacements of the disk surface under the influence of pulses of different power. For higher laser pulse powers, which are simulated by an increase of the heat flux parameter  $q_0$ , an increase in surface deflections is observed both in the center and at the edge. The dependence of the maximum displacement magnitude on the heat flux parameter is shown in Fig. 1 c.

The maximum deflection is always observed in the center, with the bend directed toward the irradiated side. The edge of the disk moves in the opposite direction. An additional factor that affects the quantitative characteristics of the thermoforming is the conditions applied at the disk edge. In the case of a fixed contour, the residual deflections for the given problem parameters are predicted to be 7.5 % smaller than in the case of a free contour.

**Discussion and conclusions**

The model developed for studying the process of laser thermoforming of a thin-walled steel structural element allows for simulating the complex coupled thermomechanical behavior of the material under the assumptions made. A dynamic formulation of the problem was applied with making use of a generalized model of physically nonlinear response of materials in a wide temperature

range consistent with the thermodynamics of irreversible processes. The obtained quantitative and qualitative results of modeling the laser thermoforming of metal sheet parts show that the creation of laser-induced thermal residual inelastic strains is the main mechanism of shape change. The evolution of inelastic deformation fields in a disk, the dependence of residual inelastic deformation distributions along the radius and thickness of the disk, the effect of laser thermal irradiation level on the magnitudes of induced deflections, and the influence of the conditions of disk contour fixation on the efficiency of thermoforming were studied. It was found that a greater variation of the disk profile can be achieved when the disk edge is free.

**Authors' contribution:** Yaroslav Zhuk – conceptualization; methodology, software; Arash Soleiman Fallah – analysis of sources, empirical data collection and validation, empirical research, preparation of literature review, and theoretical foundations of research.

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Ярослав ЖУК, д-р фіз.-мат. наук, проф., чл.-кор. НАН України  
 ORCID ID: 0000-0002-2726-8395  
 e-mail: yaroslavzhuk@knu.ua  
 Київський національний університет імені Тараса Шевченка, Київ, Україна

Араш Солейман ФАЛЛАХ, д-р філософії, проф.  
 ORCID ID: 0000-0002-0382-633X  
 e-mail: arashsol@oslomet.no  
 Університет Осло Метрополітен, Осло, Норвегія

### ТЕХНОЛОГІЯ ЛАЗЕРНОГО ТЕРМОФОРМУВАННЯ ТОНКОСТІННИХ МЕТАЛЕВИХ КОНСТРУКЦІЙНИХ ЕЛЕМЕНТІВ

Запропоновано чисельну модель для моделювання процесу лазерного термоформування (LTF). Вона розроблена на основі термодинамічно узгодженої теорії сполученої термовіскопластичності і підходить для моделювання LTF для тонкостінних металевих конструкційних елементів. У межах цієї моделі постановка задачі складається з рівняння Коші, рівнянь руху та рівняння енергетичного балансу, що зведено до рівняння теплопровідності, а також механічних і теплових граничних умов та початкових умов. Для опису поведінки матеріалу використано узагальнену модель фізично нелінійної температурно-залежної термовіскопластичності. Просторову дискретизацію осесиметричної задачі лазерного імпульсного навантаження диска виконано методом скінченних елементів. Змодельовано нестационарний процес LTF деформованої конфігурації диска. Кінцевий профіль диска отримано внаслідок термічно індукованого стану залишкових напружень і деформацій, викликаних швидким нагріванням і подальшим поступовим охолодженням матеріалу в ділянці, опроміненій лазером.

**Ключові слова:** лазерне термоформування, тонкостінні металеві елементи, теорія термов'язкопластичності, термомеханічна модель, метод скінченних елементів, залишковий напружено-деформований стан.

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